MODELLING OF HEAT EXCHANGE IN THE SYSTEM OF ATMOSPHERE – WATER – SEDIMENTS

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Aims of study

- For shallow areas of water bodies the heat flux through bottom can significantly contribute to the heat balance of flows in diurnal and seasonal time scale.
- Aim of this work was
  - to develop the mathematical model of heat exchange in system air-water-bottom sediments
  - to couple it with existing 3D circulation model for describing temperature regime of shallow water bodies.
- The periodical analytical solution of linearized system of equations was found.

3D hydrostatic free-surface circulation model THREETOX

3D numerical model for modelling different water bodies [1] includes

- Reynolds averaged equations of continuity and horizontal momentum in hydrostatic approximation
- Equations for heat and salt transport
- State equation for the water
- k-ε turbulence model

[1] Maderich V, Heling R, Bezhenar R, Brovchenko I, Jenner H, Koshebutsky V, Kuschan A, Terletska K. Development and application of 3D numerical model THREETOX to the prediction of cooling water transport and mixing in the inland and coastal waters // Hydrological Processes. – 2008. – **22**. – P. 265-277

Heat transport in 3D model

Heat transport equation

$$\frac{\partial T}{\partial t} + \vec{U} \cdot \vec{\nabla} T = \frac{\partial}{\partial z}\left(\nu_t \frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial x}\left(K_T \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_T \frac{\partial T}{\partial y}\right) + \frac{1}{\rho_0 C_{pw}} \frac{\partial I}{\partial z},$$

Free-surface boundary conditions:

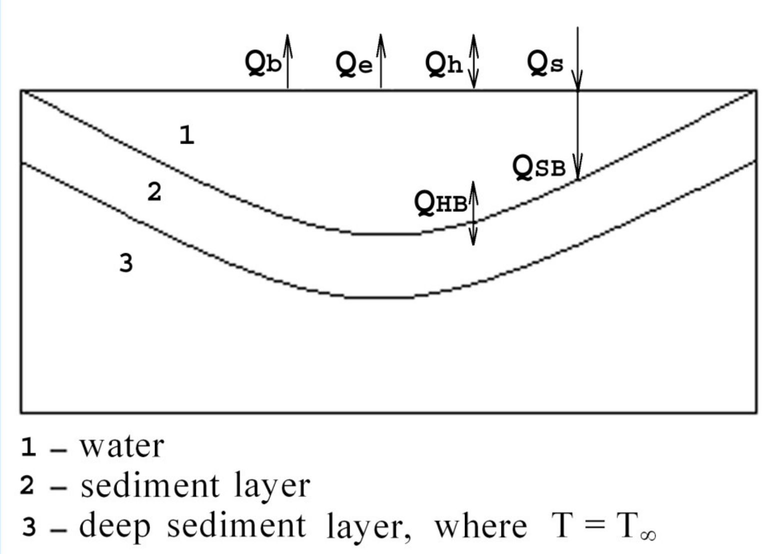
$$z = 0 : \rho_0 C_{pw} \nu_t \frac{\partial T}{\partial z} = Q$$

Bottom boundary conditions – nonzero heat flux:

$$z = -H \quad \nu_t \frac{\partial T}{\partial z} = Q_{HB}$$

I(z) is short wave radiation

Scheme of heat exchange



$$Q = Q_s + Q_b + Q_e + Q_h$$

where Q_b – long wave radiation, Q_e – flux due to evaporation, Q_h – turbulent heat flux between atmosphere and water, Q_s – flux of solar insolation at the surface

$$Q_{WB} = Q_{SB} + Q_{HB}$$

where Q_{SB} – flux of solar insolation near the bottom, Q_{HB} - turbulent heat flux between water and bottom

Conduction of heat in the sediments

Heat conduction equation

$$\frac{\partial T_B}{\partial t} = \frac{\chi_B}{\rho_B c_{pB}} \frac{\partial^2 T_B}{\partial z^2} = \chi_B \frac{\partial^2 T_B}{\partial z^2};$$

Boundary conditions

$$\chi_B \frac{\partial T_B}{\partial z}\bigg|_{z=-H} = Q_{WB}, \quad T_B\big|_{z=-\infty} = T \,.$$

Solar short-wave radiation term

$$Q_{SB} = Q_s\left(a\exp(-H/h_1) + (1-a)\exp(-H/h_2)\right),$$

where h_1 –attenuation parameter due to the red end of light spectrum , h_2 –attenuation parameter of blue-green light

Parameterization of Q_{HB}

The turbulent heat flux can be calculated using the simple bulk formula for mixed convection:

$$Q_{HB} = \rho_0 C_{pw}(\lambda_1 + \lambda_2)(T - T_B(-H))$$

where λ_1 – is the heat transfer coefficient for forced convection

$$\lambda_1 = C_{HB}\sqrt{u^2 + v^2}$$

and λ_2 – is the heat transfer coefficient for free convection

$$\lambda_2 = C_0 \left[\left(T_B(-H) - T \right) \left(g\beta\nu / \mathrm{Pr} \right) \right]^{1/3}$$

1D model of heat exchange in the system air-water-bottom sediments for shallow lake

Horizontally averaged equation of heat transfer in water with $I=0$

$$\frac{\partial T_W}{\partial t} = \frac{\partial}{\partial z} \nu_T \frac{\partial T_W}{\partial z}$$

Linearized boundary conditions are

$$z = 0 : \nu_T \frac{\partial T_W}{\partial z} = -\lambda_W (T_W - T_A)$$

$$z = -H : \nu_T \frac{\partial T_W}{\partial z} = \lambda_B (T_W - T_B)$$

Depth-averaged equation of heat budget

In shallow lake water is well mixed. Suppose that temperature of water layer is uniform through the depth: $T_W \quad \bar{T}$

After integration of one-dimensional heat transport equation through the depth

$$\bar{T} = \frac{1}{H}\int_{-H}^{0} T_W \, dz$$

we obtain heat budget equation for shallow lake

$$H\frac{\partial T_W}{\partial t} = \nu_T \left.\frac{\partial T_W}{\partial z}\right|_{z=0} - \nu_T \left.\frac{\partial T_W}{\partial z}\right|_{z=-H} = -\lambda_W (T_W - T_A) - \lambda_B (T_W - T_B)$$

System of 1D linearized equations for atmosphere-water-sediments heat exchange

Combining equations of heat transfer in water and in the bottom sediments yields

$$\frac{\partial T_B}{\partial t} = \frac{\chi_B}{\rho_B c_{pB}} \frac{\partial^2 T_B}{\partial z^2} = \chi_B \frac{\partial^2 T_B}{\partial z^2};$$

$$z = 0: \chi_B \frac{\partial T_B}{\partial z} = -\lambda_B (T_W - T_B);$$

$$z \,\square \qquad : T_B \qquad T$$

$$H \frac{\partial T_W}{\partial t} = -\lambda_W (T_W - T_A) - \lambda_B (T_W - T_B)$$

Analytical solution for periodical boundary conditions

$$T_A = T \;\; + T_0 \cos \omega t$$

$$T_W = T \;\; + \frac{T_{٠}}{c(١+a)\left(c+ac+\sqrt{٢}\right)+bc\left(\sqrt{٢}a+\sqrt{٢}b+bc\right)+١+b^{٢}}$$

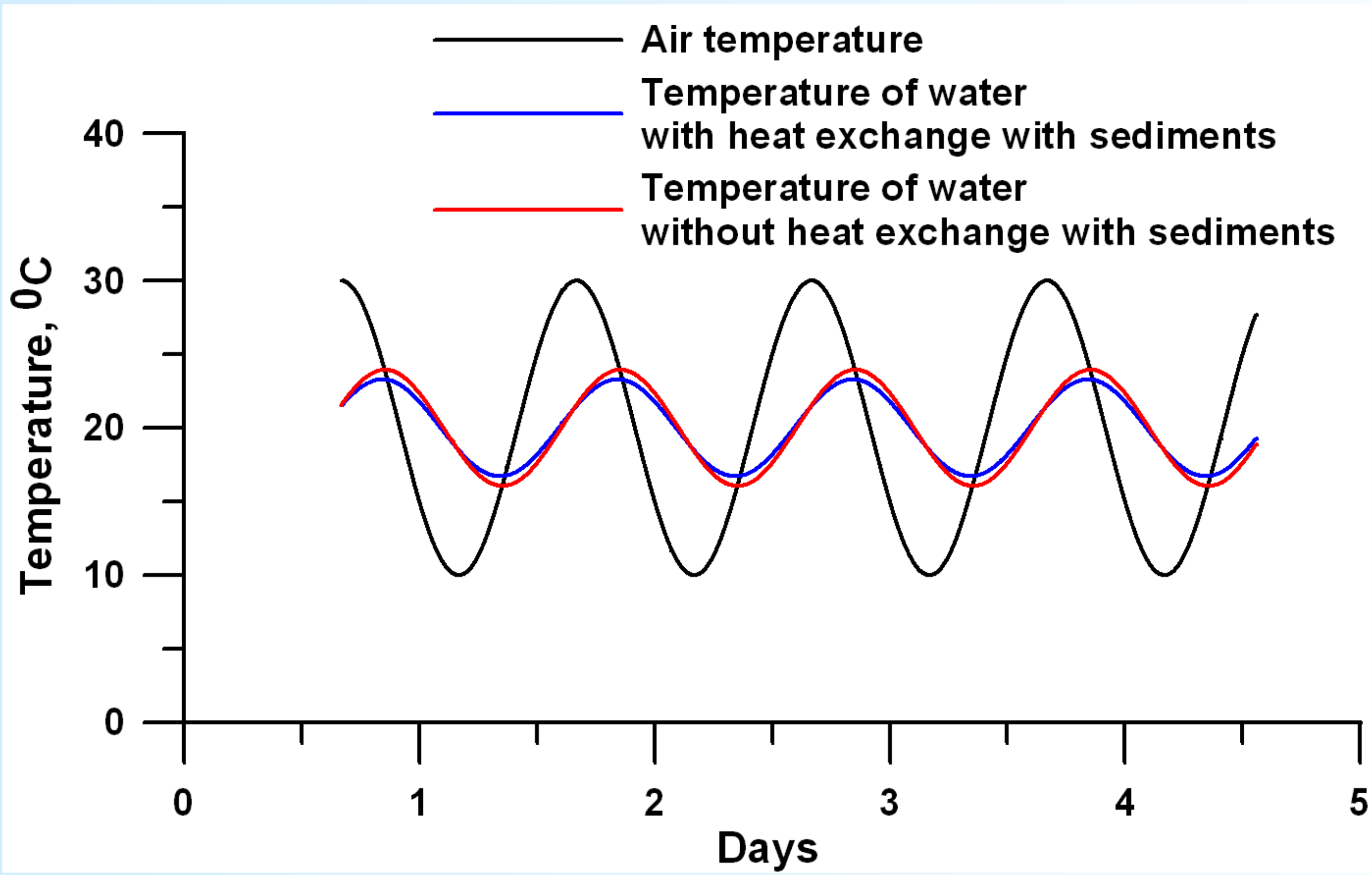
$$\times\left[\left(a+\sqrt{٢}c+\frac{\sqrt{٢}}{٢}ac+c^{٢}+ac^{٢}\right)\cos \omega t+\left(b+bc^{٢}+\sqrt{٢}bc+\frac{\sqrt{٢}}{٢}ac\right)\sin \omega t\right]$$

$$T_B = T \;\; + \frac{T_{٠}e^{-\sqrt{\frac{\omega}{٢\chi_B}}z}}{c(١+a)\left(c+ac+\sqrt{٢}\right)+bc\left(\sqrt{٢}a+\sqrt{٢}b+bc\right)+١+b^{٢}}$$

$$\times\left[\left(a\frac{\sqrt{٢}}{٢}(١+a-b)+١\right)\cos\left(\omega t-\sqrt{\frac{\omega}{٢\chi_B}}z\right)+\left(a\frac{\sqrt{٢}}{٢}(١+a+b)+b\right)\sin\left(\omega t-\sqrt{\frac{\omega}{٢\chi_B}}z\right)\right]$$

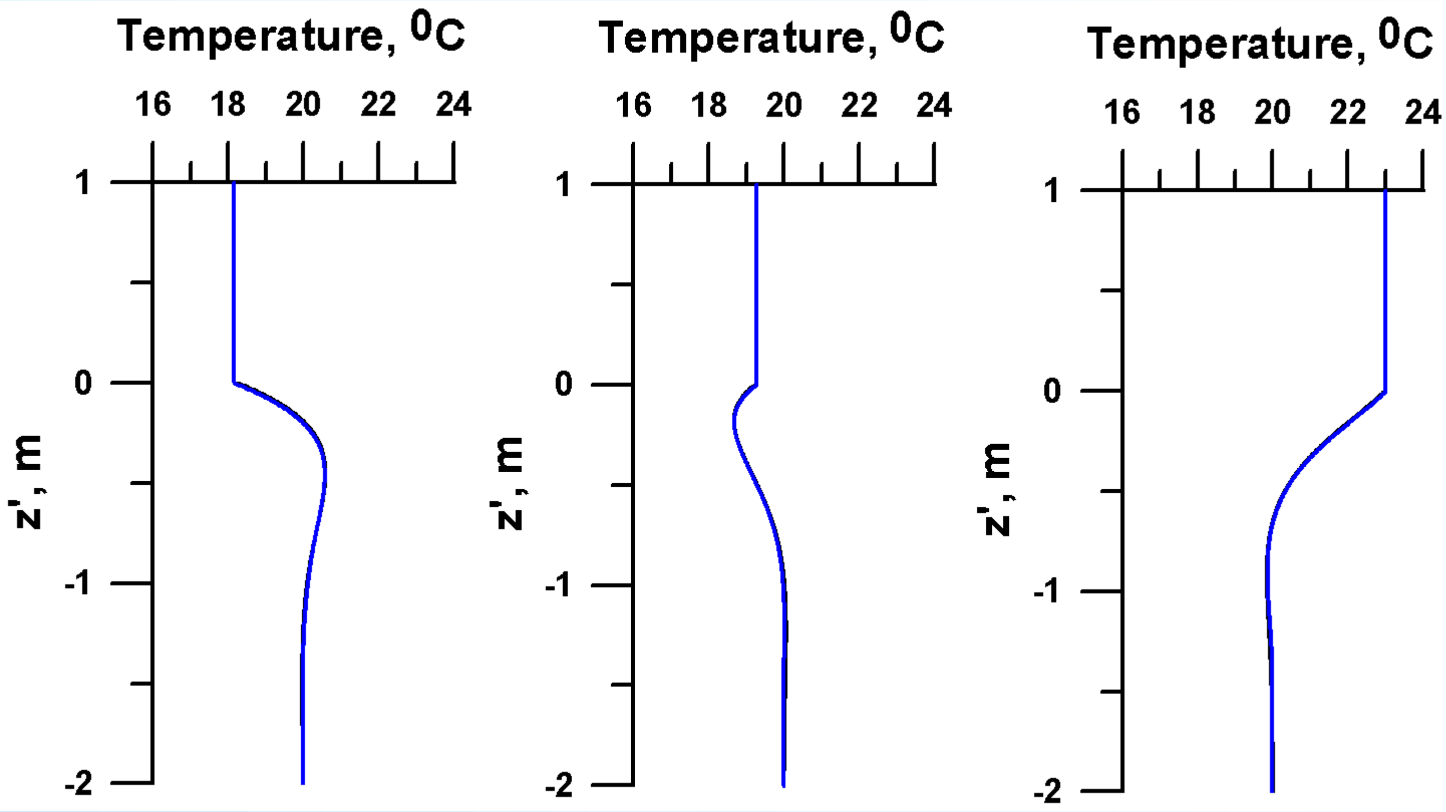
where $\quad a=\frac{\lambda_B}{\lambda_W}, \quad b=\frac{H\omega}{\lambda_W}, \quad c=\frac{\sqrt{\omega\chi_B}}{\lambda_B}$

Influence of heat exchange between water and sediments on temperature of water in diurnal scale in shallow lake



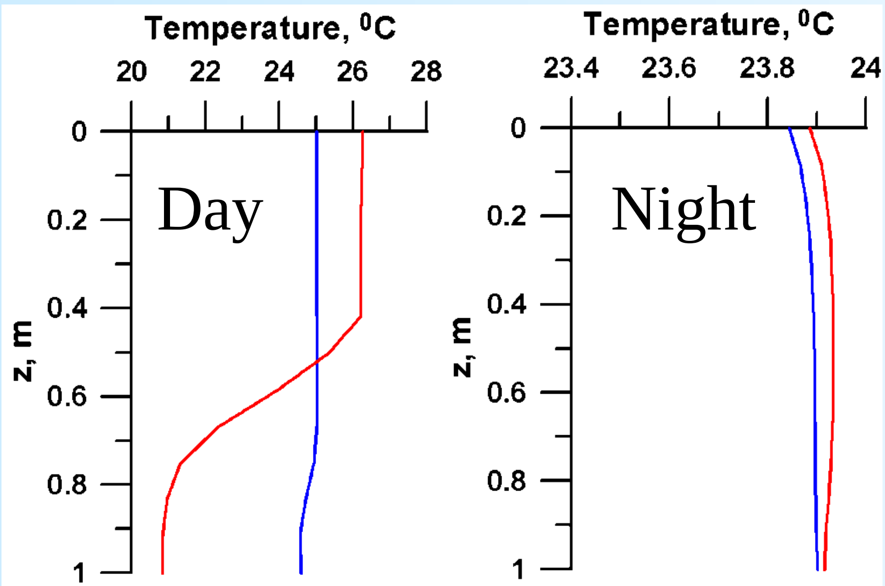
Depth of lake: 1m

Analytical vs. numerical solutions

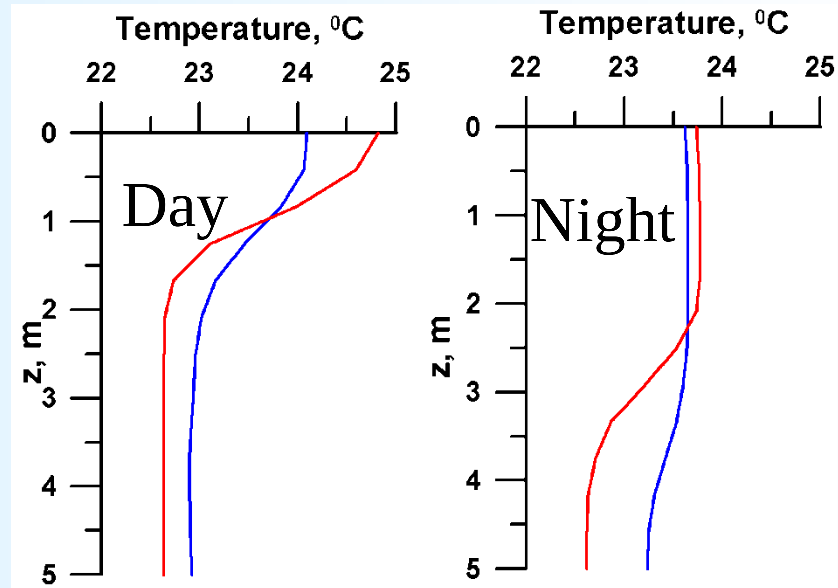


Profiles of temperature in water and sediments in time (first profile – minimum air temperature, last profile – maximum water temperature)

Profiles of temperature in the water in 3D simulations

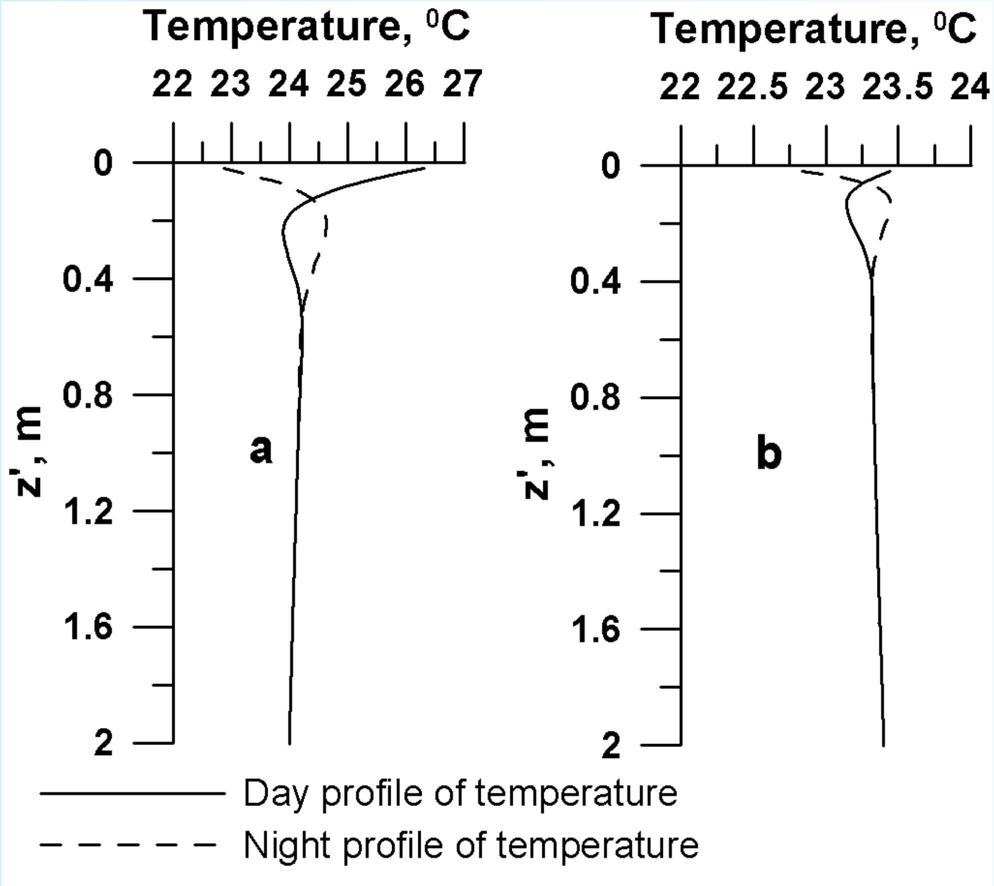


Shallow lake



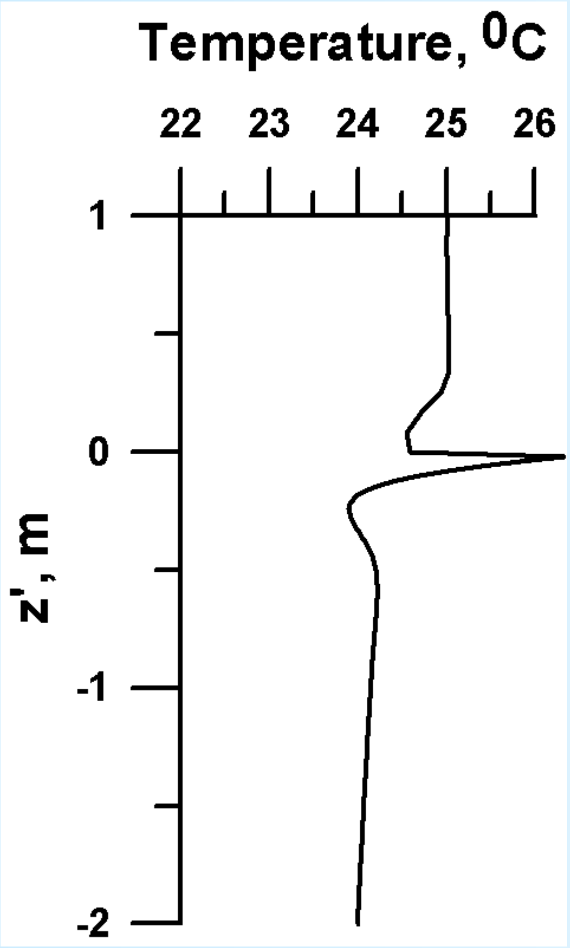
Deep lake

Profiles of the temperature in sediments in 3D simulations for shallow lake (a) and for deep lake (b)



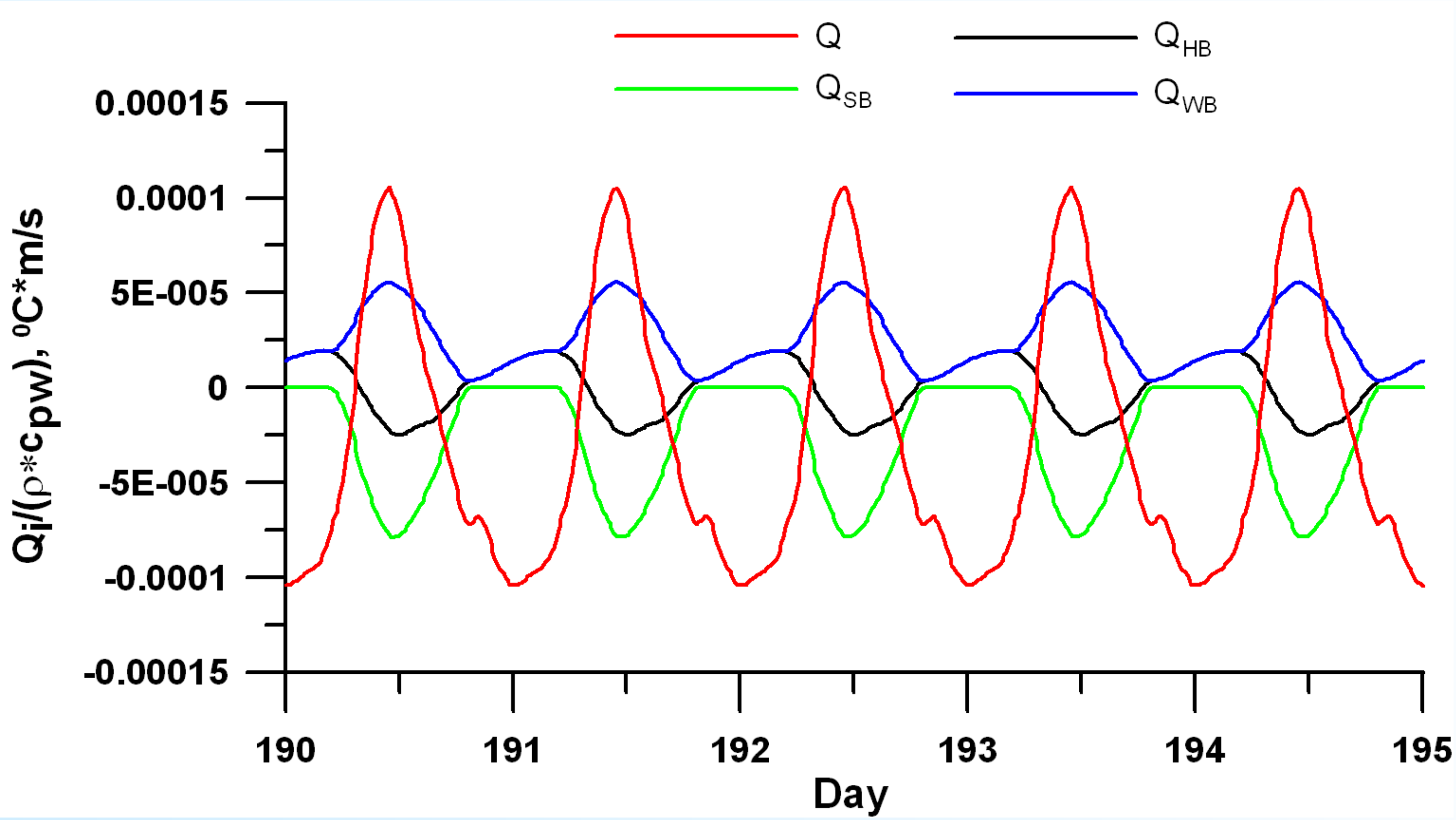
H=1m H=5m

Profile of temperature in shallow lake in 3D simulations



- The temperature of top layer of sediments is higher then near-bottom temperature of the water.
- It could be explained by heating of top layer of sediments due to sun radiation.
- The parameters of attenuation conforms to clear water.
- In the water body with high turbidity or with vegetation the difference will be much less

Heat fluxes in the shallow lake



Q – total heat flux through the water surface, Q_{SB} – flux of insolation at the bottom surface, Q_{HB} – turbulent heat flux between water and bottom, Q_{WB} – total heat flux between water and bottom

Conclusions

- The new model of heat exchange in system air-water-bottom sediments was developed and coupled with 3D model of thermo-hydrodynamics of shallow water bodies.
- The model includes conditions for mixed convection on bottom and 1D heat conduction equation.
- The analytical solutions and results of simulations showed that we deal with shallow lakes, lagoon and tidal flats we should consider heat exchange with sediments.