DYNAMICS of HARMONIE

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P.L. on dynamics in HIRLAM-B

Generalities

- Same code as ECMWF's IFS
  - IFS: global uniform-resolution version
  - ARPEGE: global non-uniform resolution
  - HARMONIE: Limited-area version
    - ALADIN: Hydrostatic version for resolutions coarser than ~10 km
    - ALARO: Non-hydrostatic version for the "grey zone"
    - AROME: Non-hydrostatic version for high resolution (<~2.5 km)
    - ECMWF_PHY: using the physics of ECMWF

Equations

$$\frac{d\mathbf{V}}{dt} + \frac{RT}{p}\nabla_\eta p + \frac{1}{m}\frac{\partial p}{\partial \eta}\nabla_\eta \phi = \varsigma$$

$$\gamma\frac{dw}{dt} + g\left(1 - \frac{1}{m}\frac{\partial p}{\partial \eta}\right) = \gamma\Omega$$

$$\frac{\partial m}{\partial t} + \nabla_\eta(m\mathbf{V}) + \frac{\partial}{\partial \eta}(m\dot{\eta}) = 0$$

$$\frac{dT}{dt} - \frac{RT}{C_p}\frac{1}{p}\frac{dp}{dt} = \frac{Q}{C_p}$$

$$\frac{dp}{dt} + \frac{C_p}{C_v}pD_3 = \frac{Qp}{C_v T}$$

$$\frac{d\phi}{dt} = gw$$

$$\frac{\partial \phi}{\partial \pi} = -m\frac{RT}{p}$$

Pressure departure and Vertical divergence

$$P = \frac{p - \pi}{\pi}$$

$$d = -g\,\frac{\rho}{m}\,\frac{\partial w}{\partial \eta}$$

The corresponding equations are

$$\frac{dP}{dt} = (1 + P)\left(\frac{1}{p}\,\frac{dp}{dt} - \frac{1}{\pi}\,\frac{d\pi}{dt}\right) = -(1 + P)\left(\frac{C_p}{C_v}\,D_3 + \frac{\dot{\pi}}{\pi}\right) + (1 + P)\,\frac{Q}{C_v T}$$

$$\frac{d\mathrm{d}}{dt} = \mathrm{d}\,\frac{1}{p}\,\frac{dp}{dt} - \mathrm{d}\,\frac{1}{T}\,\frac{dT}{dt} - \mathrm{d}\,\frac{1}{m}\,\frac{dm}{dt} - g\,\frac{p}{mRT}\,\frac{d}{dt}\left(\frac{\partial w}{\partial \eta}\right)$$

Vertical coordinate

$$\pi(x, y, \eta, t) = A(\eta) + B(\eta)\pi_s\left(x, y, t\right)$$

$$m = \frac{\partial \pi}{\partial \eta}$$

$$\eta_s = 1 \quad ; \quad \eta_T = 0 \quad \Rightarrow \quad A(0) = A(1) = B(0) = 0\,; \quad B(1) = 1$$

$$\dot{\eta}_s = \dot{\eta}_T = 0$$

Forecast fields in the code

- The “dynamical” variables (Wind, temperature, pressure departure and vertical divergence) are called collectively: GMV
- The variables (Humidity, Ozone, cloud …) which do not enter explicitly the implicit timestepping are called collectively: GFL
- The general evolution equation for GFL is

$$\frac{dM}{dt} = F$$

Where M is the mixing ratio and F the parameterized tendency

Dynamics scheme: semi-Lagrangian semi-implicit

General form of the equations

$$\frac{dX}{dt} + R = 0$$

R includes the non-advective terms of the equation plus the parameterized tendency

Linealization

$$\frac{dX}{dt} + L \cdot X + NL(X) = 0$$

NOTE: semi-analytical being implemented

Dynamics scheme: semi-Lagrangian semi-implicit two-time-level

Discretization

implicit

$$\frac{X_A^+ - X_D^0}{\Delta t} + \underbrace{\frac{1}{2}\left(L \cdot X_A^+ + L \cdot X_D^0\right)}_{\text{implicit}} +$$

$$+ \underbrace{\frac{1}{2}\left(2NL(X)_D^0 - NL(X)_D^- + NL(X)_A^0\right)}_{\text{SETTLS}} = 0$$

SETTLS

ICI (iterated centered implicit) scheme

$$\frac{X_A^+ - X_D^0}{\Delta t} + \frac{1}{2}\left(L \cdot X_A^+ + L \cdot X_D^0\right) +$$

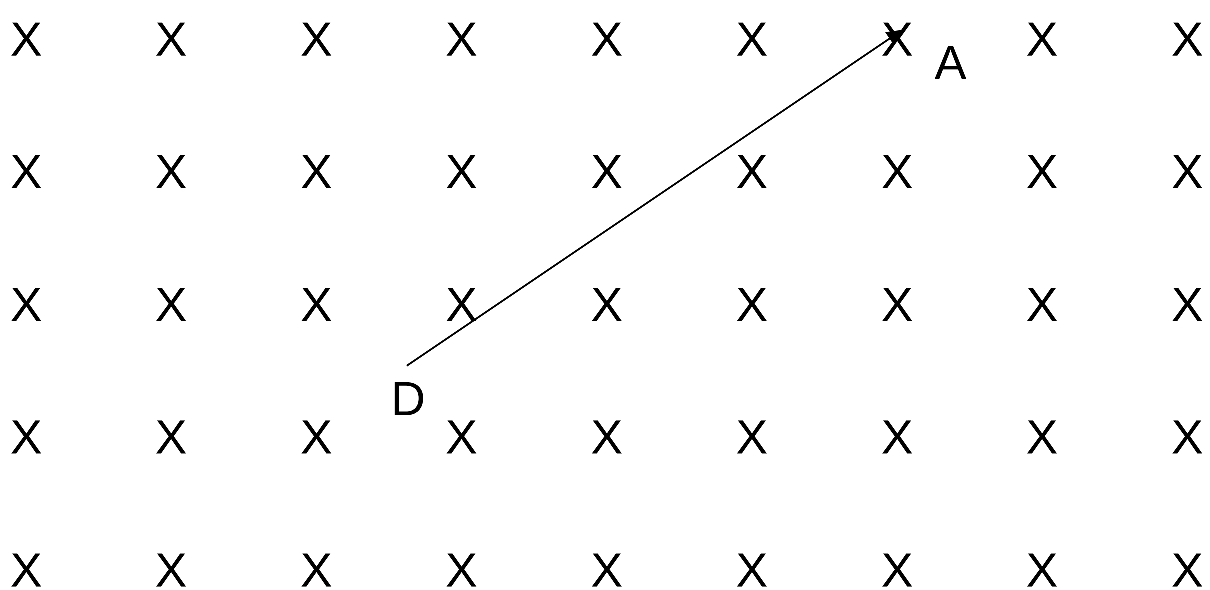
$$+\frac{1}{2}\left(NL(X)_A^{\sim +} + NL(X)_D^0\right) = 0$$

More expensive than pure semi-implicit

But sometimes needed for stability

Semi-Lagrangian trajectories

$$\frac{d\mathbf{r}}{dt} = \vec{\mathbf{V}} \Rightarrow \frac{\mathbf{r}_A^+ - \mathbf{r}_D^0}{\Delta t} = \underbrace{\frac{1}{2}\left(2\vec{\mathbf{V}}_D^0 - \vec{\mathbf{V}}_D^- + \vec{\mathbf{V}}_A^0\right)}_{\text{Average velocity during the time step}}$$



Helmholtz equation

Eliminating from the discretized set of equations
(with some constraints to be fulfilled by the operators)
all the variables except the vertical divergence,
we obtain a Helmholtz equation:

$$\left[1-(\Delta t)^2 c_*^2\left(m_*^2\nabla^2+\frac{\mathbf{L}^*}{rH_*^2}\right)-(\Delta t)^4\frac{N_*^2c_*^2}{r}m_*^2\nabla^2T^*\right]\mathrm{d}=r.h.s.$$

Which can be solved very easily in spectral space
In a projection on vertical eigenvectors

Spectral method

$$X(x, y, \eta, t) \approx \sum_{m}^{M} \sum_{n}^{N} X_m^n(\eta, t) \Phi_m^n(x, y)$$

The basis functions are chosen such that

$$\nabla^2 \Phi_m^n = E_m^n \Phi_m^n$$

They are eigenfunctions of the Laplacian operator

Basis functions

Spherical harmonics for the global version

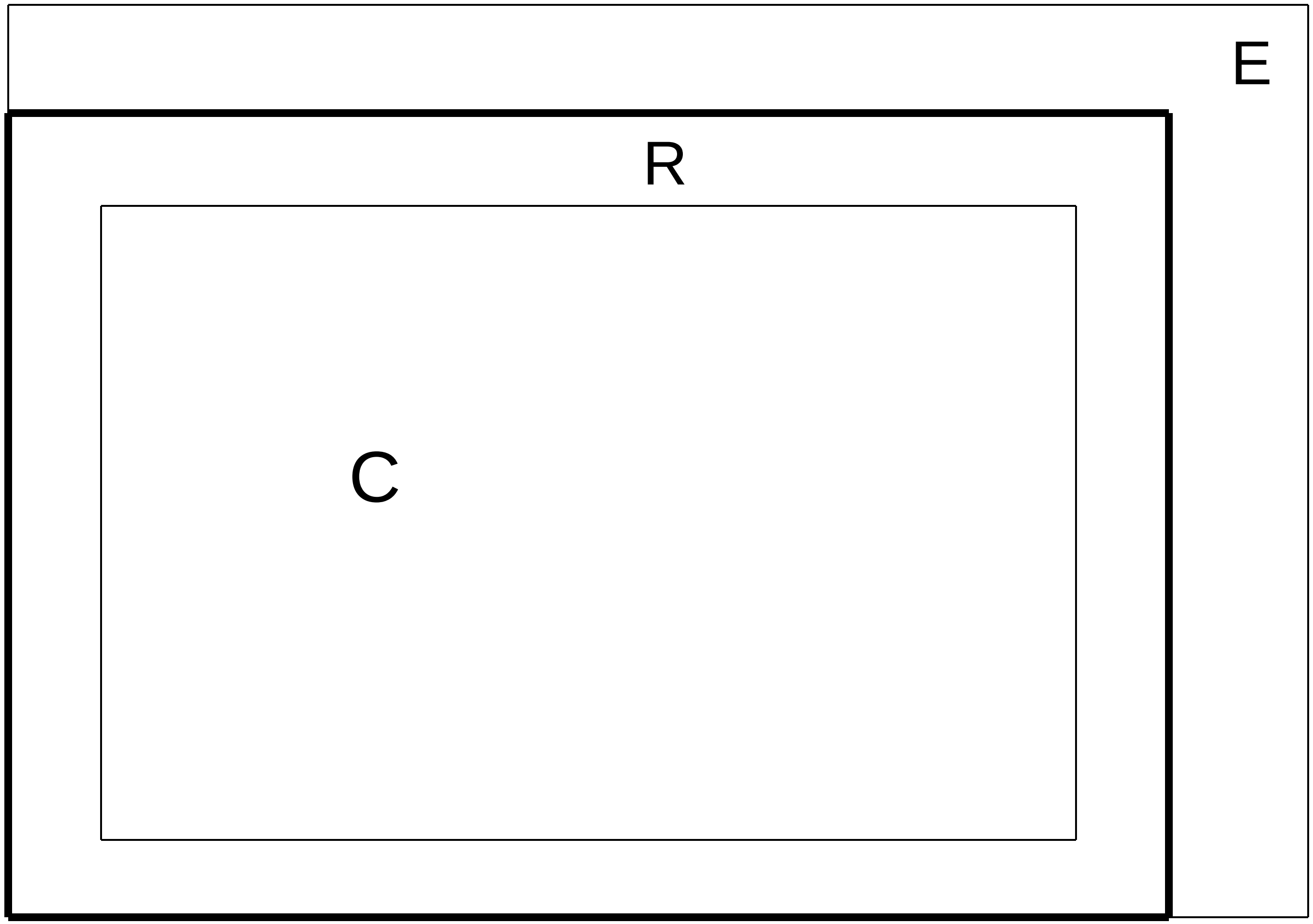
$$\Phi_m^n(\lambda, \mu) \equiv e^{im\lambda} P_m^n(\mu)$$

Bi-Fourier functions for the limited-area version

$$\Phi_m^n(x, y) \equiv e^{imx} e^{iny}$$

Only for bi-periodic fields

Extension zone and relaxation area



Computations on C+R; relaxation in R; biperiodization in E

References

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