New Accurate Methods for Modelling of the Continuity Equation

Ayoe Buus Hansen

Niels Bohr Instituttet

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Motivation

New more accurate methods for modeling of the continuity equation are important in order to achieve better meteorological weather forecasts and better climate models.

Teory 1

Desirable properties

- Accuracy
- Stability
- **Computational Efficiency**
- Transportivity and Locality
- Shape preservation
- **Conservation**
- Consistency
- Compatibility
- Preservation of Constancy
- Preservation of linear correlations between constituents

Teory 2

Equations

We have combined semi-Lagrangian cubic interpolation with cascade interpolation and modified interpolation weights.

$$w_{k-1,j} = \frac{-2\alpha + 3\alpha^2 - \alpha^3}{6} \quad , \quad w_{k,j} = \frac{2 - \alpha - 2\alpha^2 - \alpha^3}{2} \quad ,$$
$$w_{k+1,j} = \frac{2\alpha + \alpha^2 - \alpha^3}{2} \quad , \quad w_{k+2,j} = \frac{-\alpha + \alpha^3}{6} \quad .$$

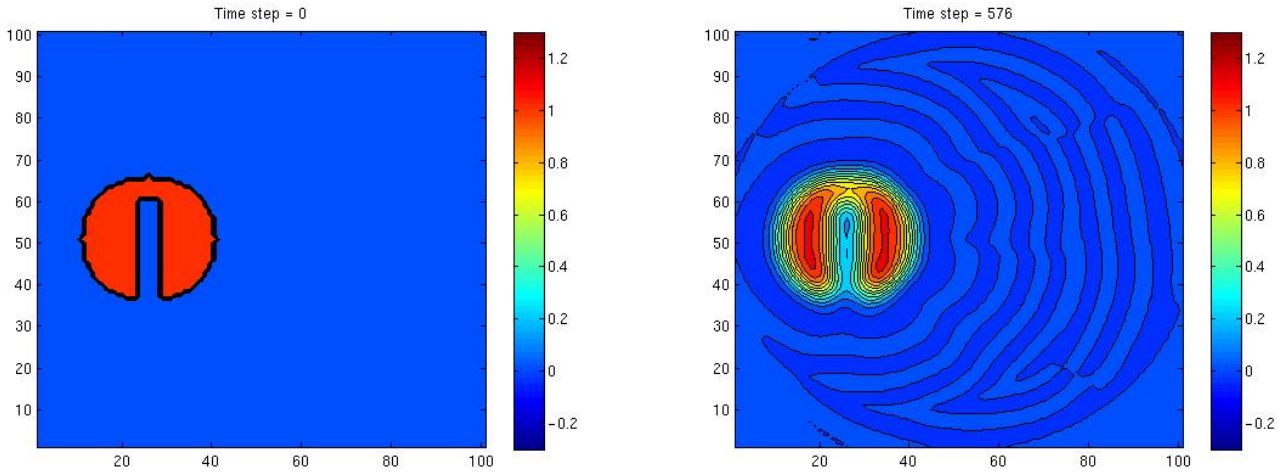
$$\hat{w}_{k,j} = \frac{w_{k,j}(\alpha_k)}{\sum_{m=1}^{K} w_{m,k}(\alpha_m)}$$

$$\begin{aligned} \psi_{i,j}^{n+1x} = \hat{w}_{k-1,i}\psi_{i-k-1}^{n} &+ \hat{w}_{k,i}\psi_{i-k}^{n} \\ &+ \hat{w}_{k+1,i}\psi_{i-k+1}^{n} + \hat{w}_{k+2,i}\psi_{i-k+2}^{n} \quad . \\ \psi_{i,j}^{n+1} = \hat{w}_{l-1,j}\psi_{j-l-1}^{n+1x} &+ \hat{w}_{l,j}\psi_{j-l}^{n+1x} \\ &+ \hat{w}_{l+1,j}\psi_{j-l+1}^{n+1x} + \hat{w}_{l+2,j}\psi_{j-l+2}^{n+1x} \quad . \end{aligned}$$

Results 1

Slotted Cylinder

A slotted Cylinder is rotated around the center of the domain six times.



It is seen that the shape of the cylinder is preserved, but the gradients are smoothed.

Results 2

dt

It was investigated whether the precision of the method was dependent on the number of time steps or the size of the time steps.

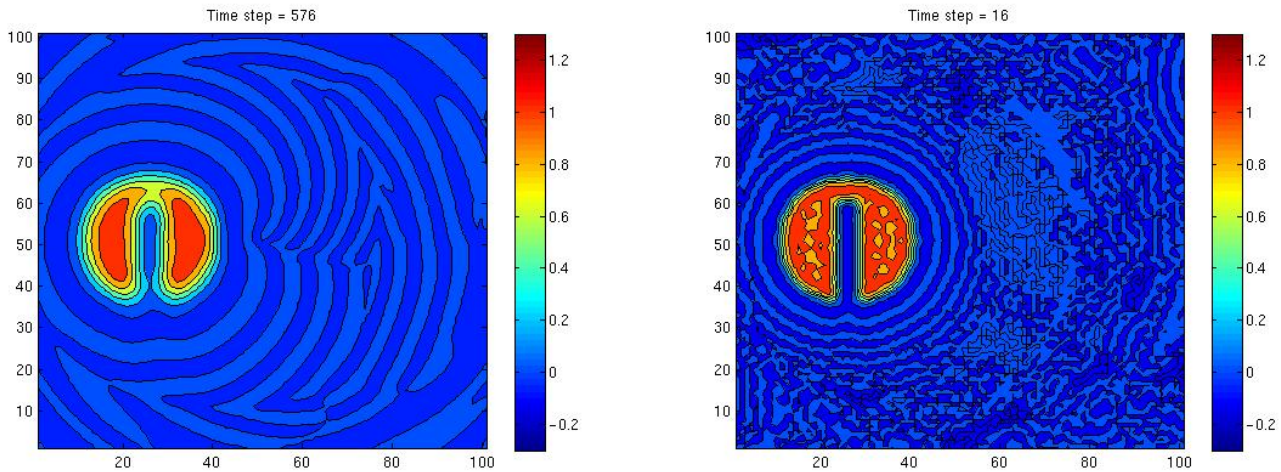


Figure: Left: dt = 300 s. Right: dt = 10800 s.

We see that the accuracy is dependent on the number of time steps, not the size.

Conclusion

Future Aspects

The method can be improved by a filter which transfers mass in case of unphysical negative values.

Desirable properties

Some fulfilled Desirable properties:

- Stability:
  The method is dependent on number of time steps, not the size of the time steps.
- **Computational Efficiency:**
  The modified interpolation weights can be used for any number of different tracers.
- **Conservation:**
  From comparison to a method without modified interpolation weights it is seen that these impose mass conservation.