An improved vertical remapping scheme

Brian Sørensen, Eigil Kaas, Ulrik Korsholm

University of Copenhagen

sorensen@gfy.ku.dk

Abstract

A new vertical remapping scheme is proposed. The scheme is being implemented into a Locally Mass Conserving Semi-Lagrangian Scheme (LMCSL), developed by Kaas (2008). In an ordinary Semi-Lagrangian scheme, the vertical levels are remapped to Eulerian model levels after each time step. This introduces an undesirable tendency to smooth sharp gradients in the vertical levels. This can be reduced by keeping the Lagrangian levels, and only interpolate the tendencies between the Lagrangian levels and Eulerian levels. Because tendencies are small compared to the absolute values, their gradients will be significantly smaller and therefore they can be interpolated with less smoothing. At each time step the Lagrangian levels will be used as model levels for the following time step. The Eulerian model levels will be kept at each step as well. This has to be done in order to calculate the physics in the model, and also in order to have some fixed levels in the semi-implicit calculations. After several time steps (dynamically calculated or a fixed number), the Lagrangian levels will be interpolated to Eulerian model levels to insure that they do not change too much, and the process will start again.

1. Introduction

Since the implementation of the new vertical remapping scheme into the Locally Mass Conserving HIRLAM model is still ongoing, the model described on this poster is a simplified version which will help in the analysis and testing of the new scheme.
The model is a simple 2D hydrostatic model (1D horizontal and 1D in the vertical). The main focus, which is the vertical remapping, will be near identical to the final implementation of the scheme.

2. Theory and model description

The basic idea behind the 2D model is a Semi-Lagrangian construction. In the Semi-Lagrangian model, the grid is a regular Eulerian grid and at each timestep the arrival cell is assumed and the trajectories are calculated back in time to obtain the departure point of the arrival cell. The value at the departure point is then calculated by cubic interpolation in the horizontal as in the LMCSL scheme.

Cubic interpolation

$$\phi(\tilde{x}_j^n, t^n) = -\frac{\alpha(1-\alpha^2)}{6}\phi_{j-p-2}^n + \frac{\alpha(1+\alpha)(2-\alpha}{2}\phi_{j-p-1}^n + \frac{(1-\alpha^2)(2-\alpha)}{2}\phi_{j-p}^n - \frac{\alpha(1-\alpha)(2-\alpha)}{6}\phi_{j-p+1}^n$$

The Lagrangian vertical levels is obtained directly by adding the mass in each arrival column from the top (Figure 1). The surface pressure is then given by the weight of the air in the arrival column. In this way it is also very easy to obtain the vertical velocity w, simply by taking the difference in pressure levels.
The trajectories are backwards in time in the horizontal and forwards in time in the vertical.

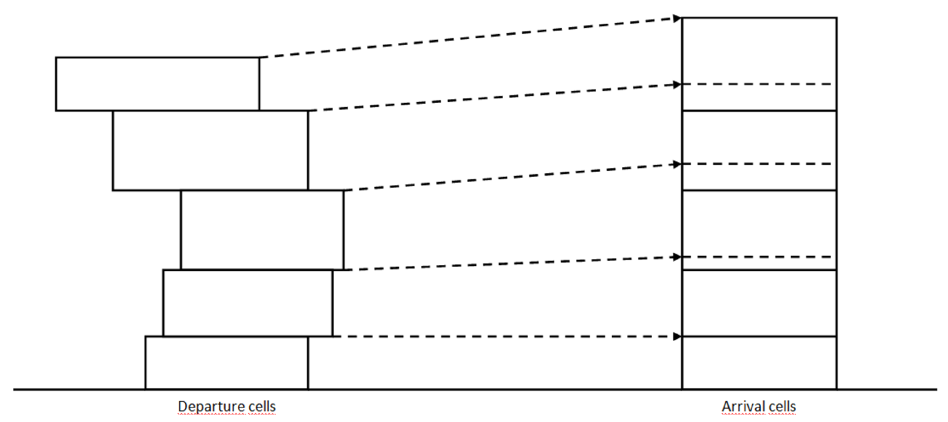

Figure 1. Departure and arrival cells.

The vertical levels are σ-levels (hybrid pressure-levels).

Hybrid pressure levels

$$P_{k+\frac{1}{2}} = A_{k+\frac{1}{2}} + B_{k+\frac{1}{2}}P_s$$

$A_{k+\frac{1}{2}}$ and $B_{k+\frac{1}{2}}$ are predefined constants and P_s is the surface pressure.

To preserve gradients the tendencies is calculated at the model levels, and is then remapped to the Lagrangian levels, determined by the advection. This remapping can be done in multiple ways, since the tendencies are quite small even linear interpolation can be considered, but higher order methods such as Piecewise Parabolic Methods (PPM) or splines will be tried to determine the optimal method.

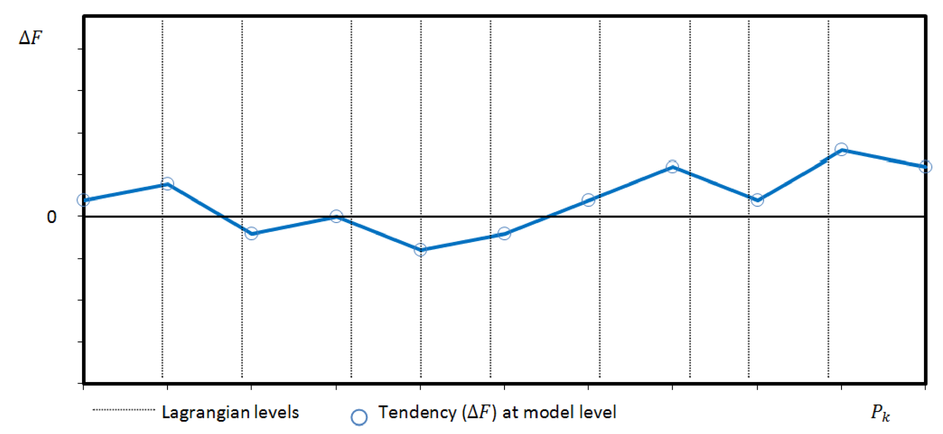

Figure 2. Tendencies at model and Lagrangian levels.

At each new timestep the departure levels will be Lagrangian, but to maintain stability, the Lagrangian levels will only be continued for a limited number of timesteps. This number can be either fixed og dynamically determined. Both methods has advantages and disadvantages.

3. Results

The results from the 2D hydrostatic model is still in process, and can unfortunately not be presented at this time.

4. Discussion

The main reason to develop this 2D hydrostatic model, is to test and analyze the different options and possibilites under more controllable conditions. The choice of remapping method must be tested, to insure balance between accuracy and efficiency. The other very important option is the number of timesteps to maintain the Lagrangian levels. Different possibilities must also be considered, it can be fixed or maybe even dynamicly calculated. If fixed the number may be limited by stability demands. If dynamic, results could be improved, but with the cost of analyzing each column at each timestep.

5. Conclusion

The theoretical analysis shows that improvements could be possible, but it is yet to be tested in the 2D Semi-Lagrangian Hydrostatic model.
The full implementation in HIRLAM, will be the real test, and will be done later this year. At this point it is too early tell the precise effect, but theoretically it should never be anything but an improvement. In computational effeciency it will be a little more inefficient due to the additional remapping of tendencies from model levels to lagrangian levels, and it will also need a small amount of additional memory. This should be compared with the additional levels, that the normal semi-lagrangian model would need to conserve the same vertical gradients, in which case it is possibly more memory efficient.

6. References

Kaas, E., 2008. An accurate and efficient transport scheme. part i: A new locally mass-conserving semi-Lagrangian solution to the continuity equation.
Lauritzen, P. H., Lindberg, K., Kaas, K.,Machenhauer, B., 2007. A mass-conservative version of the semi-implicit semi-Lagrangian HIRLAM
Durran, D. R., 1999. Numerical Methods for Wave Equations in Geophysical Fluid Dynamics.

YSSS08, NetFAM Summer School and Workshop, Zelenogorsk, Russia, 7-15 July 2008