Prediction of clouds and precipitation in atmospheric models

Cumulus congestus. Sønderjylland, 2002. Foto: Flemming Byg.

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outline

- Basic definitions and characteristics of clouds and precipitation
- Clouds in different types of models
- Known challenges and deficiences related to clouds and precipitation in NWP-models
- Cloud parameterizations in atmospheric models
- Schemes for convection, clouds and precipitation in HIRLAM (basic characteristics of Kain Fritsch & Rasch-Kristjansson and STRACO schemes)
- International research (the GCSS collaboration)
- Example of a case study with HIRLAM column model (ASTEX stratocumulus case) as studied in GCSS
- Conclusions and outlook
- Appendix: example of convective cloud cover derivation

Definition of atmospheric cloud and precipitation

1) **CLOUD**

→ A cloud is a volume of atmospheric air containing a certain amount of condensate as water or ice

alternatively

→ an atmospheric air mass with visible water or ice particles

2) **PRECIPITATION**

→ Precipitation represents a detectable flux of falling water species (rain drops, snow particles, hail, graupel, sleet) formed by microphysical processes, e.g. vapor diffusion, collission and/or coalescense of cloud drops, freezing and melting.

WHY ARE CLOUDS IMPORTANT IN METEOROLOGICAL MODELS ?

There is a mutual interaction between clouds and all the most important meteorological processes:

- Dynamics
- Turbulence
- Convection
- Radiation
- Microphysics (condensation and precipitation)
- Chemistry including aerosol processes

Absolute amount of water substance

Amount of condensed matter $q_c \in [10^{-5}, 10^{-3}] \mathrm{Kg} \cdot \mathrm{Kg}^{-1}$

Droplet concentration 100 – 1000 drops cm^{-3}

Droplet size (typical) $2\mu\mathrm{m} - 20\mu\mathrm{m}$

Cloud mass (typical) 0.1% of total mass of air

Volume of cloud drops (typically) one part out of 10^6

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Saturation inside clouds

Inside a cloud the specific humidity is as a first approximation saturated with respect to bulk water or ice:

Clausius Clapeyron equation:

$$e_s(T) = e_{s0} \cdot exp(\frac{L}{R_v}(T_0^{-1} - T^{-1}))$$

vapor pressure at $0°C$
$e_{s0} = 611\mathrm{Pa}$

water vapor gas constant
$R_v = 461.5\mathrm{J} \cdot \mathrm{Kg}^{-1} \cdot \mathrm{K}$

Latent heat of fusion
$L = 2.50 \cdot 10^6 \mathrm{J} \cdot \mathrm{Kg}^{-1}$

Freezing point
$T_0 = 273.16\mathrm{K}$

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The details of droplet or ice crystal formation very complex:

In the atmosphere there is not ` bulk water´, and water droplets/ice crystals do not automatically form –

A more refined thermodynamics is needed to understand droplet formation. This includes surface tension (σ) and the drop radius of curvature (r)

[for details see Pruppacher and Klett: Microphysics of clouds and precipitation, Chapters 4 -6, Kluwer Academic Publishers, 1997]

The details of droplet or ice crystal formation very complex:

Theory predicts very large supersaturations compared to the bulk formula before droplet formation/ice crystals would occur !

(formula already derived in 1870 by Lord Kelvin)

In reality the presence of aerosols (cloud condensation nuclei (CCN) , with diametres typically < 0.2 μm) having an affinity for water is the reason for generation of droplets at saturation vapor pressures which may be either lower of somewhat higher than the `bulk water pressures´

Saturation pressure over a droplet

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Saturation pressure over a droplet of radius r:

$$e_s^{'}(r) = e_\infty \cdot (1 + \frac{a}{r} - \frac{b}{r^3})$$

e_∞ = bulk saturation

$a = 3.3 \cdot 10^{-5} \cdot \mathrm{T}^{-1}(\mathrm{cm})$

$b = 4.3 \cdot \mathrm{i} \cdot \mathrm{M} \cdot \mathrm{m}_s^{-1}(\mathrm{cm}^3)$

i : dissociation, M : solution mass , m_s : molecular weight of solute

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Scales of occurrence (clouds)

→ horizontal dimension of order 100 m (e.g. for shallow cumulus clouds) to 100 km for large stratiform cloud sheets.

→ Cloud variables, e.g. cloud water inside clouds, often exhibit considrable variability inside clouds down to the smallest scales ~100m.
(See, e.g. for cumulus clouds Rogers and Yau, 1996, - for stratocumulus in the context of ASTEX , Albrecht et al., 1995)

→ Time scales: The shortest scales are associatet with individual shallow cumulus clouds (minutes) to large stratiform cloud sheets (many hours)

Scales of occurrence (precipitation-1)

Many studies, e.g. using data from weather radar and rain gauges have show that precipitation exhibits very significant small structures in space and time , e.g.

→ Orographical influence, not only from high mountains, but also for small hills with a height of less than 50 m, e.g. Tor Bergeron in Uppsala report No. 6 (1968): ”Unexpectedly , small orographic features are reflected in the fine structure of rainfall distribution”. – Only a 20m-30m plateau is needed in the Upsala field to produce approximately a 20 % precipitation increase.

Mesoscale structure of rain has been noted by many, e.g. Austin and Houze (J. Appl.Meteor., 925-935 ,1972):

→ Different scales can be identified, from a ’synoptic scale’, (100km * 100km) down to mesoscales (3km *3km). The associated time scales vary from typically 24 hours to 0.5 hours, respectively.

Scales of occurrence (precipitation-2)

→ The associated peak precipitation rate is increased as the spatial scale is reduced.

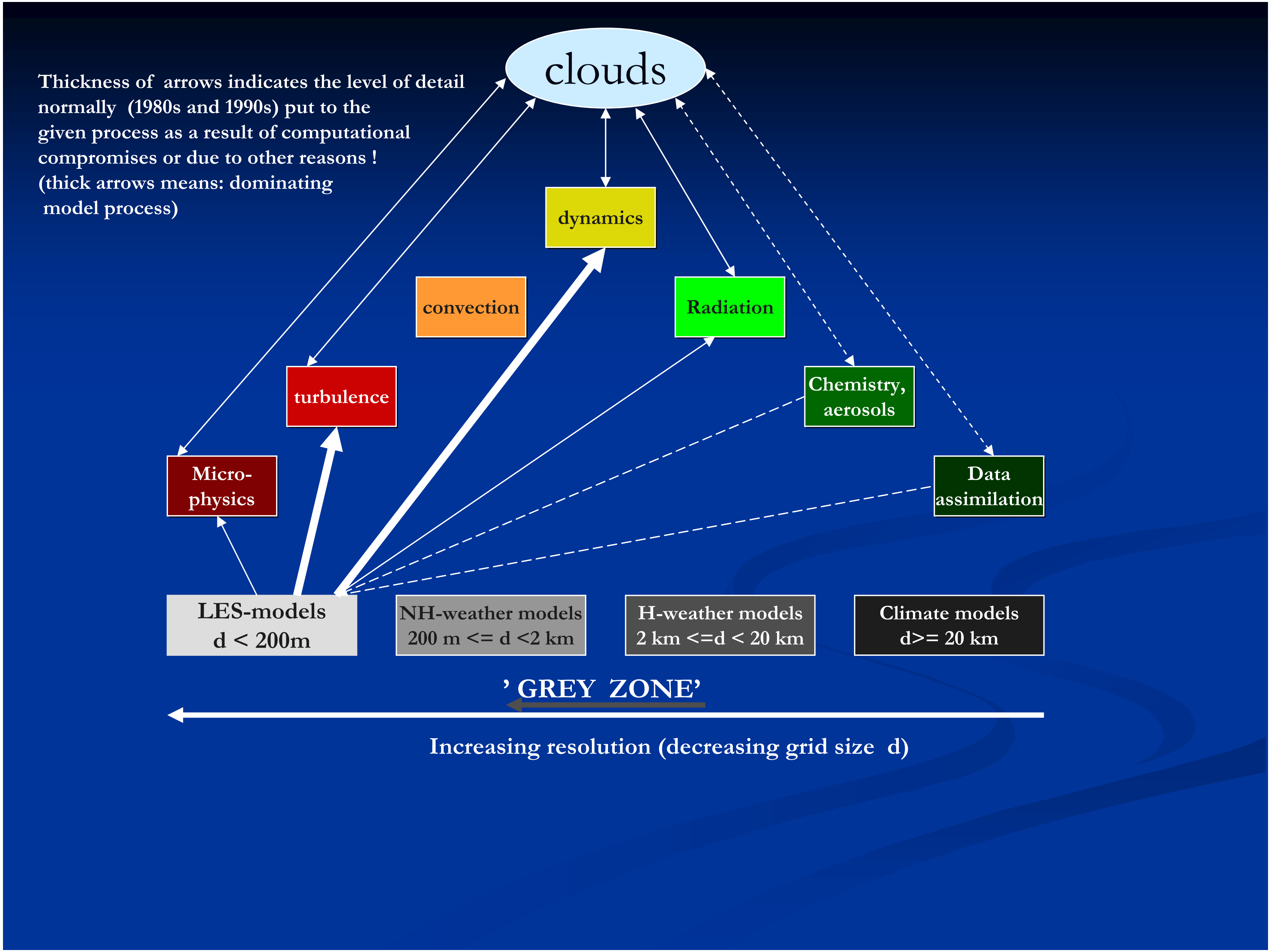
→ Heavy rain is observed in both stratiform and in showery conditions.

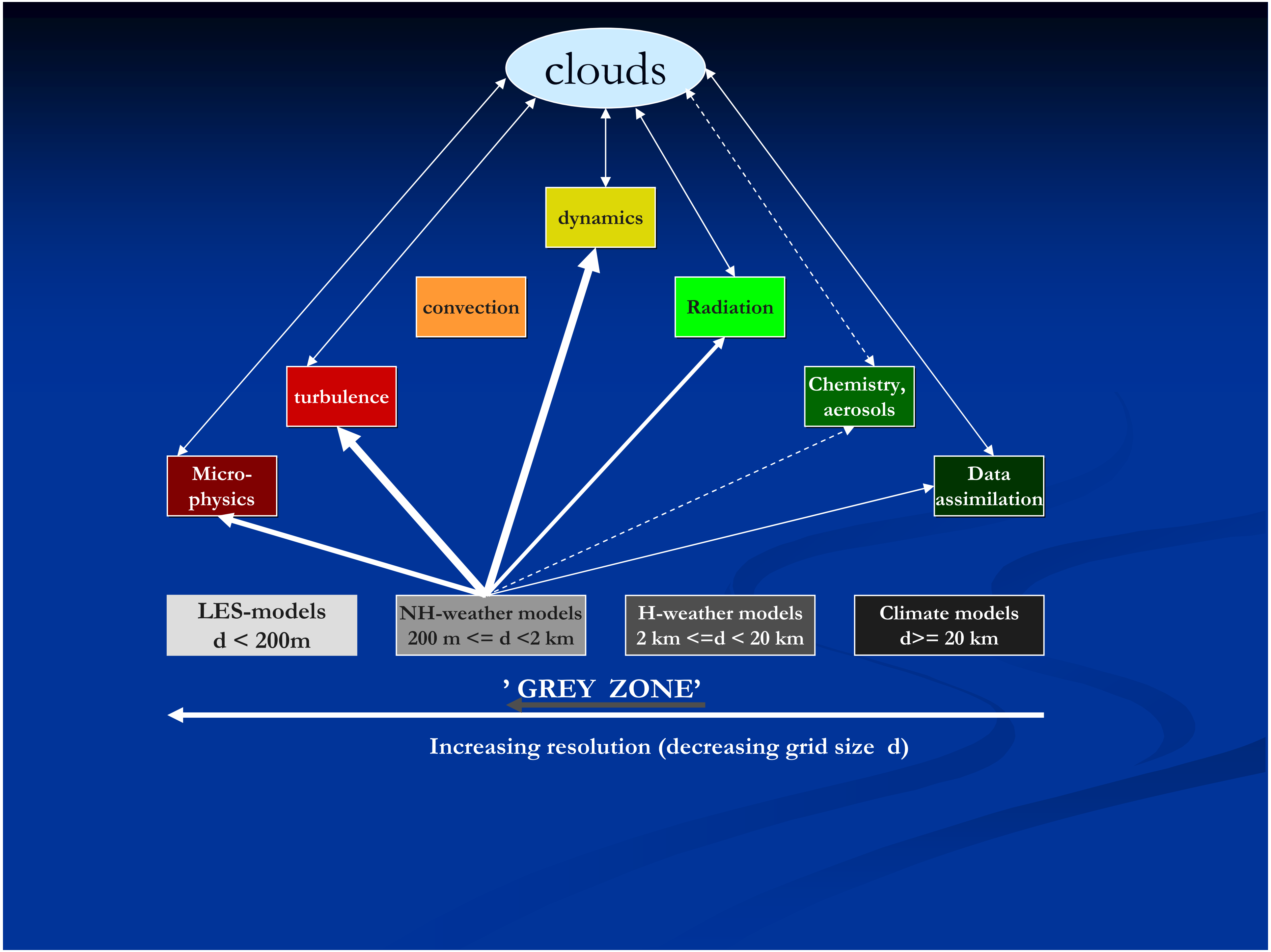
→ Very fine scale structures have also been measured using rain gaures, e.g. in Denmark . A variability of intensity, using 9 rain gauges of the same design, was detected over a 500m*500m flat field (could be 20 % - 30 %, see http://www.exigo.dk/project/summary)

Models predicting clouds and precipitation:

Common to atmospheric models which involve
real time predictions clouds and precipitation:

→Advanced cloud physics cannot alone make good predictions of clouds and precipitation ! The model must start from a realistic atmospheric state including a realistic dynamial forcing, e.g. the divergent wind field. This is because of a strong link between convergence or advection and the evolution of supersaturation and related condensation.


clouds
Thickness of arrows indicates the level of detail normally (1980s and 1990s) put to the given process as a result of computational compromises or due to other reasons !
(thick arrows means: dominating model process)
dynamics
convection
Radiation
turbulence
Chemistry, aerosols
Micro-physics
Data assimilation
LES-models d < 200m
NH-weather models 200 m <= d <2 km
H-weather models 2 km <=d < 20 km
Climate models d>= 20 km
' GREY ZONE'
Increasing resolution (decreasing grid size d)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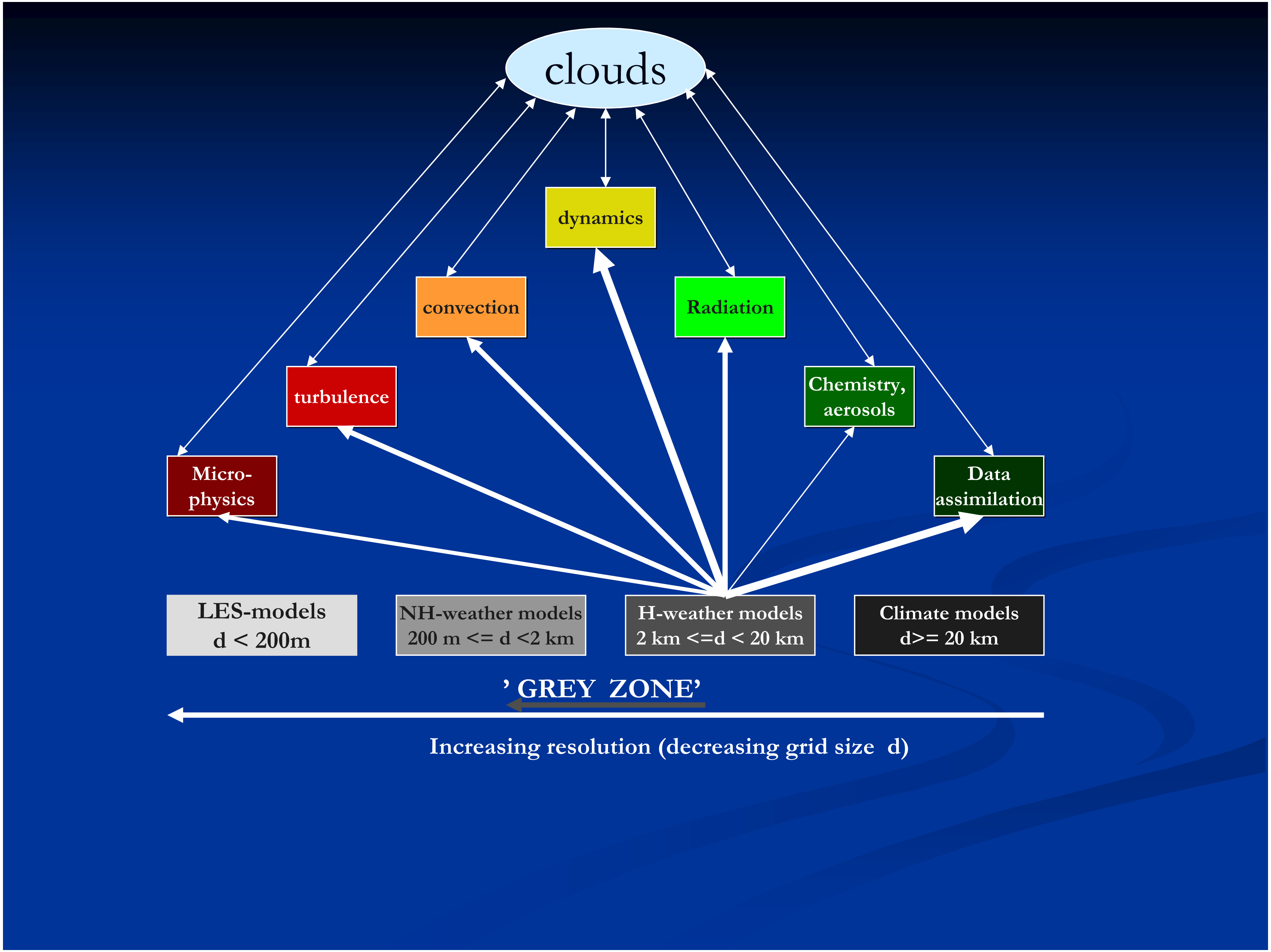
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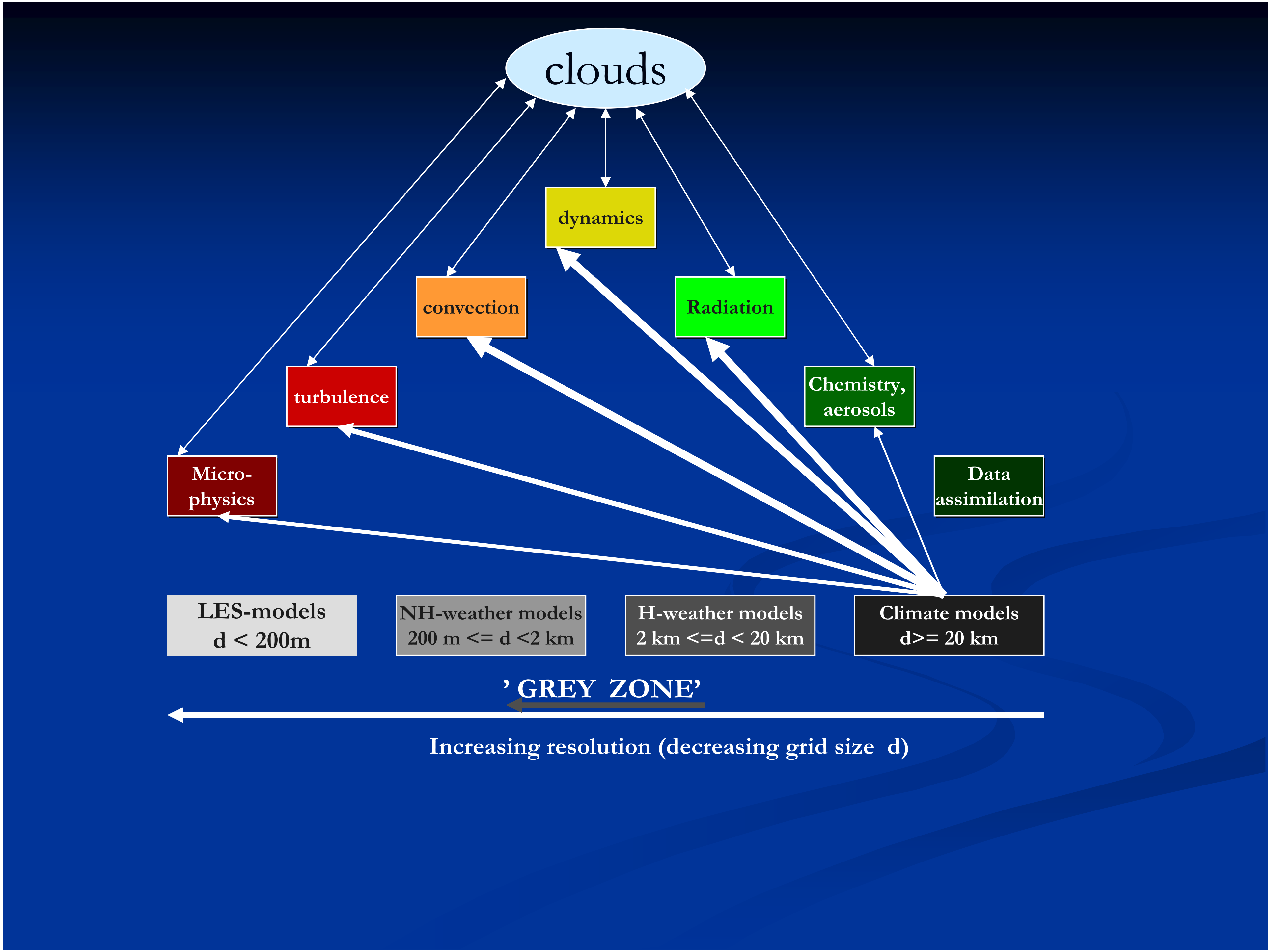
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Increasing resolution (decreasing grid size d)

Characteristics of the different models (1)

Climate models: (time prediction range : months to years)

- Coarse mesh with the risk that accuracy is insufficient: The fact that many subgrid scales need to be parameterized creates a big challenge !
- Convection needs parameterization, also mesoscale large convective complexes (if possible)
- Other physics (turbulence, radiation, microphysics need to be fairly realistic if feasible , but precipitation may be assumed to fall to the ground in one time step)
- Data-assimilation is not an issue apart from the initial state of the run
- Chemistry and aerosols are rather new in climate models and are difficult due to sources and sinks !

Characteristics of the different models (2)

H (hydrostatic) weather model: (time range: now-casting to days or weeks)

- Resolution is not sufficient to skip any parameterization of the main processes
- Assumptions related to convection and precipitation release (if released during a single time step) become questionable at high model resolution (a claimed `grey zone issue´)
- Approximations and simplicity of schemes may be acceptable for nowcasting purposes in view of the demands for fast production of products !
- Data-assimilation is essential in NWP (for clouds: radar data, Nowcasting SAF, other satellite data, GPS data and other data for humidity information)
- Inclusion of chemical/aerosol effects is rather new but in progress

Characteristics of the different models (3)

NH-weather model: (time range: now-casting to few days)

- Resolution sometimes sufficient to skip convective parameterization (special schemes combining turbulence and aspects of convection is under development , e.g. EDMF , 'Eddy-Diffusion-Mass-Flux' schemes)
- The NH model dynamics is typically computationally expensive: Different type of schemes are available: ´split-explicit' (`WRF´), semi-implicit ( `ALADIN´)
- Approximations and simplicity of schemes may be acceptable for nowcasting purposes in view of the demands for fast production of products !
- Data-assimilation essential in NWP but is less developed (so far) for very high resolution weather prediction (for clouds: radar data, Nowcasting SAF, other satellite data, GPS data and other data for humidity information)
- Inclusion of chemical/aerosol effects is new and computationally demanding.

Characteristics of the different models (4)

LES: large eddy simulation method is based on Navier-Stokes equations. The LES model concept has been used since the 1970s. A variety of different `flavours´ and applications of LES exist. – from engineering fluid dynamic problems to meteorological research models.

In meteorology:

- normally Reynolds-averaged Navier-Stokes equations,
- incompressibility and Boussinesq aproximations widely used.
- no convection scheme needed !
- in research the microphysics and radiation is much simplified, in real forecasts using LES this is not desirable.
- Initialization/data-assimilation is very difficult which has been seen in many studies (partly linked to boundary conditions, research ongoing)
- Relevant for very detailed air pollution studies and also forecasts (if feasible)
- LES are very expensive to run on large model areas due to small time steps

Known model challenges related to clouds and precipitation (1)

→to analyse properly the atmospheric initial state using data-assimilation

→to describe subgride scale physics properly, especially convective transports described by convection schemes in models using grid sizes larger than about 1 km

→Some models assume that the model precipitation falls to the surface in a single time step of the model which is unrealistic in high resolution models.

→ Lack of knowledge on actual number density and the properties of the cloud condensation nuclei – and a lack of proper model physics to take into account the effects of CCNs.

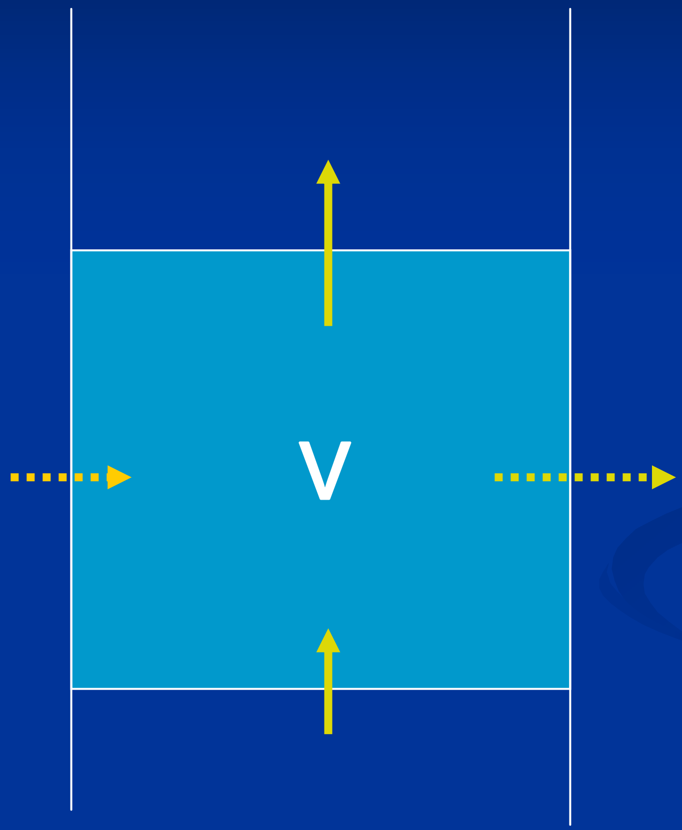
→Ice clouds is a special issue requiring `freezing nuclei´ which are sparse in the atmosphere and not well known.

Cloud physics in models:
Model physics computations are often done in a vertical column even in high resolution models

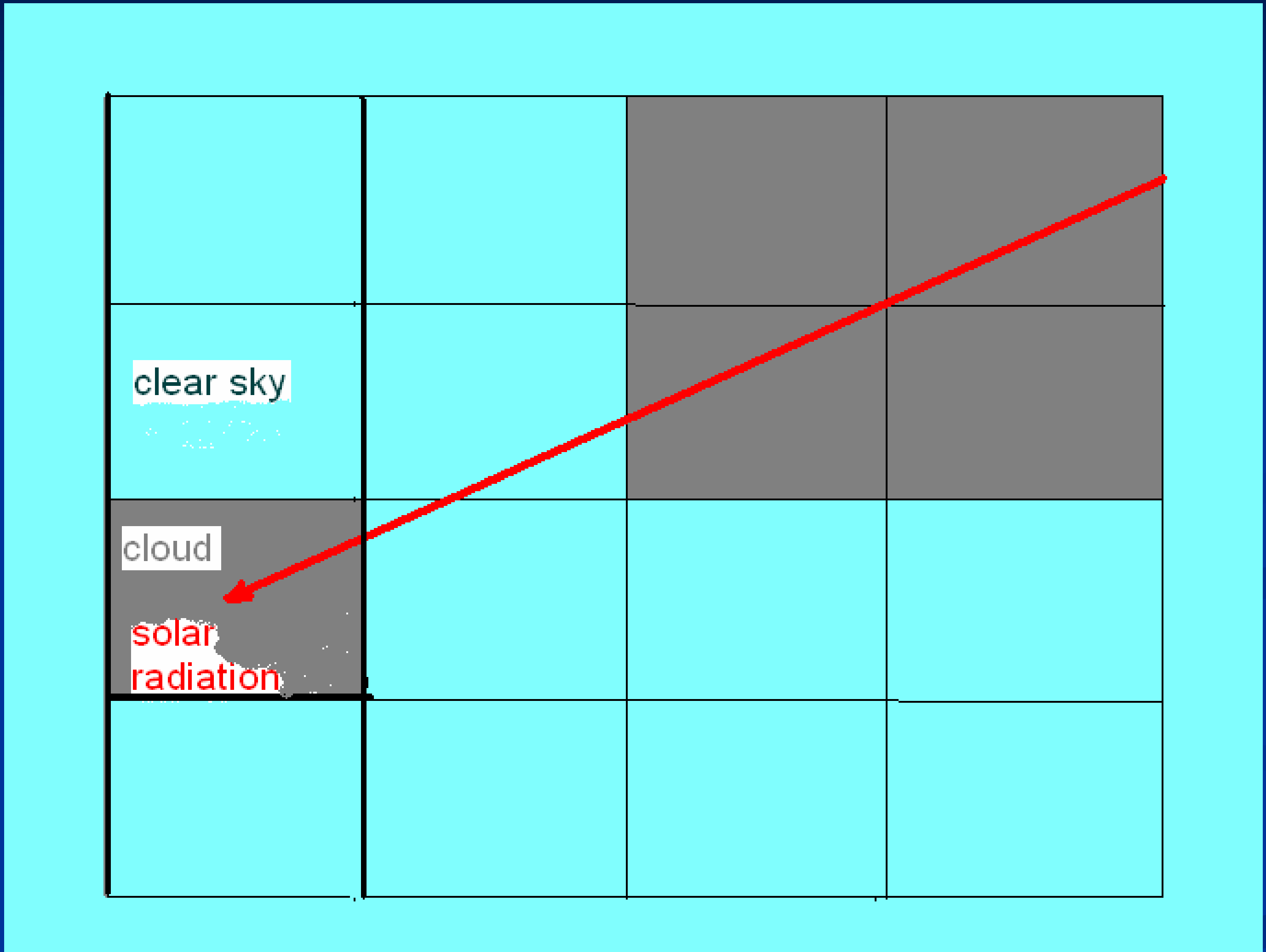
BUT

there are limitations associated with the model grid in case of `column´ physics !

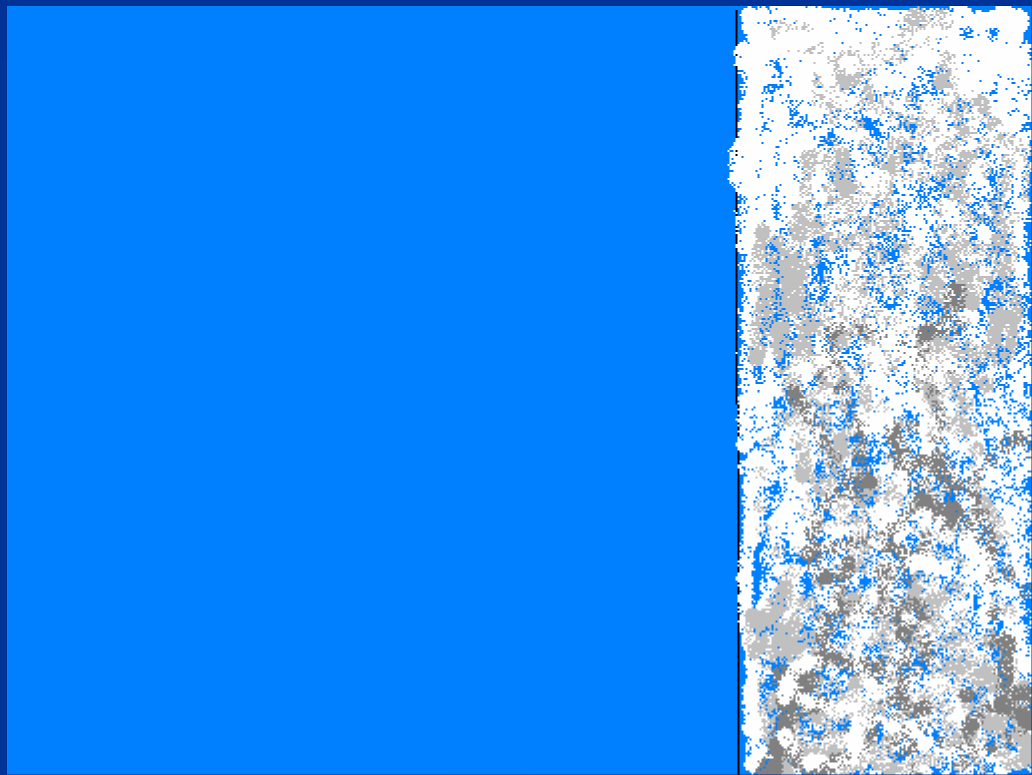
Temperature and humidity changes in a grid volume V is influenced by subgrid scale flux variations across the volume. The flux variations are often considered as varying in the vertical (`column physics´). The horizontal variation is not properly accounted for !



EXAMPLE: Column physics see a clear sky above cloud – `Correct´ physics see dense cloud giving no direct sunshine reaching the cloud !

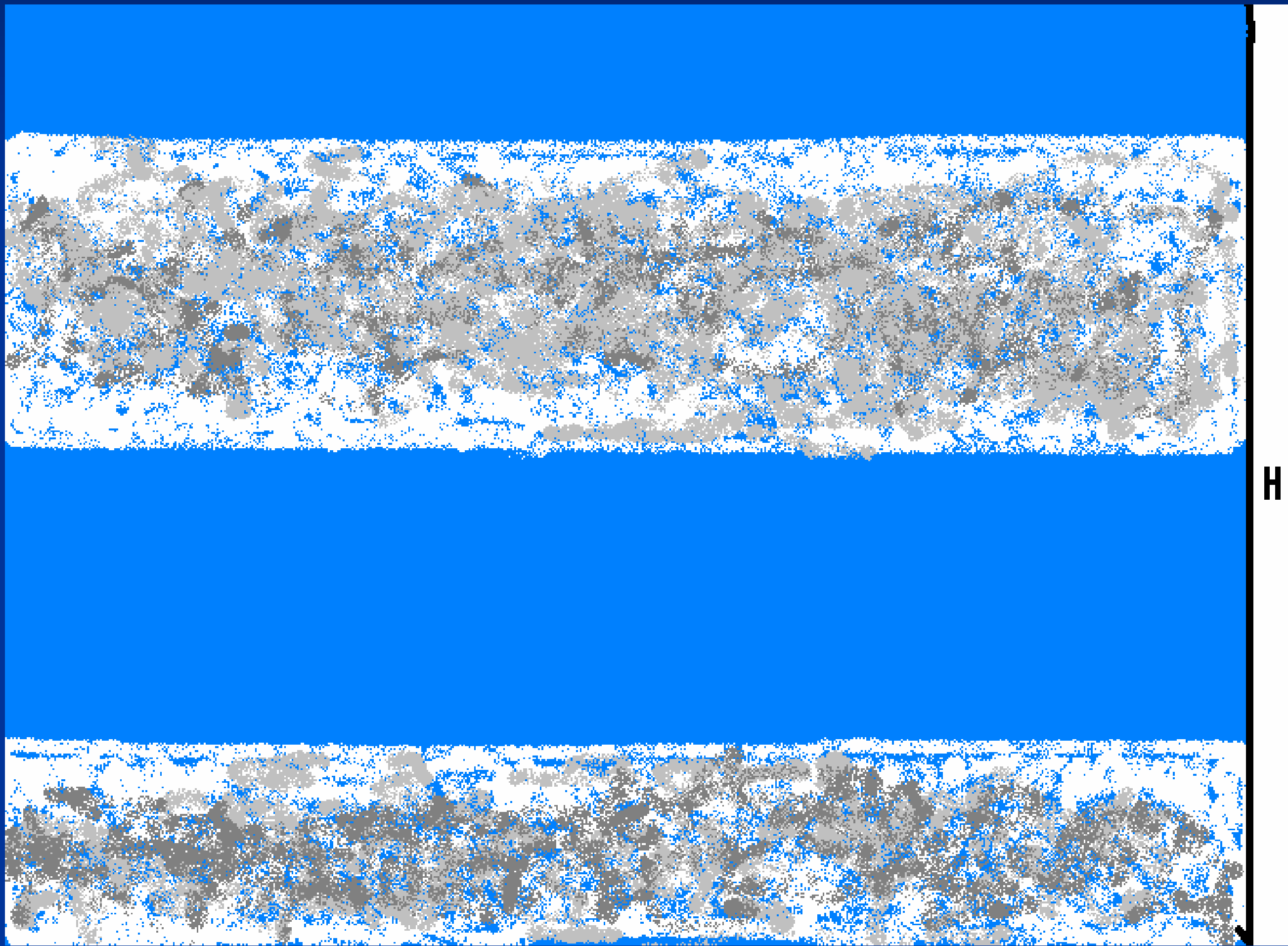


Model clouds are normally treated as fractional in horizontal and not in the vertical. This is reasonable to some extent because the horizontal grid size (e.g. 10 km) is larger than the vertical grid size (e.g. 100 m)

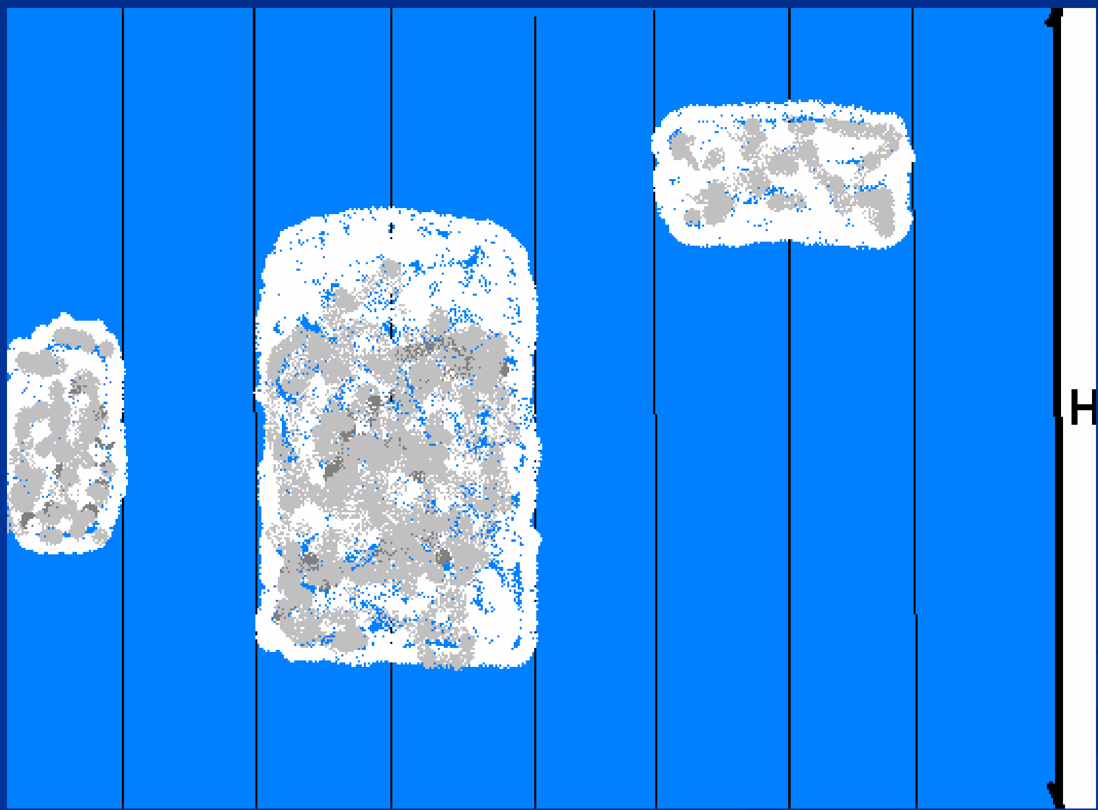


For small heigth to horizontal grid size ratio it seems more reasonable to treat cloud cover as fractional in the horizontal direction only

Real clouds may be fractional in the vertical (model grid height =H)



Real clouds may be fractional both in the horizontal and in the vertical



Types of cloud parameterizations in atmospheric models

Types of cloud parameterization in models(1)

- Diagnostic cloud parameterizations verified

by mainly synop observations, satellite data, radar data:

Example: Slingo and Ritter in the ECMWF model (1985):

Formulation based on

- relative humidity,
- `strength of inversion´ for stratocumulus type of clouds,
- function of convective precipitation rate

Types of cloud parameterizations in models(2)

- Diagnostic cloud parameterizations developed

and verified by means of high resolution LES (idea started in 1980s -1990s):

Example: Xu and Randall (J. Atmos. Sci. 53, 3084 - 3102, - 1996):

Diagnostic fomula as a function of

- Relative humidity
- Specific cloud condensate
- Saturation specific humidity

Types of cloud parameterizations (3)

- In recent years it is most common to express cloud cover from probablbility density functions (PDFs) descibing how the subgrid scale variability of prognostic variables occurs across the model grid
The PDFs may be more or less complicated (e.g. varying in time) and asymmetric. The links to the rests of model formulation need to be consistent:

- Examples in litterature since 1980s
- An example from the HIRLAM model based on a PDF is given in the Appendix - A formula for convective cloud cover at a given level is derived by solving 3 equations for , respectively, integral probability, total specific humidity and specific cloud condensate.

Special parameterization: The 'super-parameterization'.

In recent years David Randall has suggested
a multiscale model approach:

`Run a LES model inside every grid square of a coarse mesh model in order to produce subgrid scale output for the coarse mesh model´

Disadvantage:

- **It is extremely expensive to run LES model in an operational context in such a manner !**
- **The lateral boundary conditions impose potential problems for `imperfect´ interaction between the two models.**

Convection, condensation, precipitation in HIRLAM (1)

Two alternative schemes available:

1) The Kain-Fritsch for convection combined with the Rasch-Kristjansson scheme for stratiform condensation

2) STRACO (Soft TRAnsition COndensation) parameterizing both convection and stratiform condensation.

Note that

- both schemes currently assume that precipitation release is transferred to the ground in a single time step. This is a restriction which is normally made in hydrostatic models but not in very high resolution (cloud resolving models)
- the schemes have until recently operated with specific humidity and total specific cloud condensate (with a diagnostic separation between cloud water and cloud ice).
- a convection scheme is needed in order to parameterice the effect of large vertical velocities (and fluxes) connected with convection.

Convection, condensation, precipitation in HIRLAM (2)

- The stratiform schemes in both Rasch-Kristjansson (RK) and STRACO have to some extent been developed from Sundqvist's work (Sundqvist 1988, 1993)
- For RK this mainly applies to the condensation/cloud process while the microphysics including precipitation release is quite different originating from schemes in finer scale models.
- For STRACO scheme the similarity with Sundqvist developments is particularly with regard to precipitation release which has been further developed (e.g. Korsholm et al. , HIRLAM Newsletter No. 54)
- The schemes treat cloud cover in a pseudo-prognostic way (computing a tendency).

Convection, condensation, precipitation in HIRLAM (3)

- The convection schemes have a different origin.
  The Kain Fritsch scheme has been
  developed (mainly in the US by J. Kain since late 1980s)

- The STRACO scheme started in 1990s in the HIRLAM-4 project.
  It was developed on the basis of a previous ECMWF convection scheme
  introducing many changes including convective transports of cloud
  condensate. The precipitation release was kept in the framework of
  the work of Sundqvist (1988, 1993).

Both convection schemes share the basic gross concepts of convection schemes:
(triggering , cloud model, closure assumption , microphysics assumptions)
but use different methods outlined below:

Convection, condensation, precipitation in HIRLAM (4)

- **Triggering:**
  In both schemes a search throughout the depth of the troposphere is done to estimate favorable forcing conditions for initiation of convection

- **KF:** effective temperature perturbation at the lifting condensation level depending on mean vertical velocity (at LCL) and model grid size.
  Also the temperature + humidity structure close to LCL is taken into account
  Adding this temperature perturbation to the temperture of a lifted air parcel should exceed the environmental temperature to initiate convective computations (cloud model).
  Also turbulence intensity (TKE) in the boundary layer is used as a trigger for convection. - In some versions of the scheme subgrid scale vertical variance is estimated as a function of variance of orography.

- **STRACO:** temperature and humidity perturbations are estimated as a function of model resolution (decreasing with diminishing grid size) and turbulent kinetic energy (TKE).

Convection, condensation, precipitation in HIRLAM (5)

- Cloud model:

  A cloud ascent is simulated in both schemes taking into account entrainment processes between cloud and environment which may reduce cloud buoyancy and ultimately stop convection.

- KF: upward and downward cloudy mass fluxes are computed and evolves as a combined effect of positive and negative contributions (entrainment flux and detrainment flux, respectively) The total mass flux at a given level should be zero. The mas flux of environmental air is computed to provide this condition. From the mass flux and the termodynamic variables the fluxes and tendencies of the prognostic variables can be determined (excluding some precipitation and evaporation effects)

  The microphysics takes care of the rest.

  But in practice the scheme decides to rescale mass fluxes

  to reduce CAPE by a certain magnitude (explained in connection with 'closure').

Convection, condensation, precipitation in HIRLAM (6)

STRACO:

Vertical extent of convective cloud determined from entrainment parameterization depending on model resolution and environmental properties (temperature humidity, wind)

The tendencies from convection are determined as a function of the converging humidity and specified vertical functions for heating , moistening, and cloud water generation (depending on the cloud model ascent). Additional effects of ’overshooting convective eddies’ at cloud tops are included affecting all levels active in the convective process.

Convection, condensation, precipitation in HIRLAM (7)

Closure: Basic assumptions to `close' the problem.

- **KF:**

  CAPE closure. All convective mass fluxes are scaled to reduce CAPE by at least 90 percent over a convective time scale (e.g. 1800 s). The reference profile was in early versions an `undiluted ' parcel ascent, in recent version developed in CAM3 the reference is a mean cloud ascent.

- **STRACO**

  No 'rescaling' is done. Currently the moisture budget (convergence of humidity) , assumptions and computations in the cloud model plus equation for evaporation of cloud condensate generated by convection determines convective tendencies excluding microphysics associated with precipitaton.

Convection, condensation, precipitation in HIRLAM (8)

- **Microphysics (precipitation and evaporation related effects)**
- **KF : simple precipitation release as a function of cloud condensate in the convective updraft and mean vertical velocity. Some adjustments of precipitation flux from an empirical relationship for precipitation efficiency (evaporative effects)**
- **STRACO: Same framework as used for stratiform scheme (see Korsholm et al, HIRLAM Newsletter no. 54, June 2008) but the formulations enable that relevant parameters can take on different values in convective conditions.**

Convection, condensation, precipitation in HIRLAM (9)

Development issues of KF-RK and STRACO schemes:

→ Both schemes will in the future be able to use cloud water and cloud ice (and probably more variables in the future). For the STRACO scheme convective transports of extra tracer variables are currently being developed and tested.

→ In HIRLAM-model KF-RK has currently a reputation for predicting high precipitation amounts better than STRACO scheme but the reverse applies to small precipitation amounts.

→ New developments of the schemes (in CAM3 to be adopted for HIRLAM) and in STRACO seem to produce schemes with more similar behaviour than seen previously –
for STRACO some treatment of local CAPE is introduced while the CAPE closure of KF is becoming less extreme in forthcoming CAM3-version to be used in HIRLAM.

International collaboration :

GEWEX cloud system study (GCSS)

GEWEX is established under the World Climate Research Programme (WCRP)

- It is generally recognized that inadequate parameterization of clouds is one of the greatest sources of uncertainty in the prediction of weather and climate.
- Recent GCSS meeting (4th PAN-GCSS meeting) was held in Toulouse June 2008 (See the many relevant abstracts on www.knmi.nl/~siebesma/PNN-GCSS/)
- GCSS is developing better parameterizations of cloud systems for climate models by improving our understanding of the physical processes at work within the following types of cloud systems: (1) boundary layer, (2) cirrus, (3) extra tropical layer, (4) precipitating convective, and (5) polar. There are five GCSS working groups, one for each type of cloud system. Each of these working groups has adopted single-column modeling as a key research strategy, and each is also making use of cloud ensemble models.

GCSS working groups

The GCSS working groups are performing the following activities:

- Identifing and developing cloud-resolving and mesoscale models appropriate for each cloud system type.
- Specifing blueprints of minimum observational requirements for the development and validation of these models.
- Assembling, for particular cloud types, case-study data sets accessible to the community of (a) matched observations from satellites, surface and aircraft, and (b) model-derived synthetic data sets.
- Conducting workshops, including model intercomparisions using the above case study data sets.
- Using the data sets to derive a better understanding of the coupled processes within different types of cloud systems and to derive improved parameterization schemes for large-scale models.

GCSS –related work with HIRLAM

- Some GCSS related 1D cloud physics tests have been conducted .
- Initial and boundary conditions and dynamical forcing is specified in the case description.
- Model simulation results, e.g. fractional cloud cover, amount of cloud condensate etc. can be compared to the model results of LES. This type of experiments can be used for tests and developments of cloud parameterizations.

EXAMPLE :

The `ASTEX stratocumulus´ case. It has been possible to verify, e.g. that the HIRLAM physics can produce a realistic `cloud top entrainment´ for the experimental conditions of this case. The entrainment of dry air at the top implies that the whole cloud layer is ’lifted’ during the simulation time.

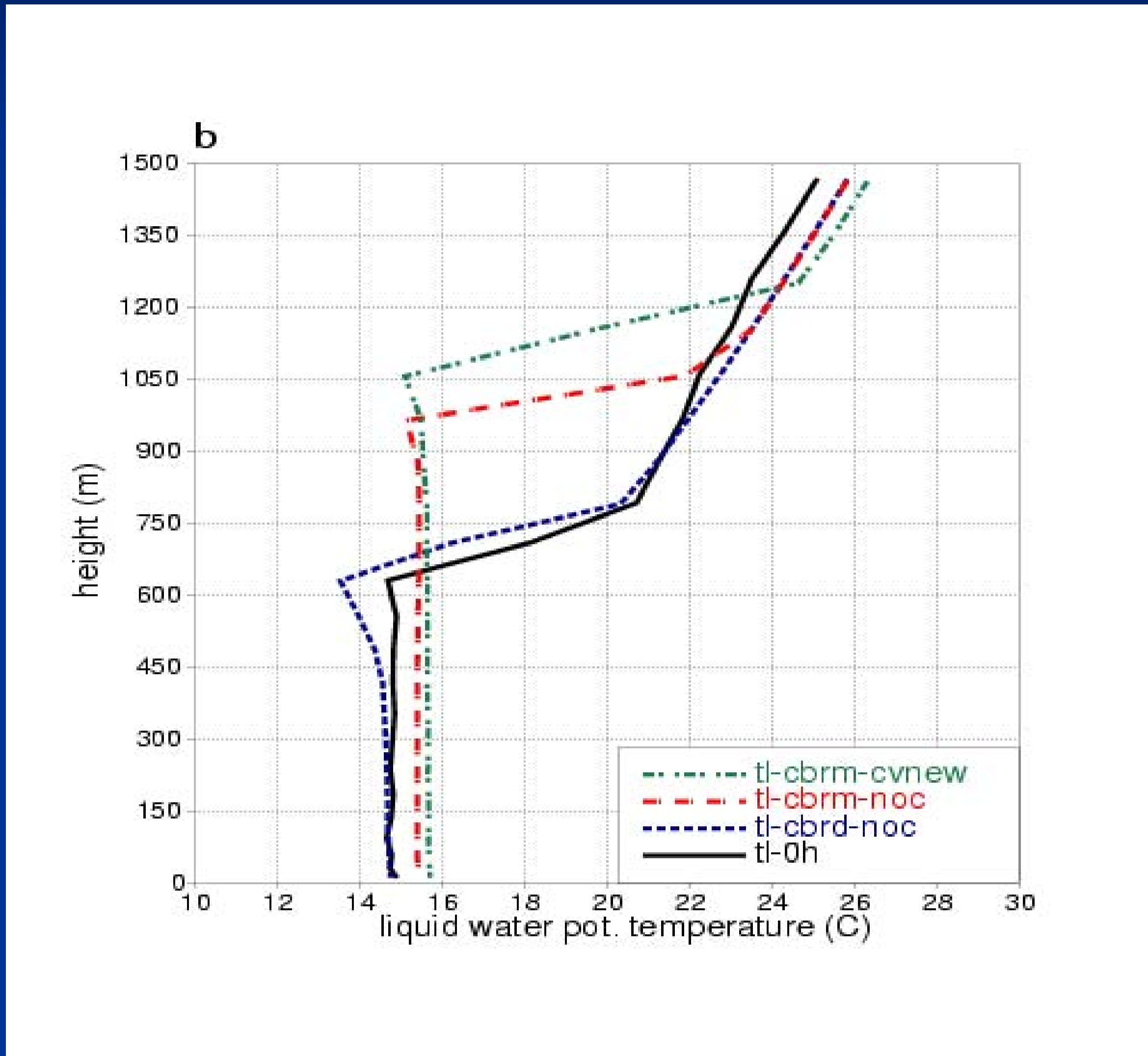
ASTEX case:

Simulations with HIRLAM physics:

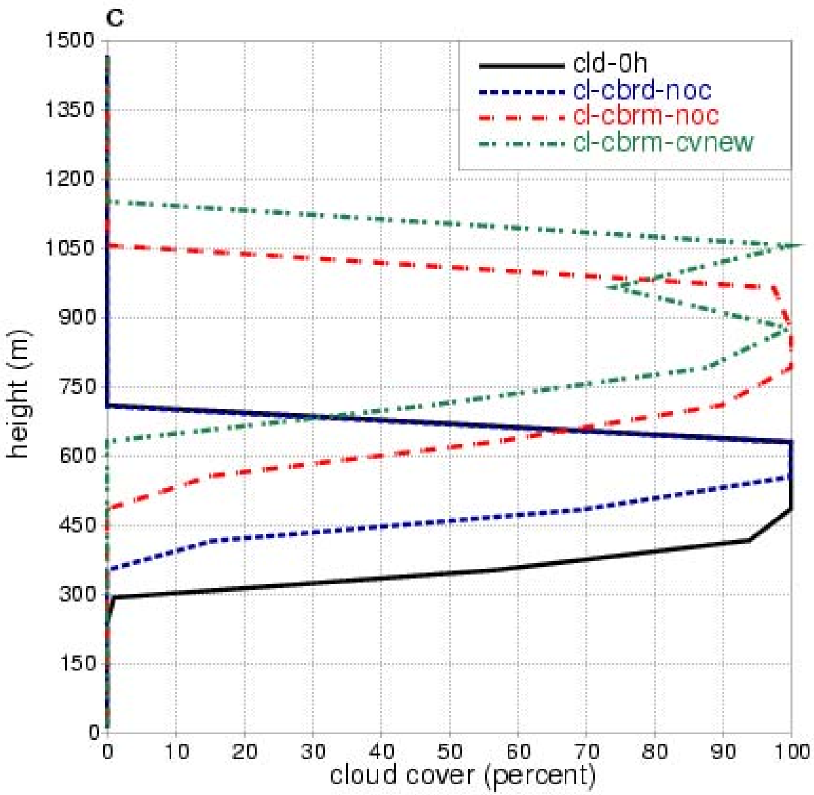
Number of levels : 80 , with 17 below 1 km, 33 below 3 km
Time step: 150s, (75s)

ASTEX: (12-13 June 1992)
-Atlantic Stratocumulus Transition EXperiment –
(Bretherton and Pincus 1995), simulation period: 24 h
Forecast length: 24h.

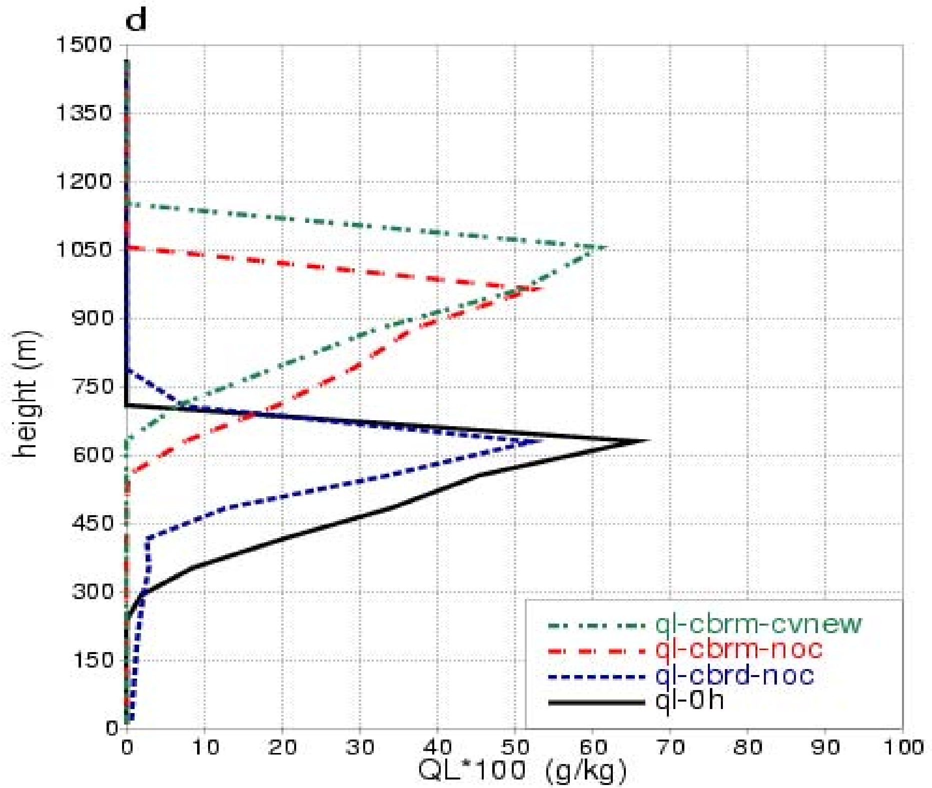
ASTEX liquid water pot. temperature (24h)



ASTEX cloud cover (24h)



ASTEX liquid water (24h)



Conclusions and outlook (1)

- Clouds are very important in relation to weather- and climate type of models
- The coming years will show to what extent new model developments related to aerosols (and chemistry) will show up in increased skills of NWP type of model predictions.
- As the model resolution increases new challenges imerge. It is important to understand the effect of various approximations (e.g. `column physics ')

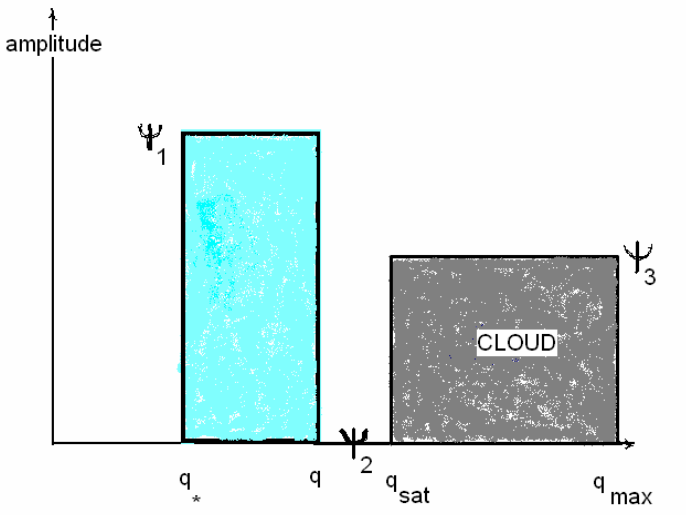
Conclusions and outlook (2)

- Data-assimilation of new variables related to aerosol and air pollution/chemistry will be important to get maximum benefit of new model extensions related to air-quality.
- Data-assimilation at very high resolution requires more research and computer power. It seems important to do research in assimilation of moisture related data at very high resolution to provide optimal conditions for cloud prediction !
- NH-models are in general computationally expensive – as a consequence it is a challenge to do frequent update short range forecasts.

Appendix:

Equations solved to determine convective cloud cover.

The area of the grey (cloud) part of the PDF shown below is equal to the convective cloud cover



i) Equation for probability integral :

$$\int_{q_*}^{\overline{q}} \psi_1 \cdot dq_t + \int_{\overline{q}}^{q_s} \psi_2 \cdot dq_t + \int_{q_s}^{q_{max}} \psi_3 \cdot dq_t = 1$$

ii) Equation for total specific humidity

$$\int_{q_*}^{\overline{q}} q_t \cdot \psi_1 \cdot dq_t + \int_{\overline{q}}^{q_s} q_t \cdot \psi_2 \cdot dq_t + \int_{q_s}^{q_{max}} q_t \cdot \psi_3 \cdot dq_t = \overline{q}_t$$

iii) Equation for specific cloud condensate

$$\int_{q_s}^{q_{max}} (q_t - q_s) \cdot \psi_3 \cdot dq_t = \overline{q}_c$$

In the above equations $q_s = q_s(T_c)$ where q_s is saturation humidity and T_c is a cloud temperature characterizing the convective cloud at the given vertical level. q_* is a minimum specific humidity for nonzero probability according to the piecewise rectangular PDF. $\overline{q}$ is the grid box mean specific humidity, $\overline{q}_t$ is a grid box mean total specific humidity, $\overline{q}_c$ is a grid box mean specific cloud condensate. q_{max} is the maximum total specific humidity with non-zero probability. ψ_1, ψ_2, ψ_3 are amplitudes of the piecewise rectangular PDF.

Based on the asymmetric rectangular PDF box structure and related equations it is possible to derive the following formula (1) for convective cloud cover if the medium box has zero amplitude

$$f_{cv} = 1 - \frac{2\,(q_s(T_c) - \overline{q})}{2q_s(T_c) - \overline{q} - q_*} \qquad (1)$$

In (1) $q_* < \overline{q}$ is the lowest occurring specific humidity which needs to be parameterized from other model variables. Currently the following parameterization is used for q_*

$$q_* = \overline{q}\frac{(1 - K_a\widehat{Q})}{(1 + K_b\frac{q_c}{q_s})} \qquad (2)$$

In (2)

$$\widehat{Q} = min(\frac{\widehat{Q_a}}{Q_{00}}, 1) \qquad (3)$$

The second term in the nominator of (2) involving K_a tentatively describes a small effect of moisture availability to the convective cloud. $\widehat{Q_a}$, constrained to be non-negative, is the vertical mean moisture supply to the convective cloud $(\mathrm{kg} \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-1})$ through humidity advection and convergence.

$K_a = 3.0 \cdot 10^{-2}$, $K_b = 6.0$, $Q_{00} = 3.0 \cdot 10^{-8} \mathrm{kg} \cdot \mathrm{kg}^{-1} \cdot s^{-1}$

The denominator of (2) expresses a dependency of total specific cloud condensate q_c through the dimensionless parameter $q_c/q_s(T_c)$

The formulation of convective cloud cover expressed by (1) and (2) is well behaved for all values of the parameters since q_* is always less than $\overline{q}$, and $\overline{q}$ is always less than $q_s(T_c)$. Hence the cloud cover will always be non-negative.

References:

Albrecht, B.A. , Bretherton, C., Johnson, D., Scuber, W.H. and A.S. Frisch (1995):
The atlantic stratocumulus transition experiment.
Bul. Amer. Meteorol. Soc., 76: 889-904

Austin , P.M. and R.A. Houze, 1972:
Analysis of the structure of precipitation Patterns in New England
J. Appl. Meteor. 11, 926-935

Bergeron, Tor, (1968):
Insight into the nature of precipitation
Uppsala Report No. 6

Bretherton, C. and R. Pincus (1995):
Cloudiness and marine boundary layer dynamics in the ASTEX lagrangian experiments, part 1 :
Synoptic setting and vertical sructure .
J. Atmos. Sci.,52, 2707 - 2723

Kain J.S., and J.M. Fritsch (1990):
A One-Dimensional Entraining/Detraining plume Model and its Application in
Convective Parameterization.
J. Atmos. Sci. Vol. 47, No.23 2784 – 2802.

Kain, J. S. (2004):
A Revised Version of the Kain-Fritsch Convective Parameterization.
J. Appl. Meteor. , 43, 170- 181

Korsholm, U. , Baklanov, A., Gross, A. and B. Sass, 2008:
Using HIRLAM to forecast chemical weather - staus and prospective
of Enviro-HIRLAM.
HIRLAM Newsletter No. 54, June 2008 [Availble on www.HIRLAM.org]

Pruppacher and Klett (1997):
Microphysics of clouds and precipitation, Chapters 4 -6,
Kluwer Academic Publishers

Rasch, P. J. and J.E. Kristjansson (1998):
Comparison of the CCM3 Model Climate Using Diagnosed and Predicted
Condensate Parameterizations
J. Climate, 1587 - 1614

Rogers, R.R. and M. K. Yau (1996):
A short course in Cloud Physics ,
Butterworth-heinemann, 290 pp

Sass, B.H. (2002):
A Research version of the STRACO cloud scheme
DMI Tech. Rep. No. 02-10.

Sass, B.H. (2007):
Idealized simulations of shallow convection using recent HIRLAM physics
DMI Sci. Rep. No. 07-02

Sundqvist, H, (1988):
Parameterization of condensation and associated clouds in models for weather prediction and general circulation simulation. Physically Based Modelling and Simulation of Climate and Climate change.
M. E. Schlesinger (Ed)
Klüwer Academic, 1, 433-461

Sundqvist, S., (1993):
Inclusion of Ice Phase of Hydrometeors in Cloud Parameterization for Mesoscale and Largescale Models.
Contr. Atm. Phys. , 66, 137-147.

Slingo , J. and and B. Ritter (1985):
Cloud prediction in the ECMWF model –
ECMWF Tech. Rep. No. 46

Xu , K. M. and D.A. Randall (1996):
A semiempirical cloudiness parameterization for use in climate models.
J. Atmos. Sci, 53, 3084 – 3102