Atmospheric Planetary Boundary Layer (ABL / PBL): theory, modelling and applications

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Part 1
Stably Stratified Atmospheric Boundary Layer (SBL)

References

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Motivation

NWP, climate and air pollution modeling require

- Surface fluxes (lower boundary conditions in all models)
  surface layer
  roughness layer
- SBL height
  in advanced surface-flux scheme (especially for shallow SBLs)
  in air-pollution modeling
- Turbulent fluxes in any stratification (to close Reynolds equations in all models)
  critical Richardson number?
  turbulent Prandtl number
  where to go?
- Depth/strength of and fluxes within capping inversions (especially in Polar regions)

State of the art

Surface fluxes

Surface layer concept: τ, F_θ, F_q = constant

Local M-O (1954) scaling: $L = -u_*^3 / F_{bs}$

Roughness length $z_{0u} \sim h_0$: no stability effect

SBL height

Local (RM,1935)⇔Z(1974): $N|_{\text{free-flow}}$ neglected

Closure

Down-gradient, Kolmogorov (1941): $K_M, K_H, K_D \sim E_K^{1/2} l_T$

TKE and ,e.g., ε-budgets: TPE disregarded

Improvements: to avoid Ri_{cr} and correct Pr_{turb}

Capping inversions low interest / no parameterization

Data Mid latitudes → residual layers (N=0) → SBL = nocturnal BL

Basic types of the SBL

- Until recently ABLs were distinguished accounting only for $F_{bs}=F_*$:

  neutral at F_*=0

  stable at F_*<0

- Now more detailed classification:

  truly neutral (TN) ABL: F_*=0, N=0

  conventionally neutral (CN) ABL: F_*=0, N>0

  nocturnal stable (NS) ABL: F_*<0, N=0

  long-lived stable (LS) ABL: F_*<0, N>0

- Realistic surface flux calculation scheme should be based on a model applicable to all these types of the ABL

MEAN PROFILES & SURFACE FLUXES
Content

- Revision of the similarity theory for the stably stratified ABL
- Analytical approximations for the wind velocity and potential temperature profiles across the ABL
- Validation of new theory against LES and observational data
- Improved surface flux scheme for use in operational models

Zilitinkevich, S. S., and Esau, I. N., 2007: Similarity theory and calculation of turbulent fluxes at the surface for stably stratified atmospheric boundary layers. *Boundary-Layer Meteorol.* **125**, 193-296.

Turbulence in atmospheric models

- turbulence closure – to calculate vertical fluxes: $\vec{\tau}$ and F_θ through mean gradients: $d\vec{U}/dz$ and $d\Theta/dz$

- flux-profile relationships – to calculate the surface fluxes: $u_*^2 = \tau_* = \tau\,|_{z=0}$, $F_* = F_\theta\,|_{z=0}$ through wind speed $U_1 = U\,|_{z=z_1}$ and potential temperature $\Theta_1 = \Theta\,|_{z=z_1}$ at a given level z_1

- Warning: In NWP and climate models, the lowest computational level is z_1~30 m

Neutral stratification (no problem)

From logarithmic wall law:

$$\frac{dU}{dz}=\frac{\tau^{1/2}}{kz},\quad \frac{d\Theta}{dz}=\frac{-F_\theta}{k_T\tau^{1/2}z},\quad U=\frac{\tau^{1/2}}{k}\ln\frac{z}{z_{0u}},\quad \Theta-\Theta_0=\frac{-F_\theta}{k_T\tau^{1/2}}\ln\frac{z}{z_{0u}}$$

k, k_T von Karman constants; z_{0u} aerodynamic roughness length for momentum;
Θ_0 aerodynamic surface potential temperature (at z_{0u}) [Θ_0-Θ_s through z_{0T}]

It follows: $\tau_1^{1/2}=kU_1(\ln z/z_{0u})^{-1}$, $F_{\theta 1}=-kk_TU_1(\Theta_1-\Theta_0)(\ln z/z_{0u})^{-2}$
$\tau_1=\tau_*$, $F_{\theta 1}=F_*$ when $z_1\approx 30$ m << h → OK in neutral stratification

Stable stratification: current theory

(i) local scaling, (ii) log-linear Θ-profile → both questionable

- When z_1 is much above the surface layer → $\tau_1 \neq \tau_*$, $F_{\theta 1} \neq F_*$
- Monin-Obukhov (MO) theory → $L = \dfrac{\tau^{3/2}}{-\beta F_\theta}$ (neglects other scales)→

$$\frac{kz}{\tau^{1/2}}\frac{dU}{dz} = \Phi_M(\xi), \quad \frac{k_T \tau^{1/2} z}{F_\theta}\frac{d\Theta}{dz} = \Phi_H(\xi), \quad \text{where} \quad \xi = \frac{z}{L}$$

- $\Phi_M = 1 + C_{U1}\xi, \ \Phi_H = 1 + C_{\Theta 1}\xi$ from z-less stratification concept

$$U = \frac{u_*}{k}\left(\ln\frac{z}{z_{u0}} + C_{U1}\frac{z}{L_s}\right), \quad \Theta - \Theta_0 = \frac{-F_*}{k_T u_*}\left(\ln\frac{z}{z_{u0}} + C_{\Theta 1}\frac{z}{L_s}\right)$$

- Ri≡ $\beta(d\Theta/dz)(dU/dz)^{-2}$ → $\mathrm{Ri}_c = k^2 C_{\Theta 1} k_T^{-1} C_{U1}^{-2}$ (unacceptable)
- $C_{U1} \sim 2$, $C_{\Theta 1}$ also ~ 2 (factually increases with $z \backslash L$)

Stable stratification: current parameterization

To avoid critical Ri modellers use empirical, heuristic correction functions to the neutral drag and heat/mass transfer coefficients

- Drag and heat transfer coefficients: $C_D = \tau / (U_1)^2$, $C_H = -F_{\theta s}/(U_1 \Delta\Theta)$
- Neutral: C_{Dn}, C_{Hn} – from the logarithmic wall law
- To account for stratification, correction functions (dependent only of Ri):

$$f_D(\mathrm{Ri}_1) = C_D / C_{Dn} \;\; \text{and} \;\; f_H(\mathrm{Ri}_1) = C_H / C_{Hn}$$

$\mathrm{Ri}_1 = \beta(\Delta\Theta) z_1 / (U_1)^2$ (surface-layer “Richardson number”) - given parameter

SS Zilitinkevich et al., 2002: Near-surface turbulent fluxes in stable stratification: Calculation techniques for use in general-circulation models. *Boundary-layer Meteorol.* **128**, 1571-1587

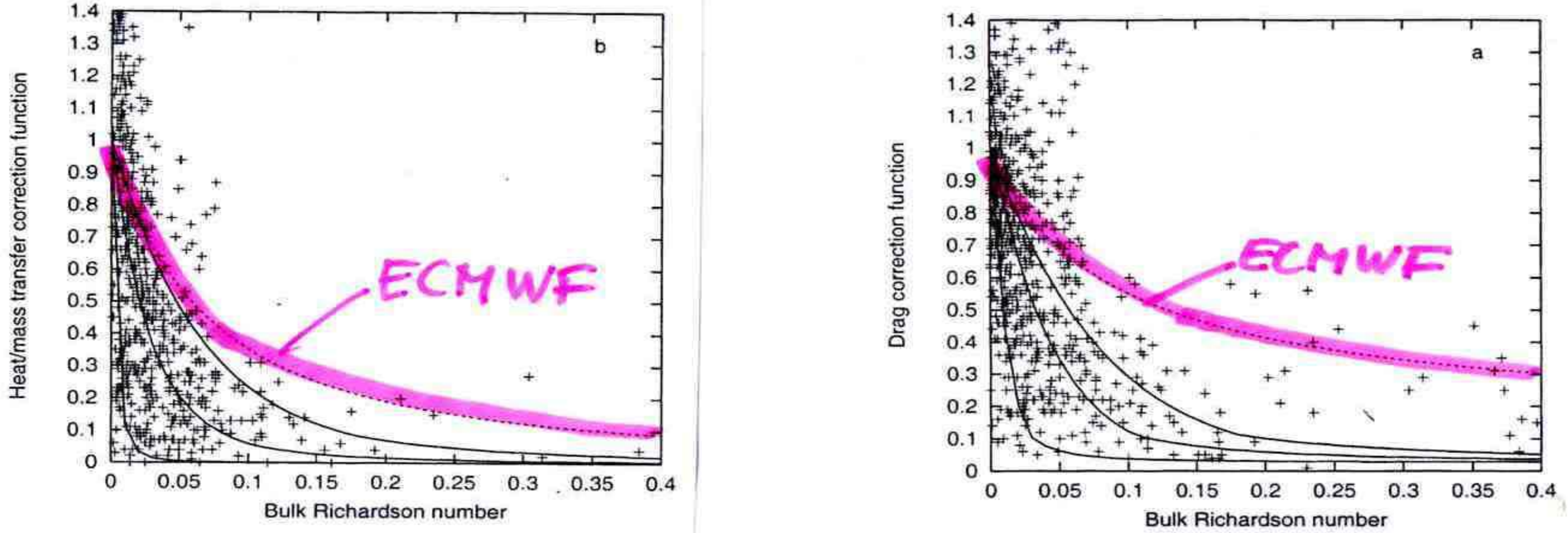

Figure 1. The correction functions (a) to the drag coefficient, f_D, and (b) to the heat and mass-transfer coefficients, $f_H = f_M$, versus the surface-layer bulk Richardson number Ri, see Eq. (7). Crosses are data from measurements at Halley, Antarctica. The correction functions from Louis *et al.* (1982) are shown by dashed lines.

$C_D \equiv \frac{\tau_s}{u^2}, \quad C_H \equiv -\frac{F_{\theta s}}{u\Delta\theta}, \quad C_M \equiv -\frac{F_{qs}}{u\Delta q}.$ $\quad Ri \equiv \frac{(\beta\Delta\theta + 0.61 g \Delta q) z_1}{u^2}$ $\quad f_D = C_D / C_{Dn}, \quad f_H = C_H / C_{Hn}, \quad f_M = C_M / C_{Mn}$

SS Zilitinkevich et al., 2002: Near-surface turbulent fluxes in stable stratification: Calculation techniques for use in general-circulation models. *Boundary-layer Meteorol.* **128**, 1571-1587

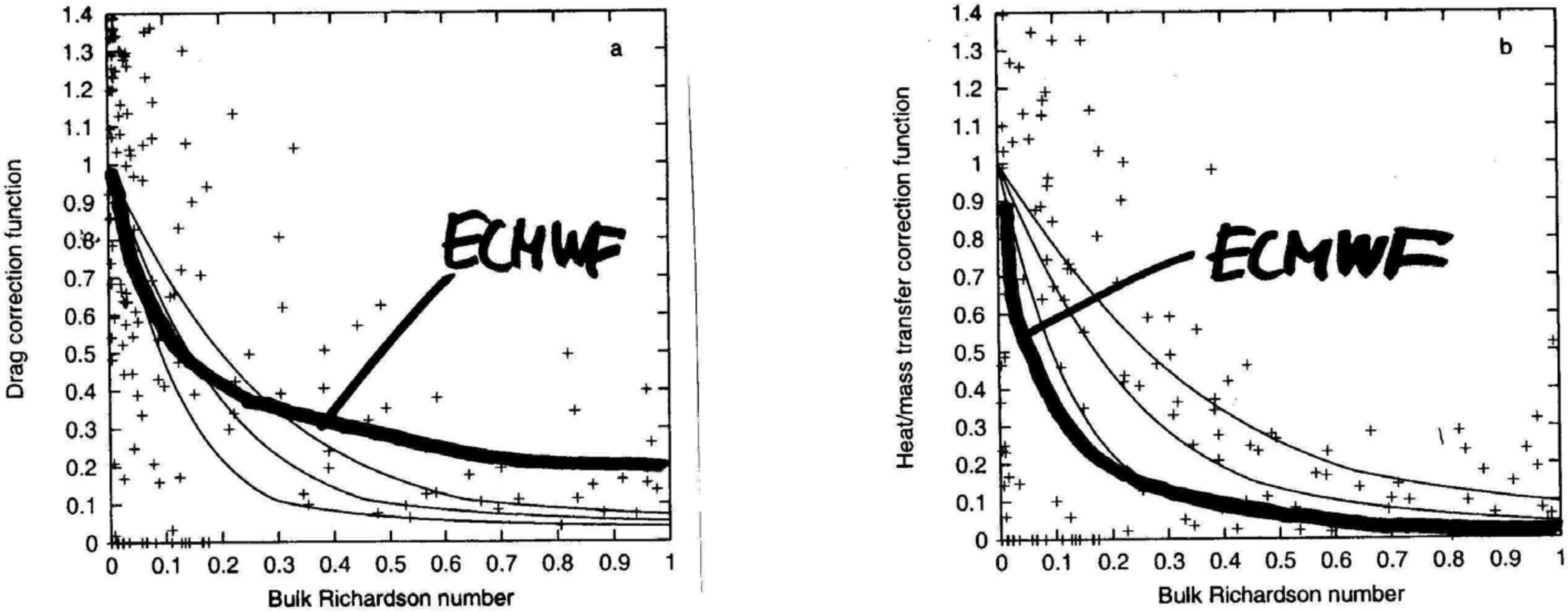

Figure 2. The same as in Fig. 1, but for Sodankyla, Arctic Finland: (a) f_D and (b) $f_H = f_M$. Crosses are measurements at this site.

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Stable stratification: revised theory

Zilitinkevich and Esau (2005)→ two additional length scales besides *L*:

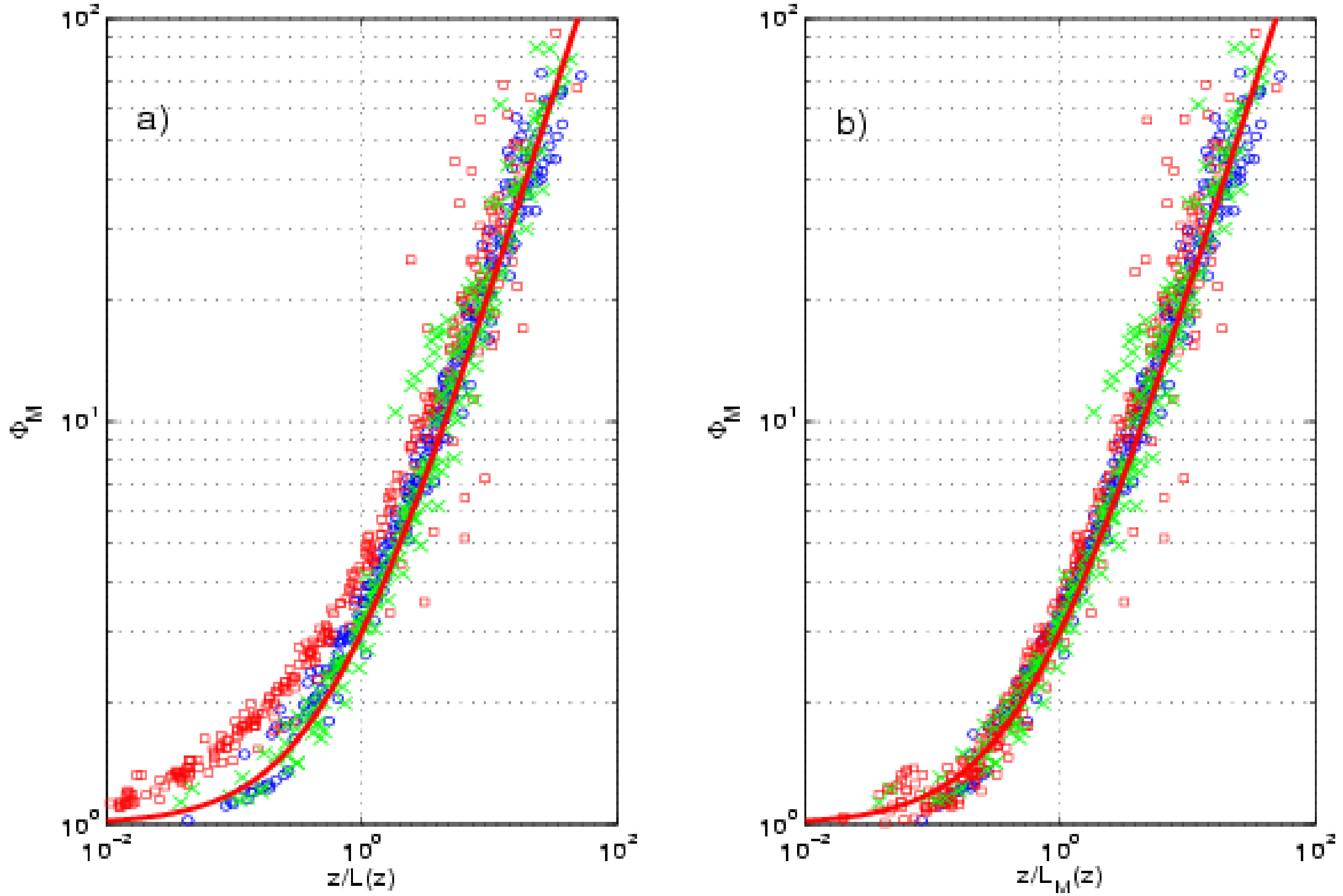
$$L_N = \frac{\tau^{1/2}}{N}$$ non-local effect of the free flow static stability

$$L_f = \frac{\tau^{1/2}}{|f|}$$ the effect of the Earth's rotation

N is the Brunt-Väisälä frequency at $z>h$ (N ~10^{-2} s^{-1}), f is the Coriolis parameter

Interpolation: $$\frac{1}{L_*} = \left[\left(\frac{1}{L}\right)^2 + \left(\frac{C_N}{L_N}\right)^2 + \left(\frac{C_f}{L_f}\right)^2\right]^{1/2}$$ where C_N=0.1 and C_f=1

$kz\tau^{-1/2}dU/dz$ vs. z/L (a), z/L_* (b) x nocturnal; o long-lived; □ conventionally neutral



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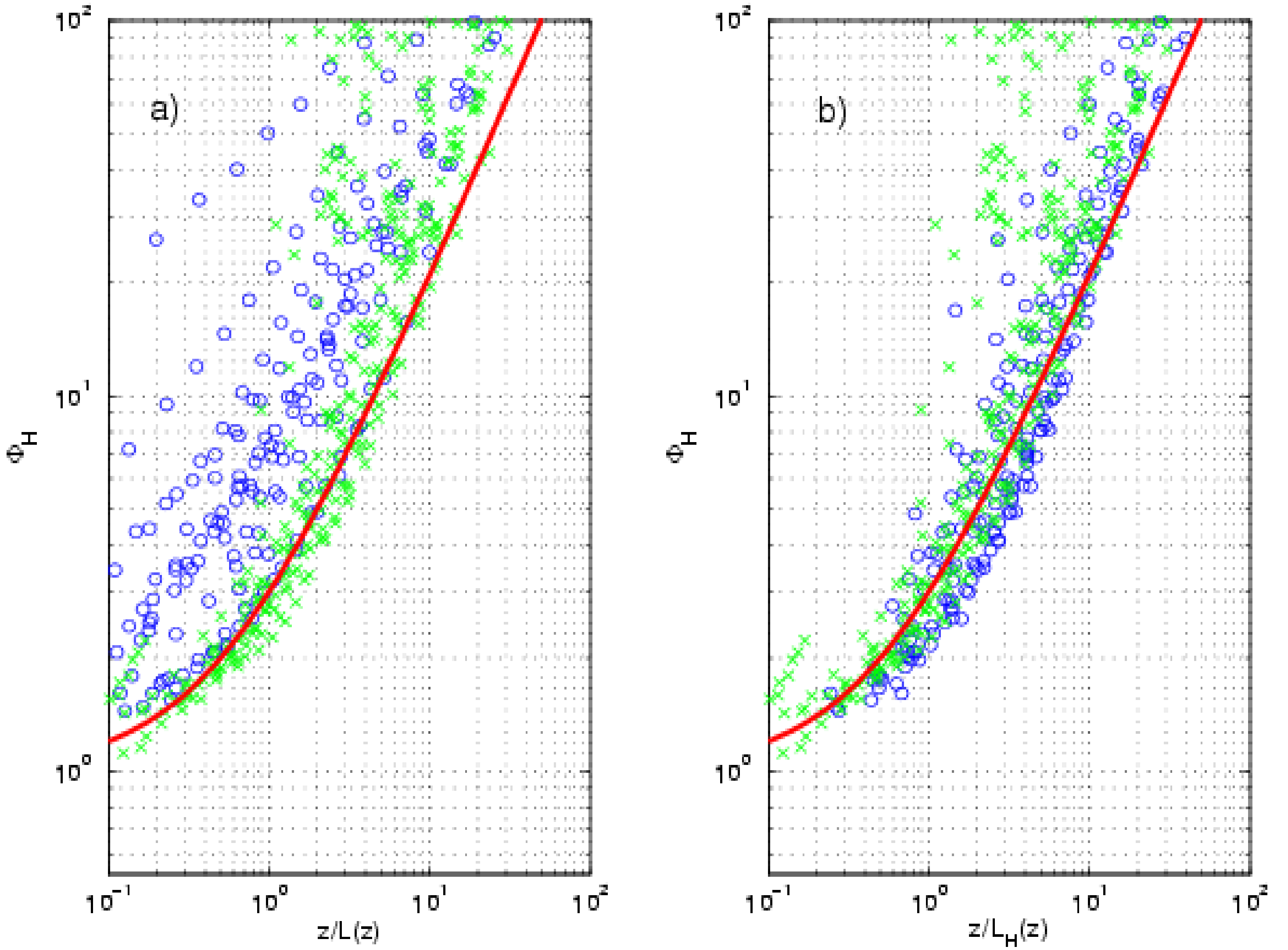
$\Phi_H = (k_T \tau^{1/2} z/F_\theta) d\Theta/dz$ vs. z/L (a), z/L_* (b) x nocturnal; o long-lived



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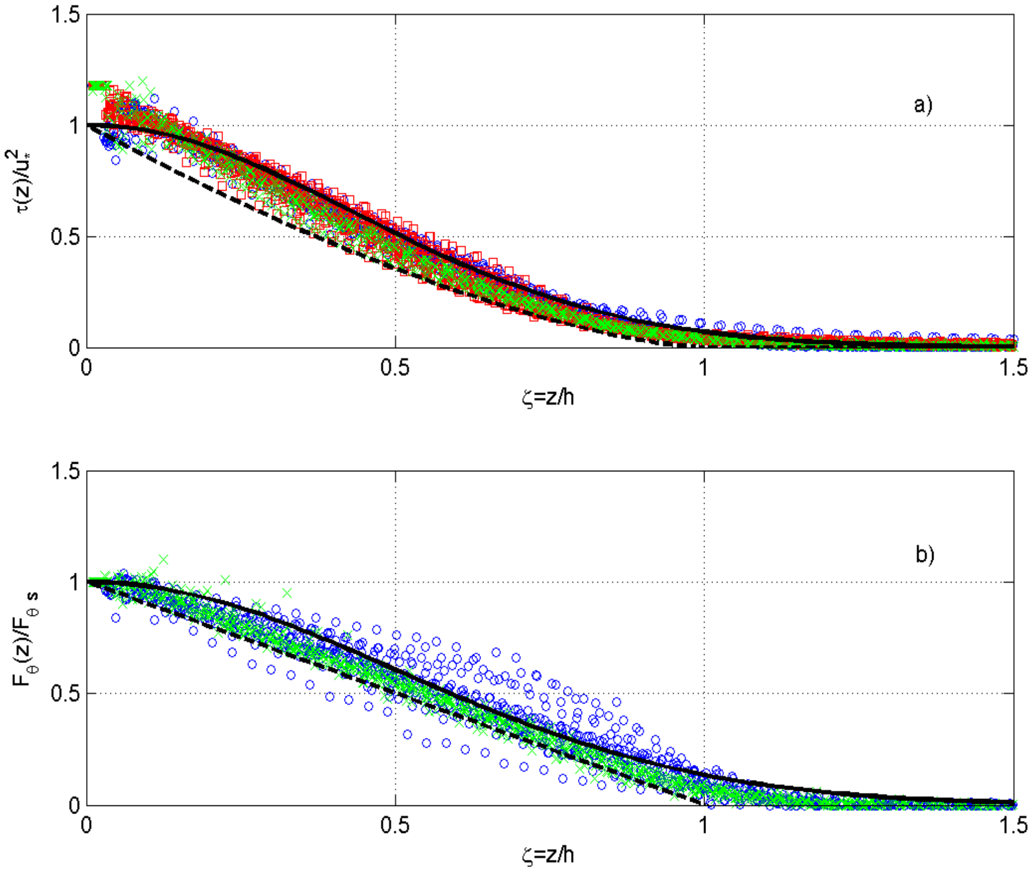
Vertical profiles of turbulent fluxes

LES turbulent fluxes: solid lines $\tau / u_*^2 = \exp(-\frac{8}{3}\varsigma^2)$, $F_\theta / F_{\theta s} = \exp(-2\varsigma^2)$

Approximation based on atmospheric data (e.g. Lenshow, 1988): dashed lines



New mean-gradient formulation (no critical Ri)

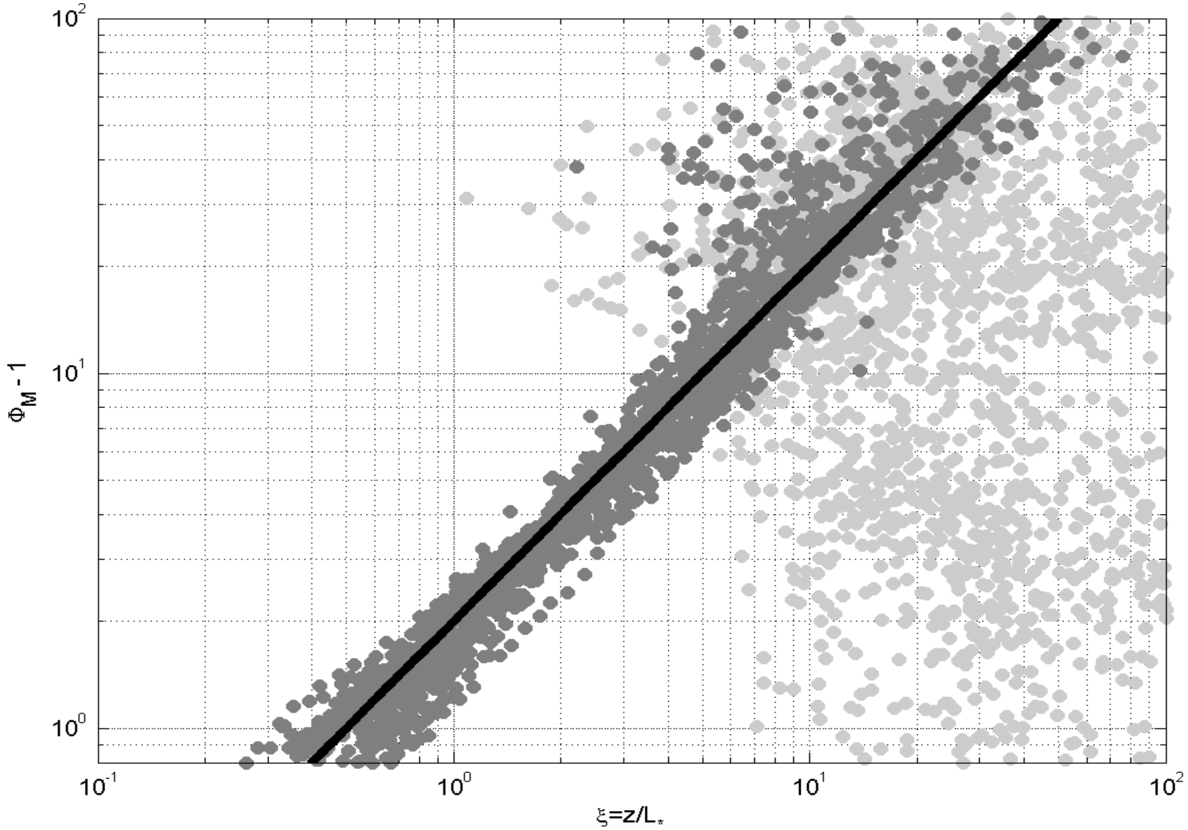
Flux Richardson number is limited: $\mathrm{Ri}_f = \frac{-\beta F_\theta}{\tau dU/dz}$ > $\mathrm{Ri}_\mathrm{f}^\infty \approx 0.2$

Hence asymptotically $\frac{dU}{dz} \to \frac{\tau^{1/2}}{\mathrm{Ri}_f^\infty L}$, and interpolating $\Phi_M = 1 + C_{U1}\xi$

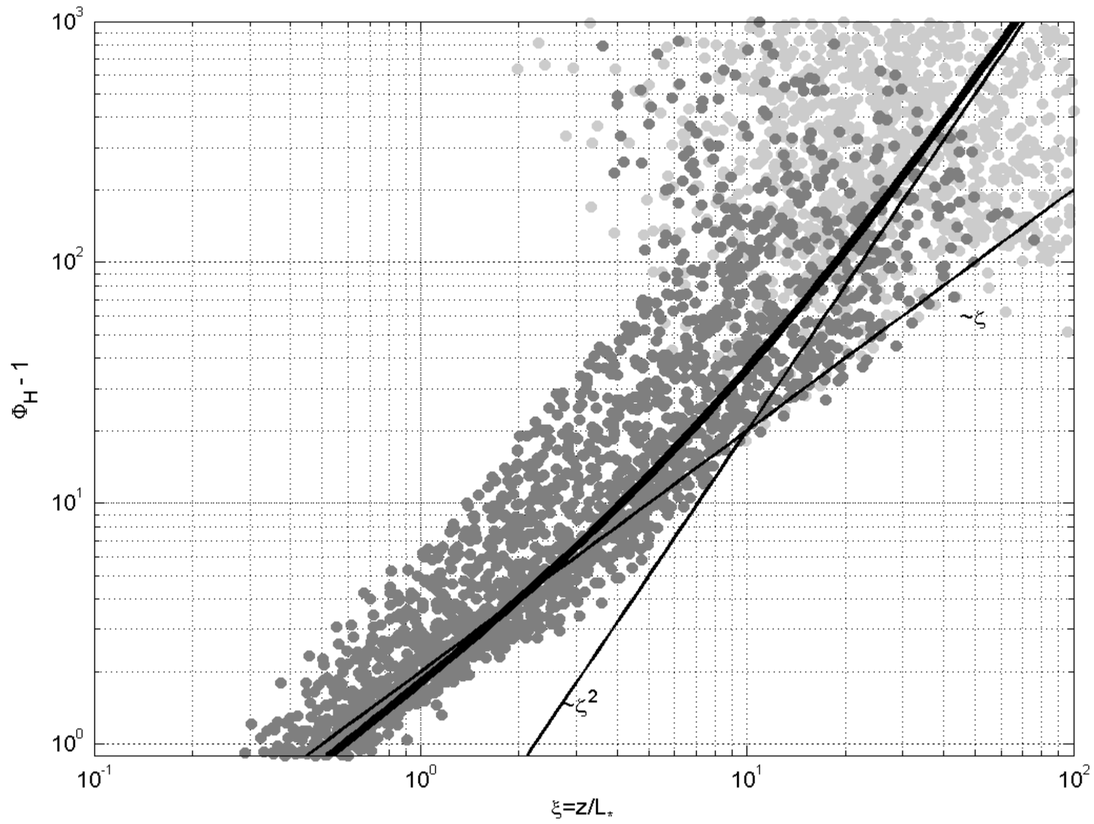
Gradient Richardson number becomes $\mathrm{Ri} \equiv \frac{\beta d\Theta/dz}{(dU/dz)^2} = \frac{k^2}{k_T}\frac{\xi\Phi_H(\xi)}{(1+C_{U1}\xi)^2}$

To assure no Ri-critical, ξ-dependence of Φ_H should be stronger then linear.

Including CN and LS ABLs: $\Phi_M = 1 + C_{U1}\frac{z}{L_*}$, $\Phi_H = 1 + C_{\Theta 1}\frac{z}{L_*} + C_{\Theta 2}\left(\frac{z}{L_*}\right)^2$



Φ_M vs. $\xi = z / L_*$, after LES DATABASE64 (all types of SBL). Dark grey points for *z*<*h*; light grey points for *z*>*h*; the line corresponds to $C_{U1} = 2$.



Φ_H vs. $\xi = z / L_*$ (all SBLs). Bold curve is our approximation: $C_{\Theta 1} = 1.8$, $C_{\Theta 2} = 0.2$; thin lines are $\Phi_H = 0.2\xi^2$ and traditional $\Phi_H = 1+2\xi$.

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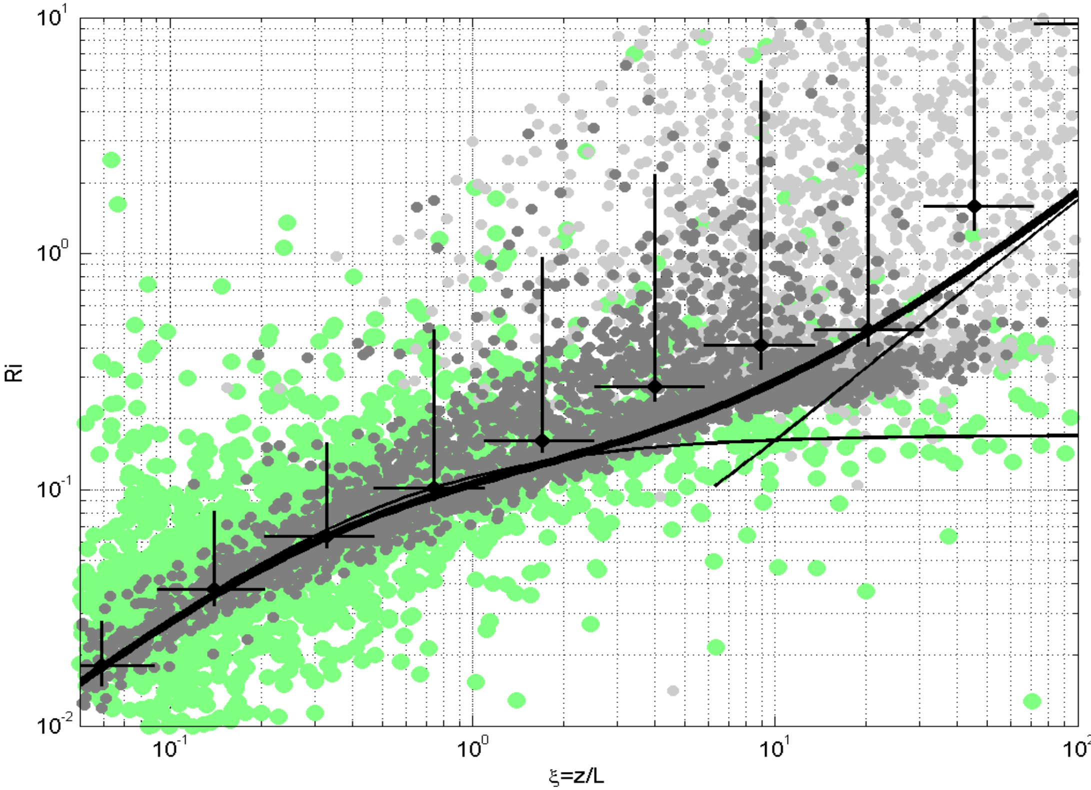


Ri vs. $\xi = z / L$, after LES and field data (SHEBA - green points). Bold curve is our model with C_{U1}=2, $C_{\Theta 1}$=1.6, $C_{\Theta 2}$=0.2. Thin curve is Φ_H=1+2ξ.

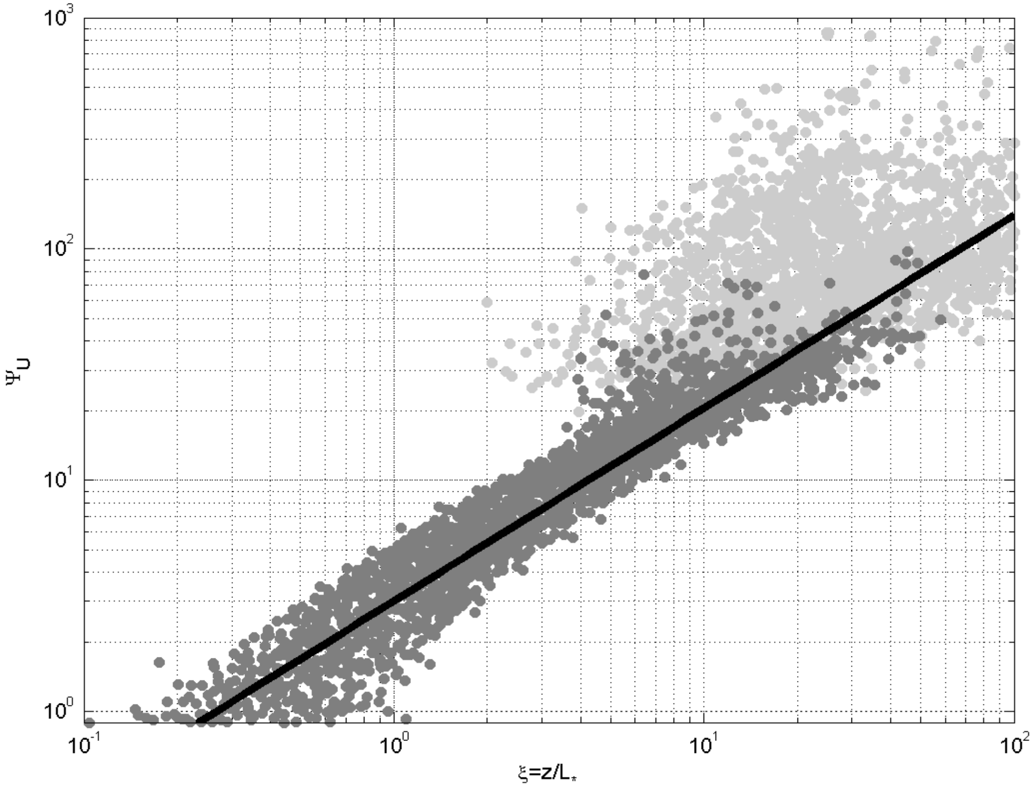
Mean profiles and flux-profile relationships

We consider wind/velocity and potential/temperature functions

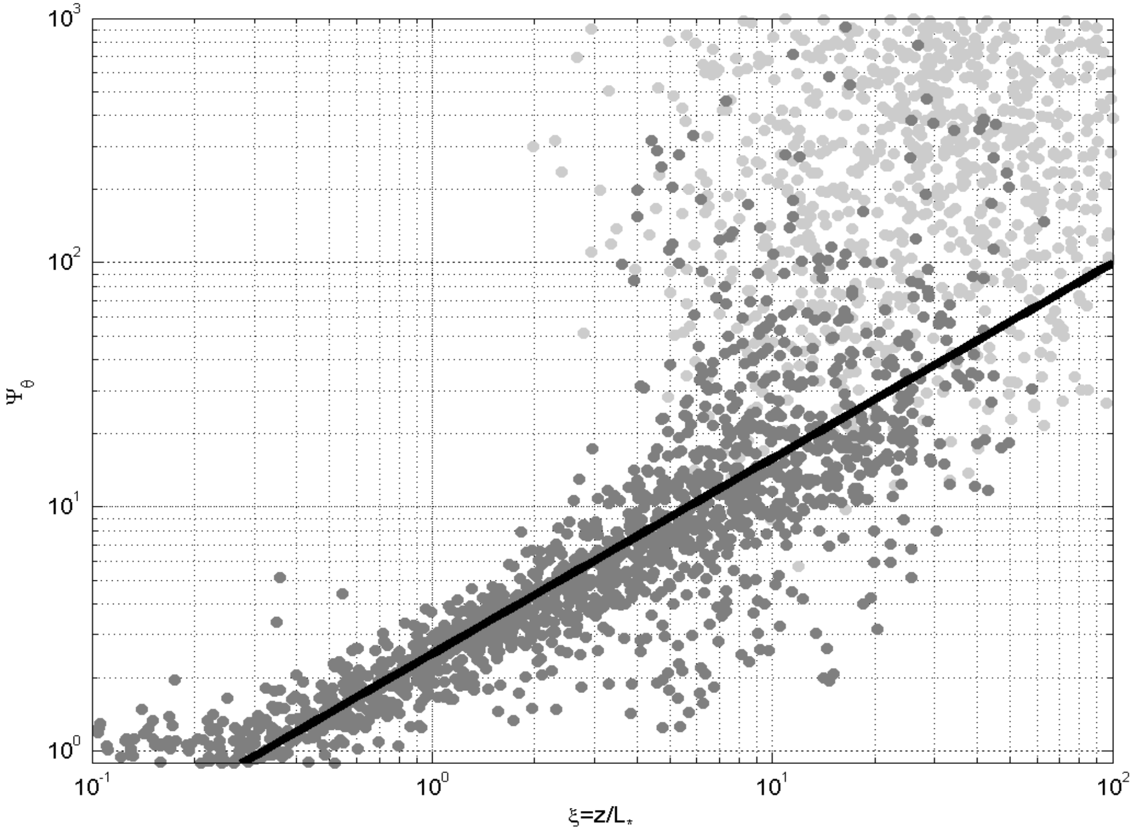
$$\Psi_U = \frac{kU(z)}{\tau^{1/2}} - \ln\frac{z}{z_{0u}} \quad \text{and} \quad \Psi_\Theta = \frac{k_T \tau^{1/2}\left[\Theta(z) - \Theta_0\right]}{-F_\theta} - \ln\frac{z}{z_{0u}}$$

Our analyses show that Ψ_U and Ψ_Θ are universal functions of $\xi = z / L_*$

$$\Psi_U = C_U \xi^{5/6}, \quad \Psi_\Theta = C_\Theta \xi^{4/5}, \text{ with } C_U\text{=3.0 and } C_\Theta\text{=2.5}$$



Wind-velocity function $\Psi_U = k\tau^{-1/2}U - \ln(z/z_{0u})$ vs. $\xi = z/L_*$, after LES DATABASE64 (all types of SBL). The line: $\Psi_U = C_U \xi^{5/6}$, C_U=3.0.



Pot.-temperature function $\Psi_\Theta = k\tau^{-1/2}(\Theta - \Theta_0)(-F_\theta)^{-1} - \ln(z/z_{0u})$

(all types of SBL). The line: $\Psi_\Theta = C_\Theta \xi^{4/5}$ with C_U=3.0 and C_Θ=2.5.

Analytical wind and temperature profiles (SBL)

$$\frac{kU}{\tau^{1/2}} = \ln\frac{z}{z_{0u}} + C_U\left(\frac{z}{L}\right)^{5/6}\left[1+\frac{(C_N N)^2+(C_f f)^2}{\tau}L^2\right]^{5/12}$$

$$\frac{k_T\tau^{1/2}(\Theta-\Theta_0)}{-F_\theta} = \ln\frac{z}{z_{0u}} + C_\Theta\left(\frac{z}{L}\right)^{4/5}\left[1+\frac{(C_N N)^2+(C_f f)^2}{\tau}L^2\right]^{2/5}$$

where C_N=0.1 and C_f=1. Given *U*(*z*), $\Theta(z)$ and *N*, these equations allow determining τ, F_θ, and $L=\tau^{3/2}(-\beta F_\theta)^{-1}$, at the computational level *z*.

Algorithm

Given τ, F_θ, surface fluxes are calculated using empirical dependencies

$$\frac{\tau}{\tau_*} = \exp\left[-\frac{8}{3}\left(\frac{z}{h}\right)^2\right], \quad \frac{F_\theta}{F_*} = \exp\left[-2\left(\frac{z}{h}\right)^2\right]$$ (Figures above)

The equilibrium ABL height, h_E, is determined diagnostically (Z. et al., 2006a):

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2 \tau_*} + \frac{N|f|}{C_{CN}^2 \tau_*} + \frac{|f \beta F_*|}{C_{NS}^2 \tau_*^2}$$ (C_R=0.6, C_{CN}=1.36, C_{NS}=0.51)

The actual ABL height, after prognostic equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U} \cdot \nabla h - w_h = K_h \nabla^2 h - C_t \frac{u_*}{h_E}(h - h_E)$$ (C_t = 1)

Given *h*, the free-flow Brunt-Väisälä frequency is $N^4 = \frac{1}{h} \int_h^{2h} \left(\beta \frac{\partial \Theta}{\partial z}\right)^2 dz$

Conclusions (mean profiles & surface fluxes)

Background: Generalised scaling accounting for the free-flow stability,
No critical Ri (TTE closure)
Stable ABL height model

Verified against
LES DATABASE64 (4 ABL types: TN, CN, NS and LS)
Data from the field campaign SHEBA

Deliverable 1: analytical wind & temperature profiles in SBLs

Deliverable 2: surface flux scheme for use in operational models

Requested: (i) roughness lengths and (ii) ABL height

STRATIFICATION EFFECT ON THE ROUGHNESS LENGTH

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Reference

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre, 2007: The roughness length in environmental fluid mechanics: the classical concept and the effect of stratification. Submitted to *Boundary-Layer Meteorology.*

Content

- Roughness length and displacement height:

$$u(z) = \frac{u_*}{k}\left[\ln\frac{z-d_{0u}}{z_{0u}} + \Psi_u\left(\frac{z}{L}\right)\right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics: neutral-stability z_0 = level, at which $u(z)$ plotted vs. $\ln z$ approaches zero; $z_0 \sim 1/25$ of typical height of roughness elements, h_0

- Meteorology / oceanography: h_0 comparable with MO length $L = \frac{u_*^3}{-\beta F_{\theta s}}$

- Stability dependence of the actual roughness length, z_{0u}:

  $z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification

Surface layer and roughness length

Self similarity in the surface layer (SL) $5h_0 < z < 10^{-1}h$

Height-constant fluxes: $\tau \approx \tau |_{z=5h_0} \equiv u_*^2$

u_* and z serve as turbulent scales: $u_T \sim u_*,\ l_T \sim z$

Eddy viscosity $(k \approx 0.4)$ K_M $(\sim u_T l_T) = k u_* z$

Velocity gradient $\partial U / \partial z = \tau / K_M = u_* / kz$

Integration constant: $U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$

z_{0u} (redefined constant of integration) is “roughness length”

“Displacement height” d_{0u} $U = k^{-1} u_* \ln\left[(z - d_{u0}) / z_{u0}\right]$

Not applied to the roughness layer (RL) $0 < z < 5h_0$

Parameters controlling z_{0u}

<u>Smooth surfaces:</u> viscous layer → $z_{0u} \sim \nu / u_*$

<u>Very rough surfaces:</u> pressure forces depend on:

obstacle height h_0

velocity in the roughness layer $U_R \sim u_*$

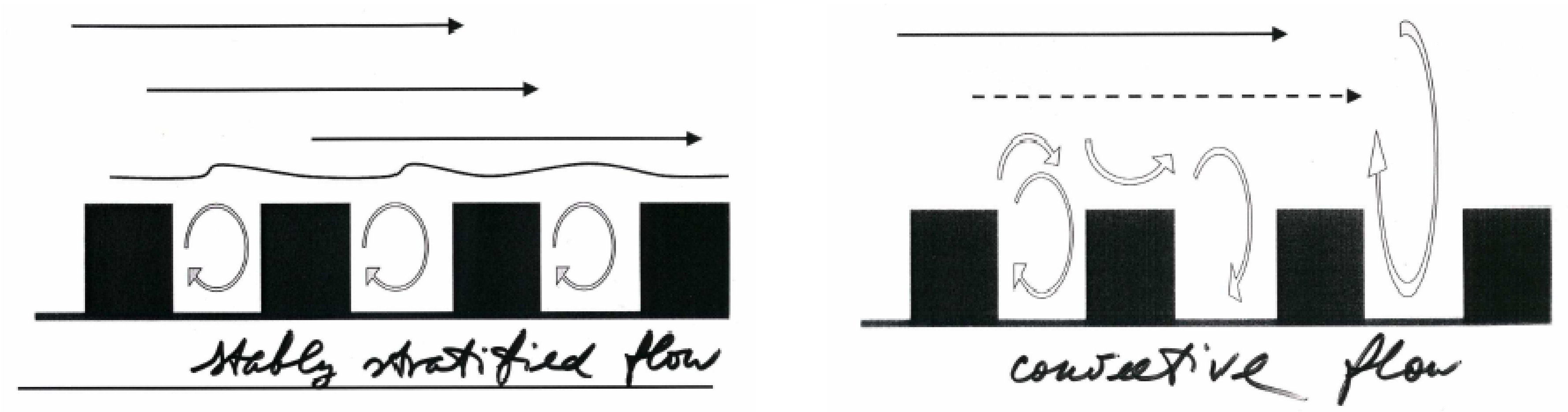
$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

No dependence on u_*; surfaces characterised by $z_{0u} =$ constant

<u>Generally</u> $z_{0u} = h_0 f_0(\mathrm{Re}_0)$ where $\mathrm{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0

Stability Dependence of Roughness Length



For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L, the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL $(0 < z < 5h_0)$
with 90% of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(\mathrm{RL}) = K_M(h_0 + 0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ classical formula $z_{0u} \sim h_0$

Stable: $K_M = k u_* z (1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = k u_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} z^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$

Recommended formulation

Neutral ⇔ stable $$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

Neutral ⇔ unstable $$\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$

Experimental datasets

Sodankyla Meteorological Observatory, Boreal forest (FMI)

h ≈ 13 m, measurement levels 23, 25, 47 m

BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

h ≈ 14.6 m, measurement levels 3.6, 11.3, 14.7, 17.9, 22.4, 31.7 m

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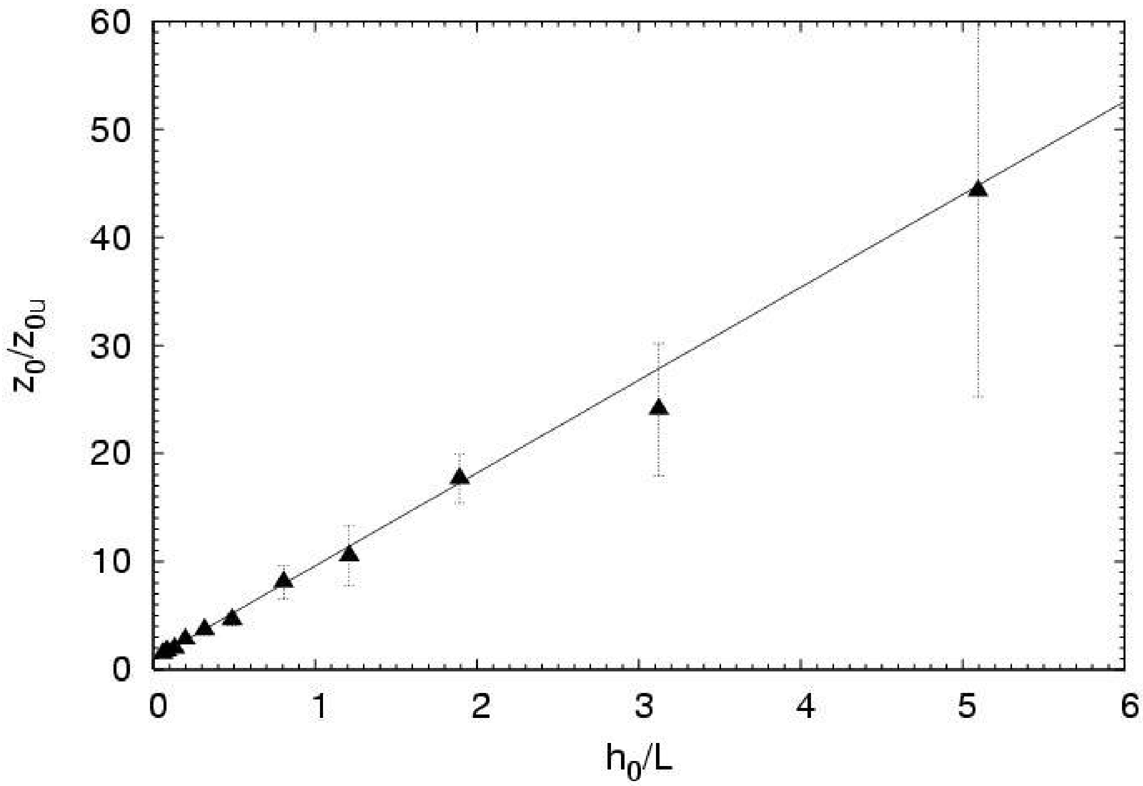
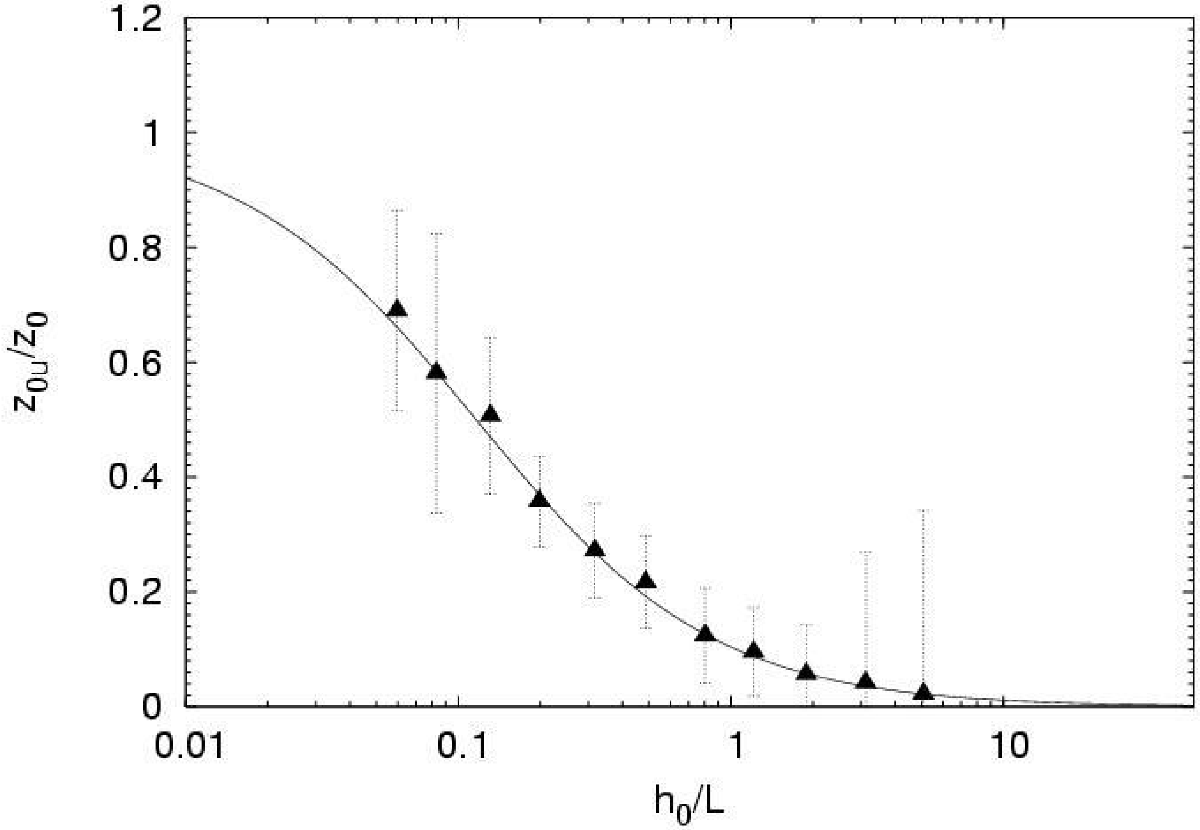
Stable stratification



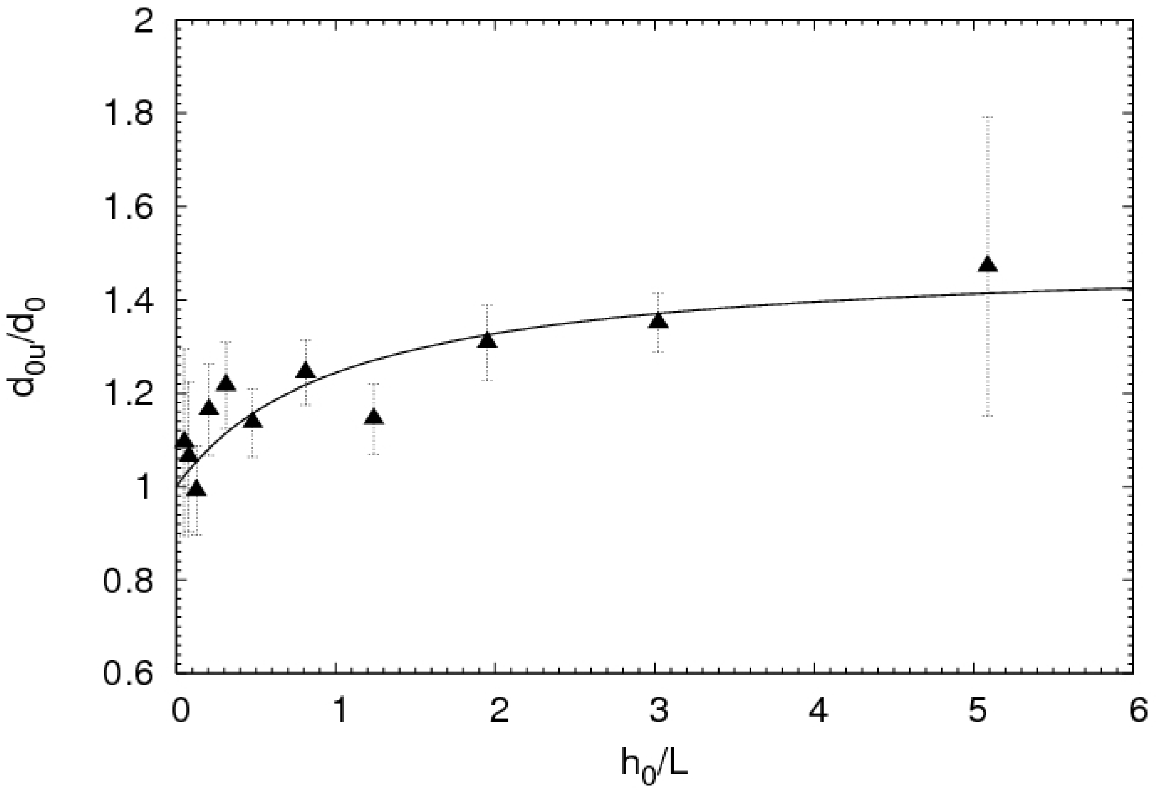
Bin-average values of z_0 / z_{0u} (neutral- over actual-roughness lengths) versus h_0/L in stable stratification for Boreal forest (h_0=13.5 m; z_0 =1.1±0.3 m). Bars are standard errors; the curve is $z_0 / z_{0u} = 1 + 8.13 h_0 / L$.

Stable stratification



Bin-average values of z_{0u}/z_0 (actual- over neutral-roughness lengths) versus h_0/L in stable stratification for boreal forest (h_0=13.5 m; z_0 =1.1±0.3 m). Bars are standard errors; the curve is $z_{0u}/z_0 = (1 + 8.13 h_0 / L)^{-1}$.

Stable stratification



Displacement height over its neutral-stability value in stable stratification. Boreal forest (h_0 = 15 m, d_0= 9.8 m).

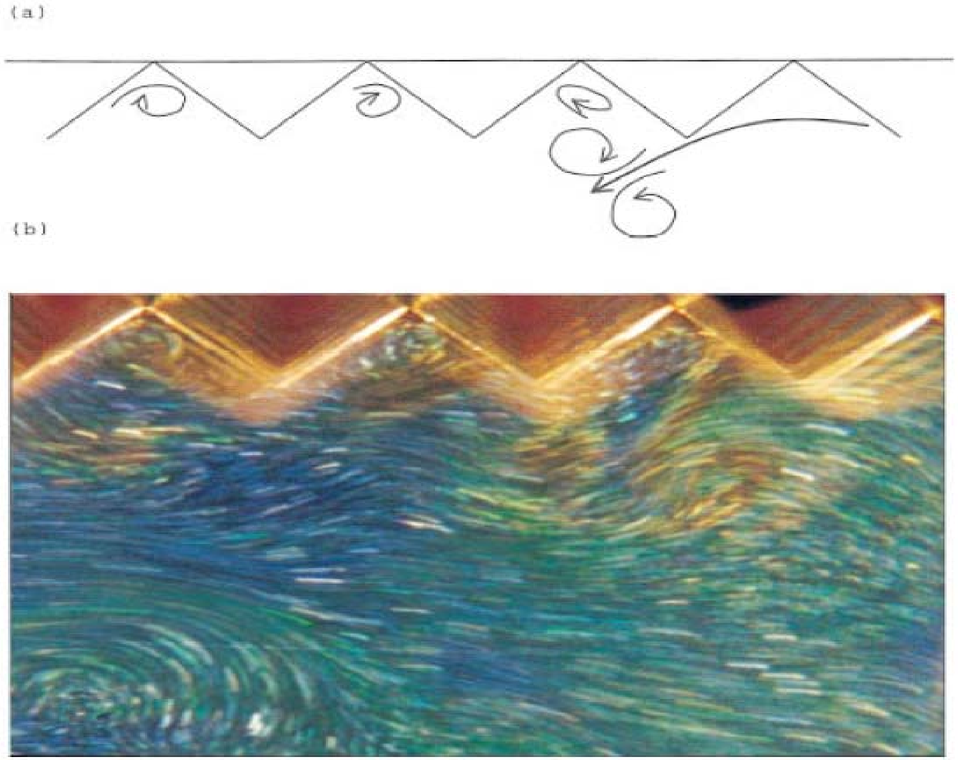
The curve is $$d_{0u}/d_0 = 1 + 0.5(h_0/L)(1.05 + h_0/L)^{-1}$$

Unstable stratification

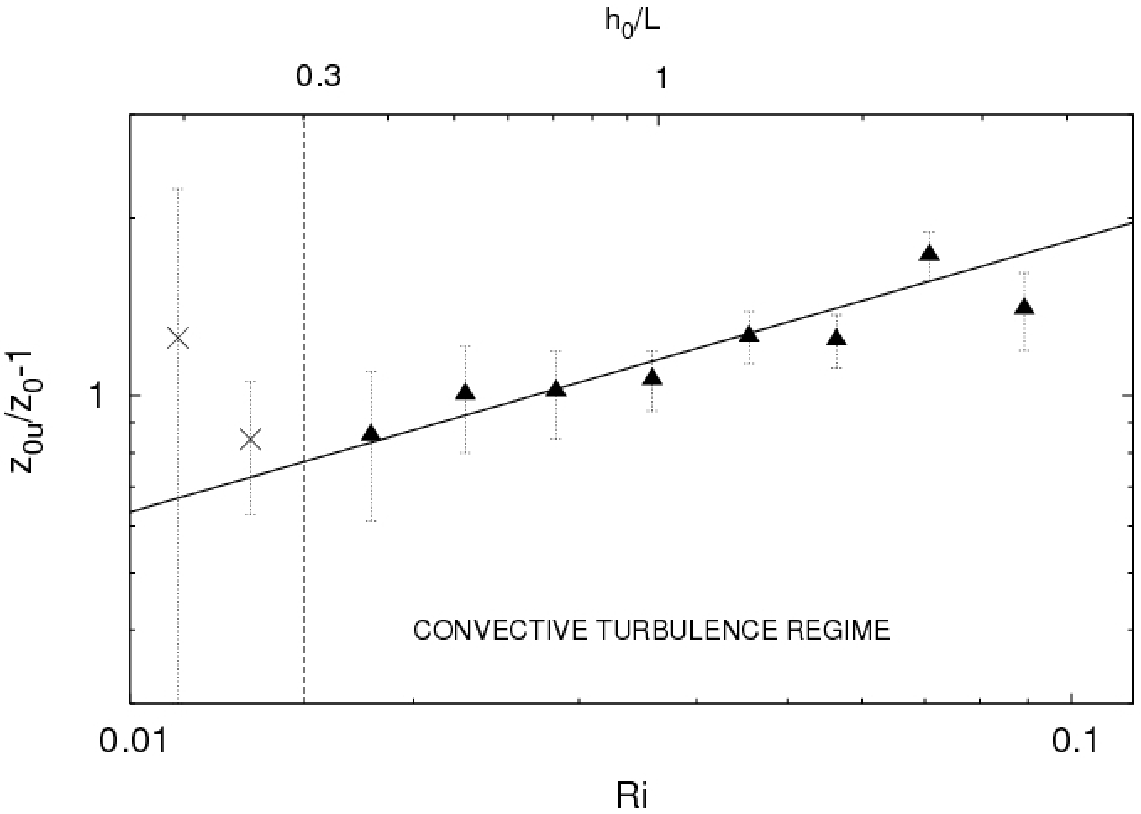
Convective eddies extend in the vertical causing $z_0 > z_{0u}$

VOLUME 81, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1998

Y.-B. Du and P. Tong, Enhanced Heat Transport in Turbulent Convection over a Rough Surface

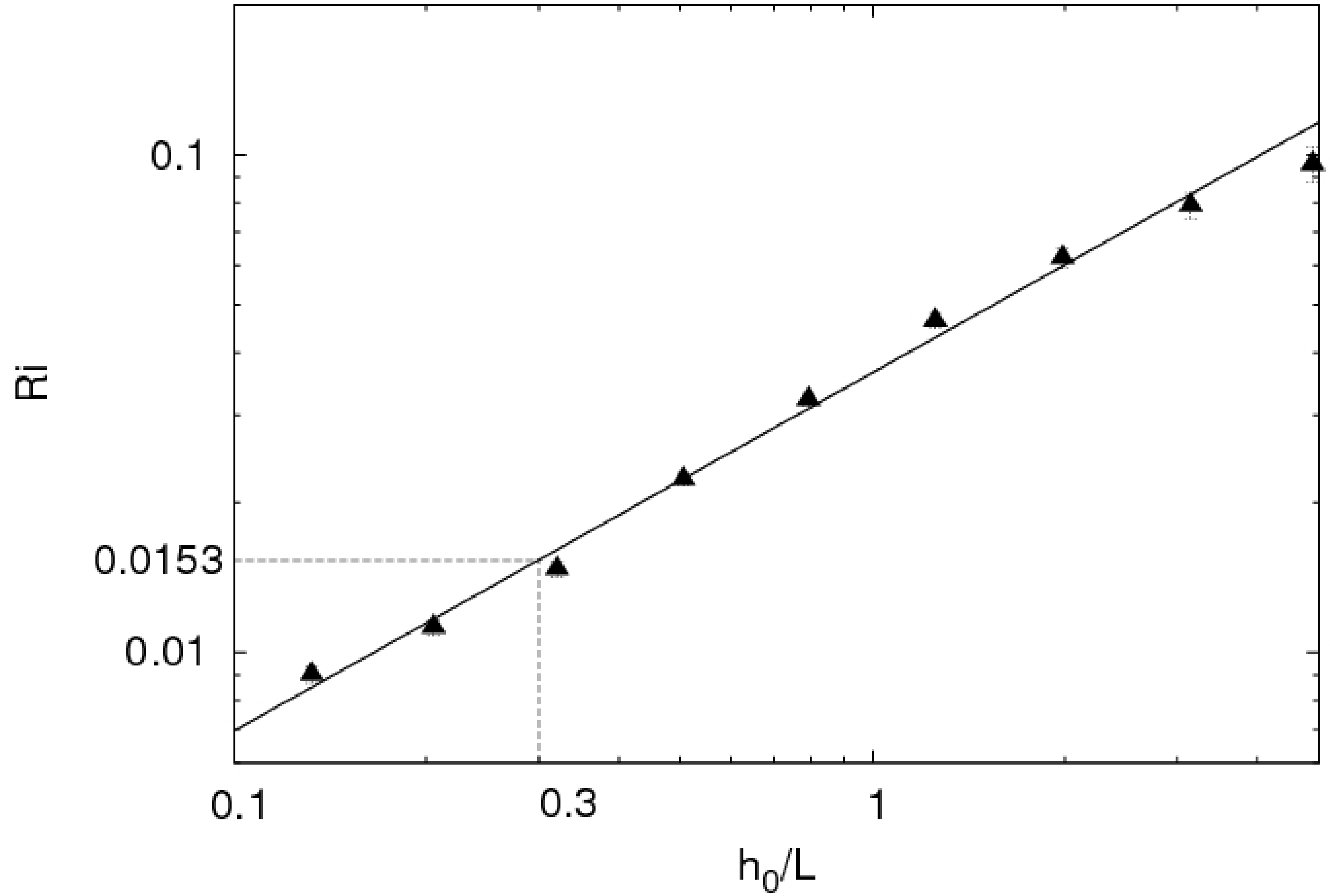


Unstable stratification



Unstable stratification, Basel, z_0/z_{0u} vs. Ri = $(gh_0/\Theta_{32})(\Theta_{18}-\Theta_{32})/(U_{32})^2$
Building height =14.6 m, neutral roughness z_0 =1.2 m; BUBBLE, Rotach et al., 2005).
h_0/L through empirical dependence on Ri on (next figure)
The curve ($z_0/z_{0u} = 1 + 5.31\mathrm{Ri}^{6/13}$) confirms theoretical $z_{0u}/z_0 = 1 + 1.15(h_0/\text{-}L)^{1/3}$

Unstable stratification



Empirical $\mathrm{Ri} = 0.0365\,(h_0/-L)^{13/18}$

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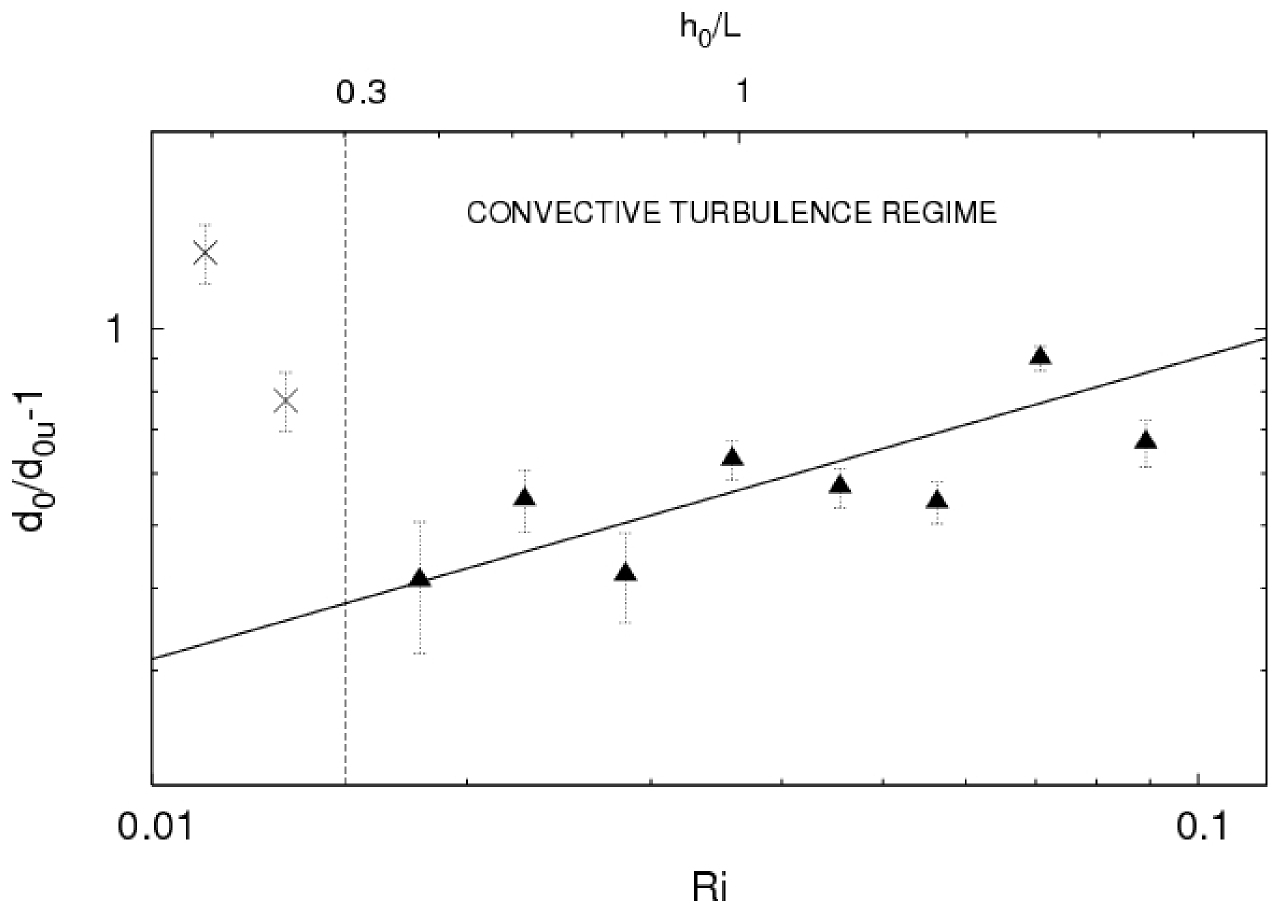
Unstable stratification



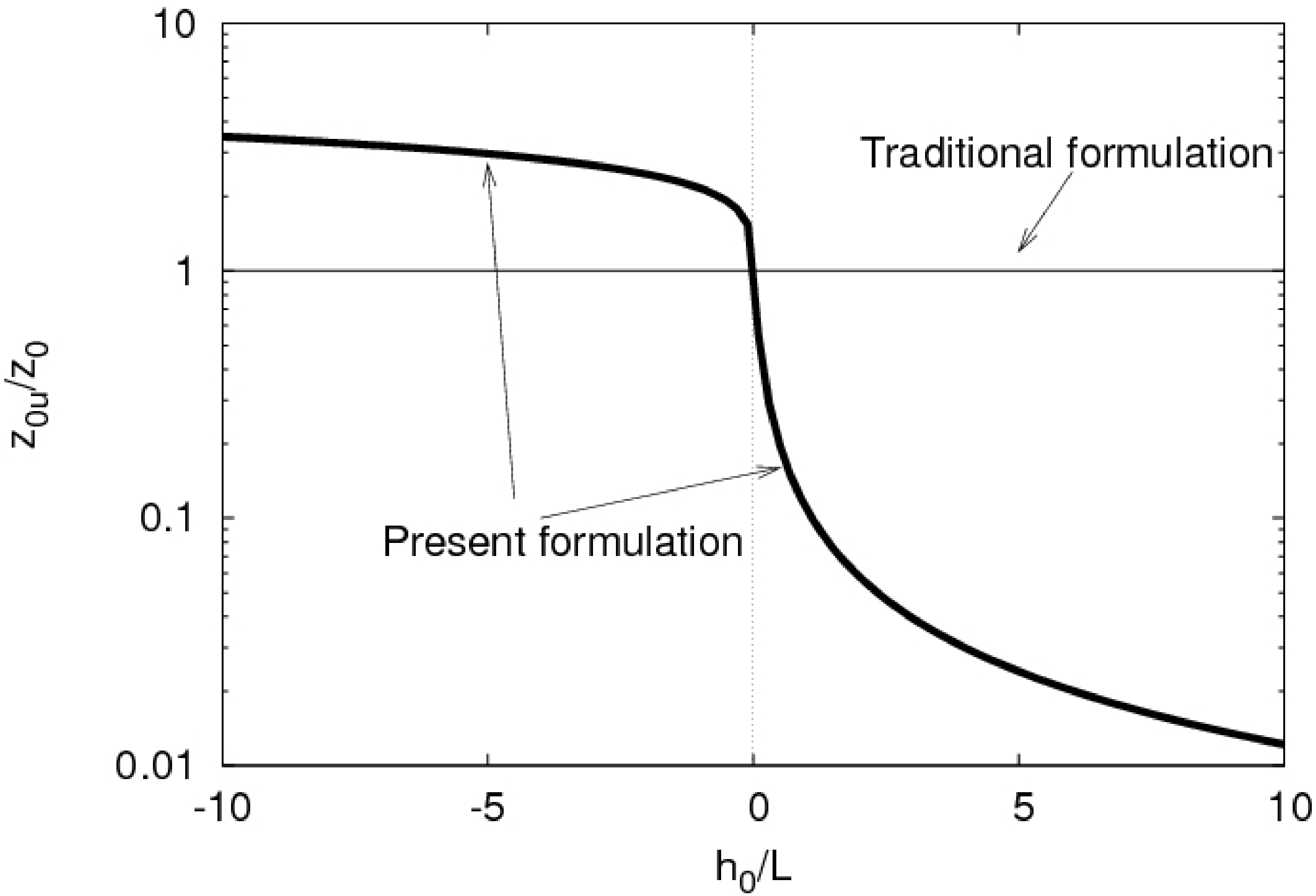
Displacement height in unstable stratification (Basel): $d_0 / d_{ou} - 1$ versus Ri

The line confirms theoretical dependence: $d_{0u} = \dfrac{d_0}{1 + C_{DC}(h_0 / -L)^{1/3}}$

STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the “meteorological interval” -10 < h_0/L <10 after new theory and experimental data

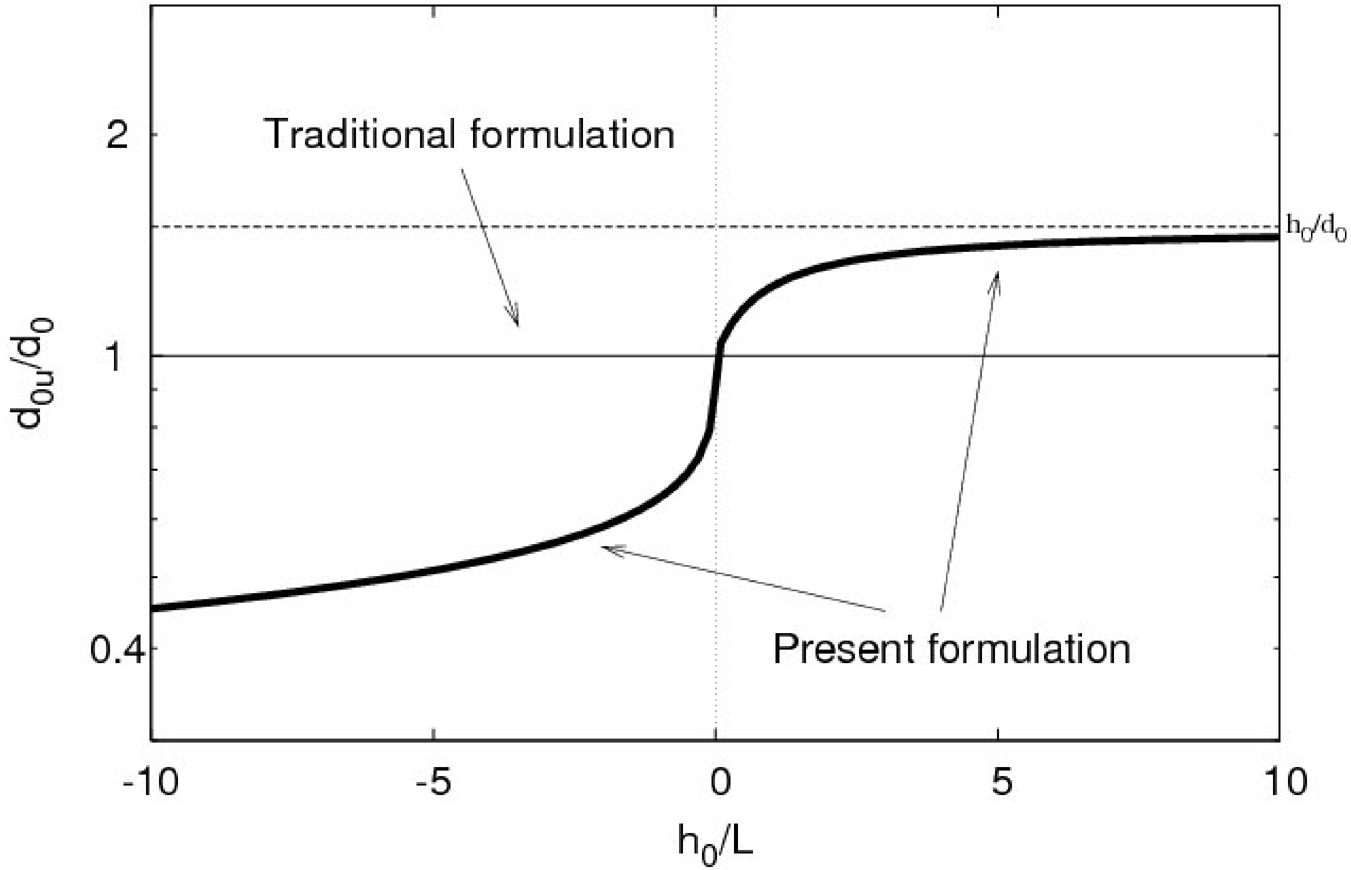
<u>Solid line</u>: z_{0u}/z_0 versus h_0/L <u>Thin line:</u> traditional formulation $z_{0u} = z_0$



STABILITY DEPENDENCE OF THE DISPLACEMENT HEIGHT

in the "meteorological interval" -10 < h_0/L <10 after new theory and experimental data

Solid line: d_{0u}/d_0 versus h_0/L Dashed line: the upper limit: $d_0 = h_0$



Conclusions (Roughness length)

- Traditional concept: roughness length and displacement height fully characterised by geometric features of the surface

- New theory and data: essential dependence on hydrostatic stability especially strong in stable stratification

- Applications: to urban and terrestrial-ecosystem meteorology

- Especially: urban air pollution episodes in very stable stratification

NEUTRAL and STABLE ABL HEIGHT

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Factors controlling PBL height

Basic factors:

- Deepening due shear-generated turbulence
- Swallowing by earth's rotation and negative buoyancy forces:
  (i) flow-surface interaction, (ii) free-flow stability atmosphere

Additional factors:

- baroclinic shears (enhances deepening)
- large-scale vertical motions (both ways))
- temporal and horizontal variability

Strategy:

Basic regimes → theoretical models → general formulation

Scaling analysis

Ekman (1905): $h_E \sim \sqrt{K_M / |f|}$; K_M in three basic regimes:

$$h_E^2 \sim \frac{K_M}{|f|}, \qquad K_M \sim u_T l_T \sim \begin{cases} u_* h_E & \text{for TN} \\ u_* L_N & \text{for CN} \\ u_* L & \text{for NS} \end{cases}$$

$l_T \sim$ h_E in TN $\quad L_N = u_* N^{-1}$ in CN $\quad L = -u_*^3 F_{bs}^{-1}$ in NS

Basic formulations

$$h_E \sim \begin{cases} u_* |f|^{-1} \text{ Rossby, Montgomery (1935) TN} \\ u_* |fN|^{-1/2} \text{ Pollard et al. (1973) CN} \\ u_*^2 |fB_s|^{-1/2} \text{ Zilitinkevich (1972, 74) NS} \end{cases}$$

Dominant role of the smallest scale

$$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N\,|\,f\,|}{(C_{CN} u_*)^2} + \frac{|\,fB_s\,|}{(C_{NS} u_*^2)^2}, \qquad C_R, C_{CN}, C_{NS} = \text{constant}$$

Four parameters u_*, f, N, B_s; hence two dimensionless numbrers:

$$\mu = u_* \,|\, fL \,|^{-1} \text{ and } \mu_N = N \,/\, |\, f \,|$$

More generally, h_E dependents also on

- geostrophic shear $\Gamma = |\,\partial \mathbf{u}_g / \partial z\,|$ (increases h_E: Z & Esau, 2003)
- vertical velocity w_h ($\pm w_h t_{PBL}$, $t_{PBL} \sim h_E / u_*$: Z & Baklanov, 2002).

Hence, additional (usually unavailable) parameters:

$$\mu_\Gamma = \Gamma / N \quad \text{and} \quad \mu_w = w_h / u_*$$

How to verify *h*-equations?

Stage I: TN $h_E = C_R u_* / f$ transitions TN → CN and TN → NS

$$\left(\frac{u_*}{f h_E}\right)^2 = \begin{cases} C_R^{-2} + C_{CN}^{-2}\mu_N & \text{TN - CN} \\ C_R^{-2} + C_{NS}^{-2}\mu & \text{TN - NS} \end{cases}$$

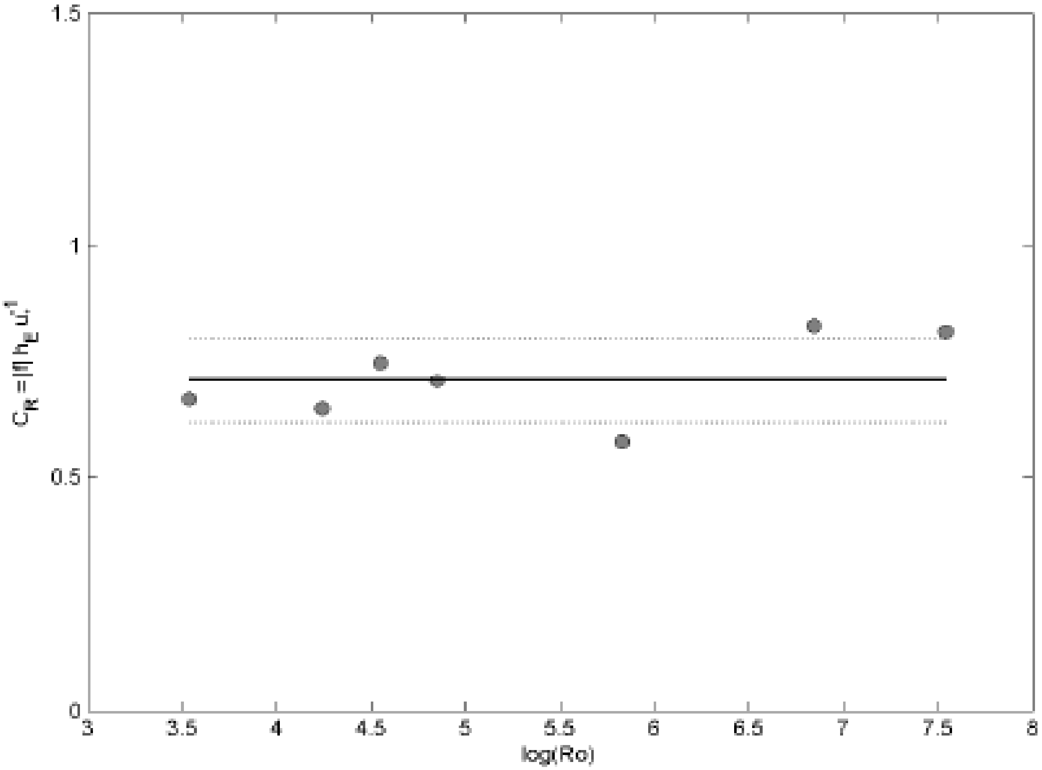
to determine constants C_R, C_{CN}, C_{NS} from selected high-quality data

---

Stage II: Substitute constants in $$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N|f|}{(C_{CN} u_*)^2} + \frac{|fB_s|}{(C_{NS} u_*^2)^2}$$

and verify against all available data

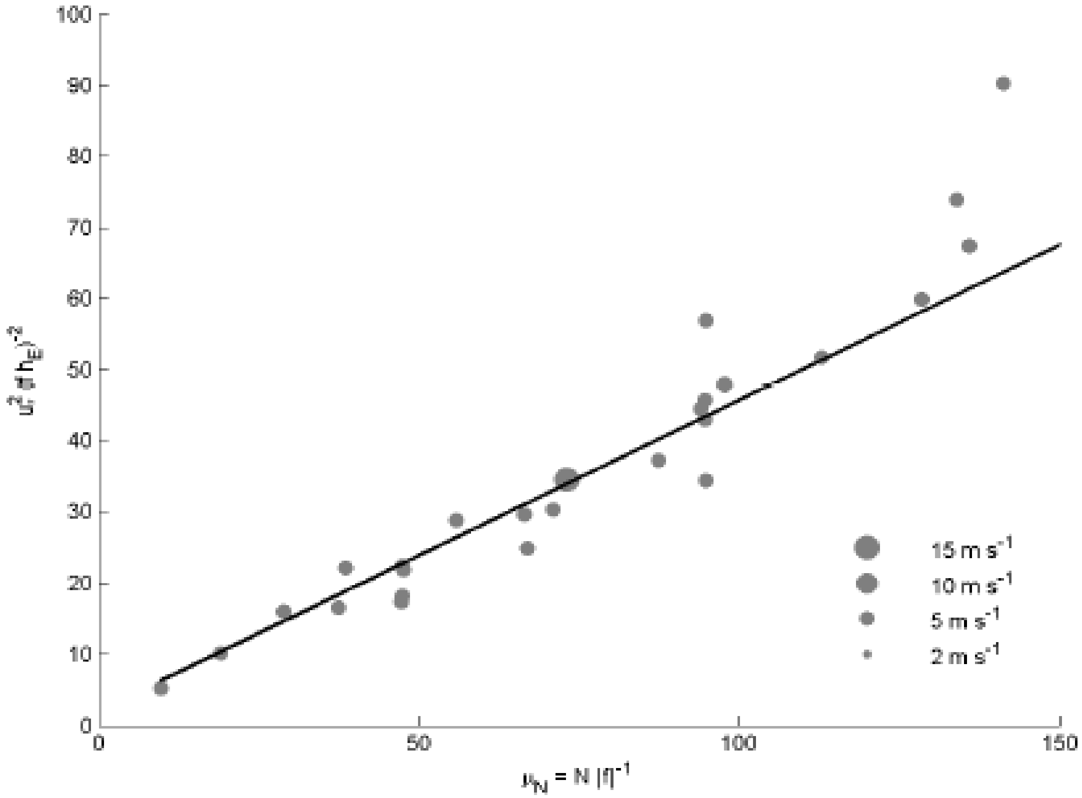
Stage I: Truly neutral ABL



Stage I **TN ABL**: C_R vs. Ro=$U_g(|f|z_{0u})^{-1}$ after LES

Bold line: C_R=0.7$\pm$0.1. Dotted line: standard deviation

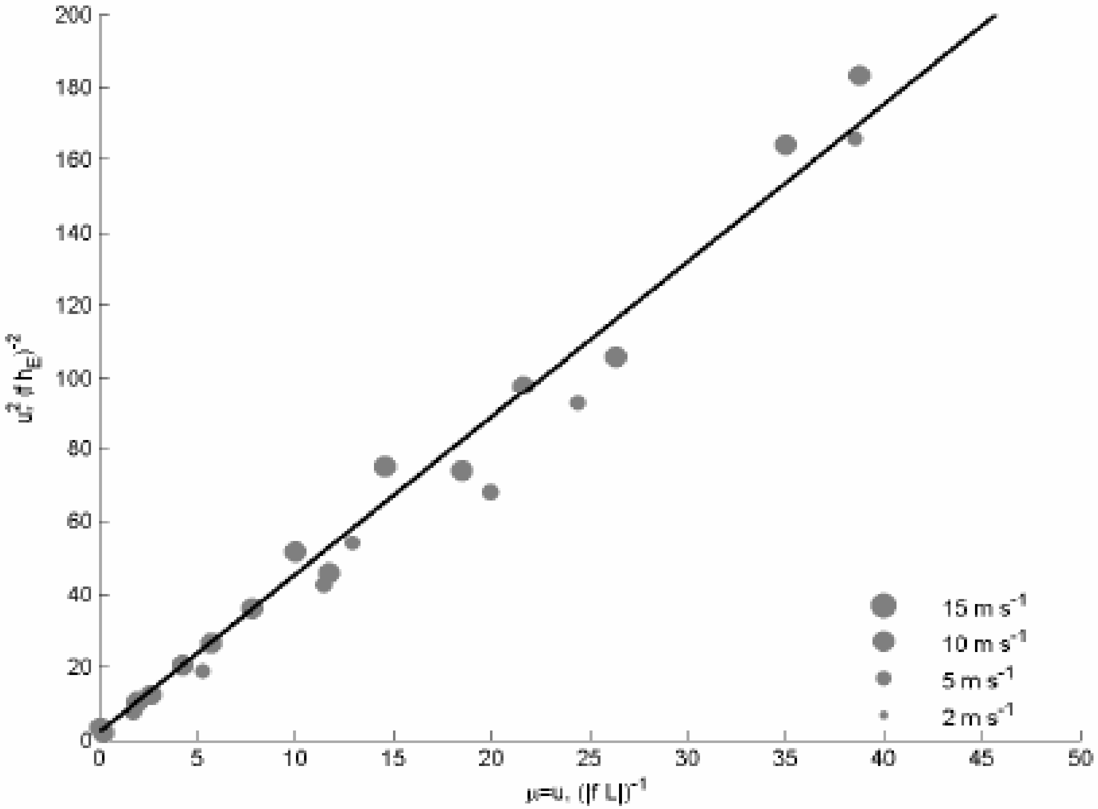
Stage I: Transition TN→CN ABL



Stage I **Transition TN• CN**: $u_*^2 (f h_E)^{-2}$ vs. $\mu_N = N / | f |$, after LES:

Theory: $u_*^2 (f h_E)^{-2} = C_R^{-2} + C_{CN}^{-2} \mu_N$. Empirical C_R=0.6, C_{CN}=1.36

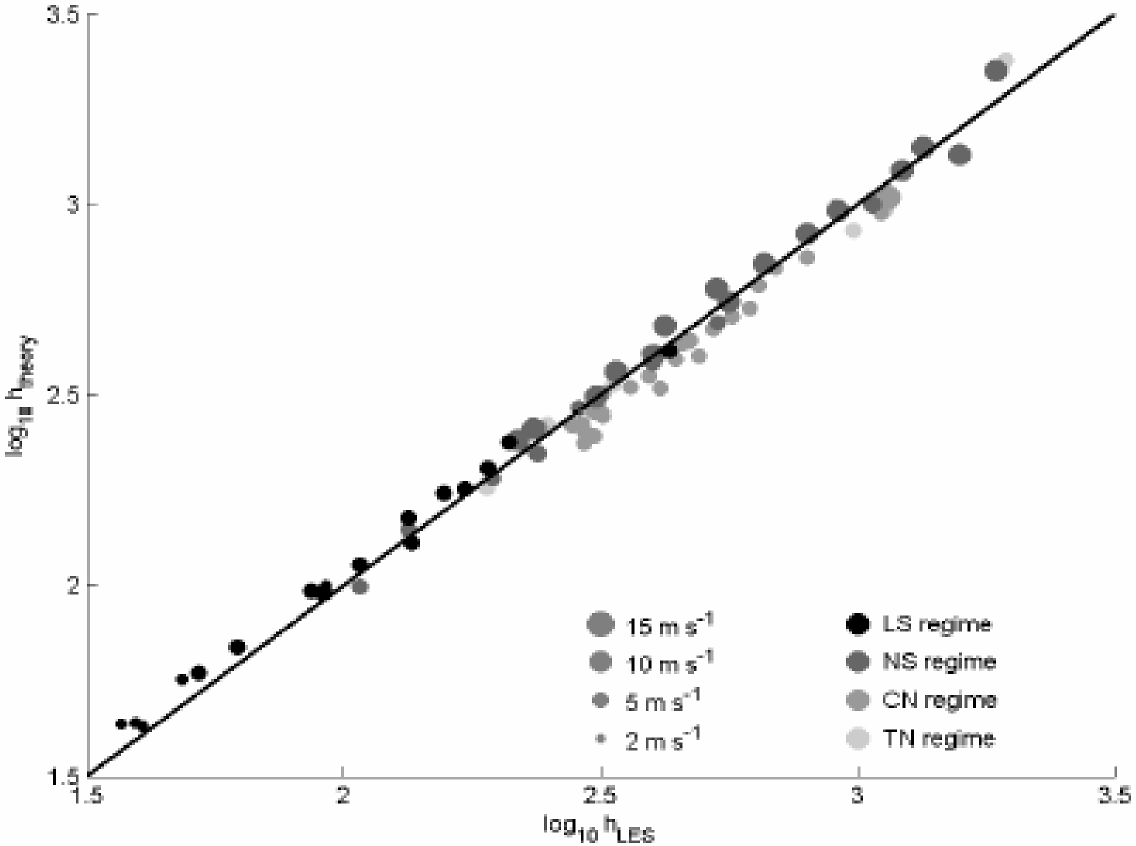
Stage I: Transition TN→NS ABL



Stage I **Transition TN •** **NS**: $u_*^2 (fh_E)^{-2}$ vs. $\mu = u_* \mid fL \mid^{-1}$, after LES.

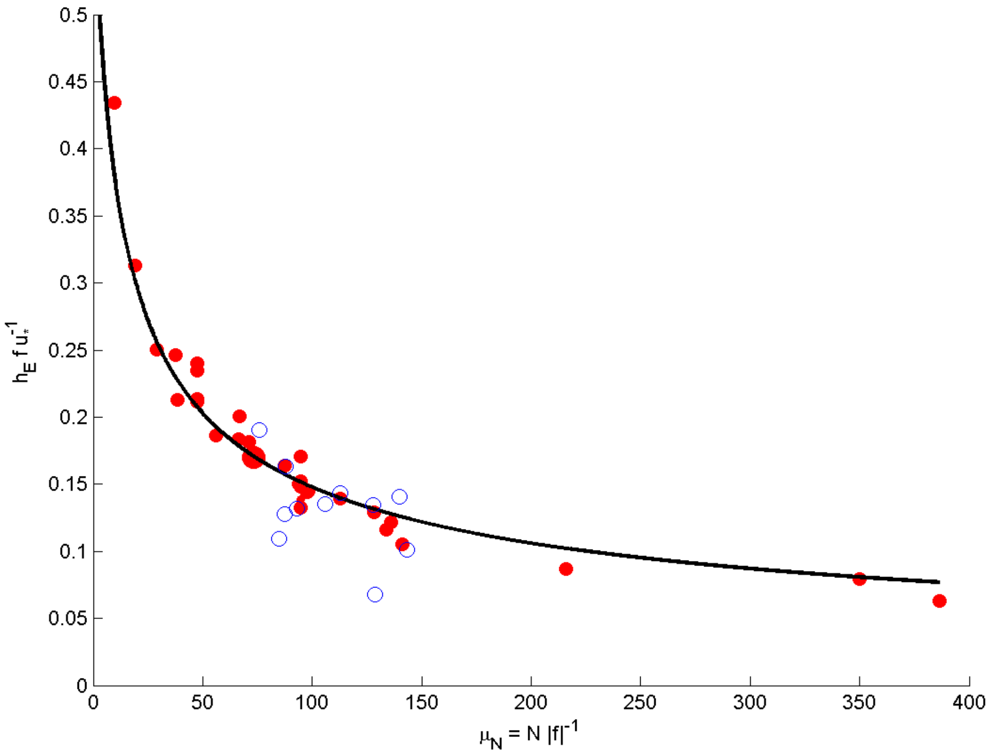
Theory: $u_*^2 (fh_E)^{-2} = C_R^{-2} + C_{NS}^{-2}\mu$, empirical C_R=0.6, C_{NS}=0.51

Stage II: General case



Stage II: **Correlation:** h_{theory} vs. h_{LES} after all available LES data

The height of the conventionally neutral (CN) ABL



Z & Esau, 2002, 2007: the effect of free-flow stability (N) on CN ABL height, h_E,, (LES – red; field data – blue; theory – curve). Traditional theory overlooks this dependence and overestimates h_E up to an order of magnitude.

Conclusions (SBL height)

- h_E, depends on many factors → multi-limit analysis / complex formulation
- difficult to measure: baroclinic shear (Γ), vertical velocity (w_h), h_E itself
- hence use LES, DNS and lab experiments
- baroclinic ABL: substitute $u_T = u_*(1 + C_0\Gamma/N)^{1/2}$ for u_* in the 2nd term of

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2 \tau_*} + \frac{N|f|}{C_{CN}^2 \tau_*} + \frac{|f\beta F_*|}{C_{NS}^2 \tau_*^2} \qquad (C_R=0.6,\ C_{CN}=1.36,\ C_{NS}=0.51)$$

- account for vertical motions: $h_{E-\mathrm{corr}} = h_E + w_h t_T$, where $t_T = C_t h_E / u_*$
- generally prognostic (relaxation) equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U} \cdot \nabla h - w_h = K_h \nabla^2 h - C_t \frac{u_*}{h_E}(h - h_E) \qquad (C_t = 1)$$

End

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