Compact turbulent scheme with 3 parameters:

Compact stability dependence model for turbulent schemes with prognostic TKE and without critical Richardson number: 3 parameter system of functions for the whole range of Richardson numbers

Ivan Bašták Ďurán*
Jean-François Geleyn**
Filip Váňa***

* CHMI Prague, ivanbastak@gmail.com
** CNRM Météo-France Toulouse
***ECMWF Reading

Parameterization of Stable Boundary Layer in NWP Models
Finnish Meteorological Insitute, Helsinki, December 3 - 5 2012

Turbulent scheme properties:

- prognostic TKE
- one stability parameter: gradient Richardson number Ri
- no critical gradient Richardson number Ri_{cr}
- valid for whole range of Richardson numbers (also unstable stratification)
- as compact as possible
- possible extension to TOMs

Prognostic TKE equation

$$
\begin{aligned}
\frac{\partial e}{\partial t} &= Adv(e) + \overbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}^{\text{TKE diffusion}} \\
&\quad \underbrace{+ K_m \left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}_{\text{shear}} \underbrace{- \frac{g}{\theta} K_h \frac{\partial \overline{\theta}}{\partial z}}_{\text{buoyancy}} \underbrace{- C_\epsilon \frac{(e)^{\frac{3}{2}}}{L}}_{\text{dissipation}}
\end{aligned}
$$

$$
K_M = L C_K \sqrt{e}\chi_3(\tau, S^2, N^2), \qquad K_H = L C_K C_3 \sqrt{e}\phi_3(\tau, S^2, N^2)
$$

$e = \frac{1}{2}(\overline{u' \cdot u' + v' \cdot v' + w' \cdot w'})$ =TKE, $K_{M/H}$ - exchange coefficients for momentum and heat, K_E- auto-diffusion coefficient for TKE, χ_3,ϕ_3 - stability functions,

C_K,C_ϵ - closure constants, C_3 - inverse Prandtl number at neutrality, L - mixing length, $S^2 = \left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]$, $N^2 = \frac{g}{\theta}\frac{\partial \overline{\theta}}{\partial z}$, $\tau = \frac{2L}{C_\epsilon \sqrt{e}}$ - TKE dissipation time scale

Three parameter system without Ri_{cr}:

$$\boxed{\begin{aligned} \chi_3 &= \frac{1 - \frac{Ri_f}{R}}{1 - Ri_f} \\ \phi_3 &= \frac{1 - \frac{Ri_f}{Ri_{fc}}}{1 - Ri_f} \\ \frac{Ri}{Ri_f} &= \frac{Ri_{fc}(R - Ri_f)}{C_3\, R\,(Ri_{fc} - Ri_f)} \end{aligned}}$$

$$Ri_f \equiv Ri\,\frac{C_3\phi_3}{\chi_3}$$

$$\& \quad \nu \equiv (C_K\, C_\epsilon)^{\frac{1}{4}} = \nu\,(C_3, R)$$

C_3 - inverse Prandtl number at neutrality

Ri_{fc} - critical flux Richardson number

R - parameter describing the effect of the flows anisotropy

Ri_f - flux Richardson number,
ν - closure constant [influences overall intensity of turbulence]
Ri_{cr} - critical gradient Richardson number

Diagnostic TKE equation -filter $(S^2, N^2) \rightarrow Ri$

$$
\begin{aligned}
0 &= K_H S^2 - K_H N^2 - C_\epsilon \frac{(e)^{\frac{3}{2}}}{L} \\
&\Downarrow \\
e &= \frac{C_K}{C_\epsilon} L^2 S^2 f,\ f = \left(\chi_3(\tau, S^2, N^2) - Ri\, C_3 \phi_3(\tau, S^2, N^2)\right) \\
&\Downarrow \ \text{filter} \qquad\qquad\qquad\qquad = \chi_3(\tau, S^2, N^2)\,(1 - Ri_f) \\
S^2 &= S^2(Ri, \tau),\ N^2 = N^2(Ri, \tau) \\
&\Downarrow \\
&\chi_3(Ri),\ \phi_3(Ri),\ f(Ri)
\end{aligned}
$$

$Ri = \frac{N^2}{S^2}$-gradient Richardson number, Ri_f -flux Richardson number

Critical gradient Richardson number Ri_{cr}

- suppression of turbulence:
  $e \to 0 \Rightarrow f(Ri) \to 0 \Rightarrow (\chi_3(Ri) \to 0 \vee Ri_f \to 1)$
- There is no Ri_{cr} but 'weak mixing turbulence' according to:
  - measurements
  - LES simulations
  - QNSE theory
  - EFB theory

CCH02 stability functions (with $\alpha_3 = 0$):

$$K_M = e\tau\, S_M^0(\lambda, F)\,\chi_3, \qquad \chi_3 = \chi_3\left(\tau, S^2, N^2, \lambda, F, O_\lambda\right)$$

$$K_H = e\tau\, S_M^0(\lambda, F)\, C_3(\lambda, F)\,\phi_3, \quad \phi_3 = \phi_3\left(\tau, S^2, N^2, \lambda, F, O_\lambda\right)$$

parameters:

- λ - influences time scale $\tau_{p,v} = \lambda\tau$ of return-to-isotropy part of the pressure correlation term for momentum flux
- F - affects value of mean shear–turbulence interactions part of the pressure correlation term for momentum flux
- $O_\lambda = 0.5\,\lambda_0 C_4$:
  - λ_0 affects value of buoyancy–turbulence interactions part of the pressure correlation term for heat flux
  - C_4 controls conversion between TPE and TKE

S_M^0, α_3 - constants

Turbulent Potential Energy (TPE)-EFB theory:

$$\cancel{\frac{d\overline{\theta'^2}}{dt} + \frac{\partial \overline{w'\theta'^2}}{\partial z}} = -2\overline{w'\theta'}\frac{\partial\overline{\theta}}{\partial z} - 2\epsilon_\theta$$

diagnostic $\Downarrow$

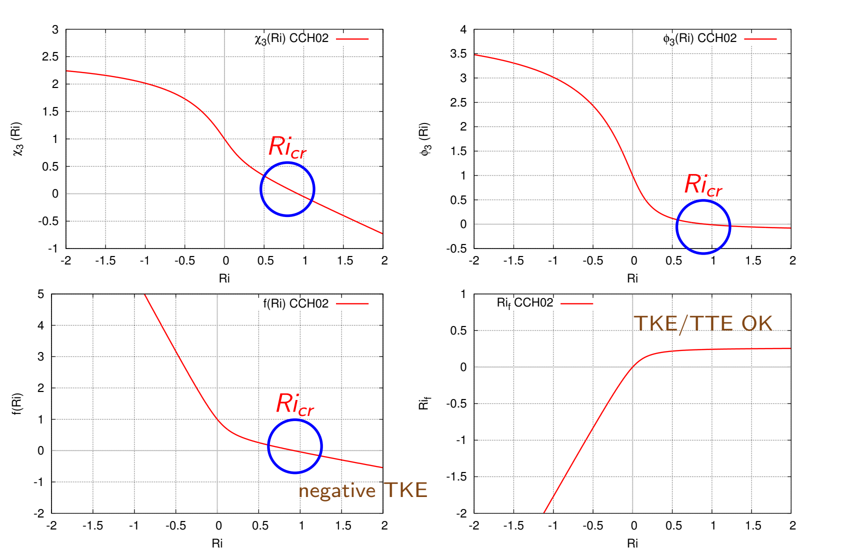
$$\underbrace{K_H N^2}_{\text{buoyancy}} = \underbrace{\frac{C_3\, C_\epsilon}{C_4}\frac{TPE\, e^{\frac{1}{2}}}{L}}_{\text{dissipation}}, \quad TPE = \frac{g}{\theta}\left(\frac{\partial\theta}{\partial z}\right)^{-1}\frac{\overline{\theta'^2}}{2}$$

Turbulent Total Energy (TTE): $TTE = e + TPE$

$$e = TTE\,\frac{1 - Ri_f}{1 - (1 - C_p)\, Ri_f}, \quad C_p = \frac{2\, C_3}{C_4}$$

C_p, C_4 - closure constants

CCH02 stability functions ($\alpha_3 = 0$):



Elimination of Ri_{cr}:

A system:

- damping of return-to-isotropy part of the pressure correlation term for heat flux (original CCH08 idea):

$$\lambda_5(Ri) = \lambda_5^0(1 + \sigma_t(Ri))$$

while $\sigma_t(Ri) \propto Ri$ for $Ri \to \infty$ in CCH02

$\Rightarrow$ faster decrease of ϕ_3 with increasing Ri

B system:

- modification in buoyancy–turbulence interactions part of pressure correlation term for momentum:

$$\lambda_4 = 0$$

$\Rightarrow$ disables direct influence of heat flux on momentum flux

$\Rightarrow$ $\chi_3 = \chi_3(\tau, S^2)$

σ_t- turbulent Prandtl number, λ_5^0 - constant λ_5 in original CCH02 system

Both A and B system lead to three parameter system:

$$\chi_3 = \frac{1 - \frac{Ri_f}{R}}{1 - Ri_f}$$

$$\phi_3 = \frac{1 - \frac{Ri_f}{Ri_{fc}}}{1 - Ri_f}$$

$$\frac{Ri}{Ri_f} = \frac{Ri_{fc}(R - Ri_f)}{C_3\, R\, (Ri_{fc} - Ri_f)}$$

$$\Rightarrow \quad f(Ri) = 1 - \frac{Ri_f}{R}$$

$$\& \quad C_K\, C_\epsilon = 2\, S_M^0\, (\lambda, F)$$

$C_3(\lambda, F)$ - inverse Prandtl number at neutrality

$Ri_{fc}(\lambda, F, O_\lambda)$ - critical flux Richardson number

$R(\lambda, F)$ - parameter describing the effect of the flows anisotropy

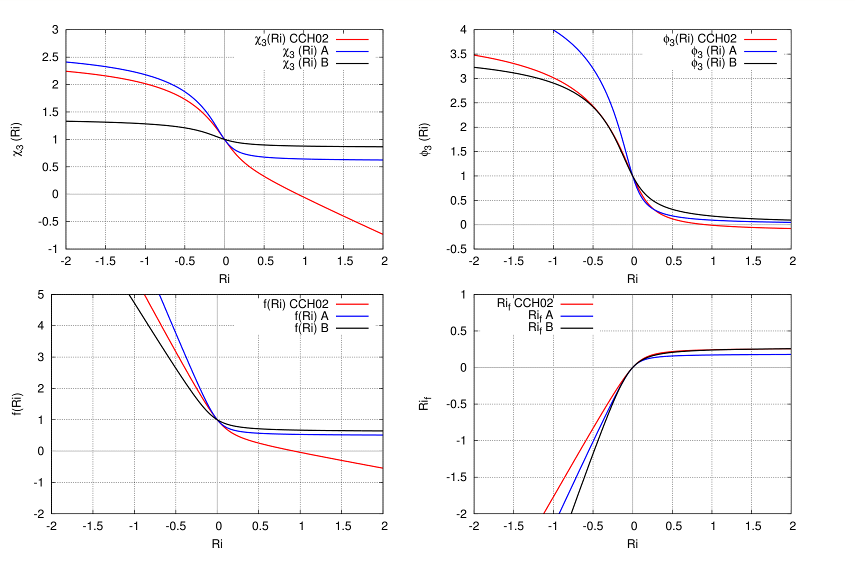
A and B system similitudes:

- in $S^0_M(\lambda, F)$
- in $C_3(\lambda, F)$
- relations for stability functions χ_3, ϕ_3 and Ri_f

A and B system differences:

- in $Ri_{fc}(\lambda, F, O_\lambda)$
- in $R(\lambda, F)$
- relations for 'remaining' fluxes: $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, $\overline{u'v'}$, $\overline{u'\theta'}$, $\overline{v'\theta'}$
- decompositon of ϕ_3 in to anisotropy and conversion part $\Rightarrow$ impact on TOMs parametrisation

Stability functions comparison:



QNSE scheme:

- QNSE=Quasi Normal Scale Elimination
- spectral analyses of the flow
- valid mainly for stable stratification ($Ri > 0$)
- no analytical form of stability functions - data points
- no Ri_{cr}

Fitted QNSE scheme:

$$\text{for } Ri \geq 0 \quad \chi_3(Ri) = \frac{1 + 0.75Ri(1 + a.Ri)}{1 + 3.23Ri(1 + a.Ri)} ,$$
$$\text{prolongation for } Ri < 0 \quad \chi_3(Ri) = \frac{1 - b.Ri}{1 - (b - 2.48).Ri} ,$$

$a = 13.0$, $b = 4.16$ - tuning constants

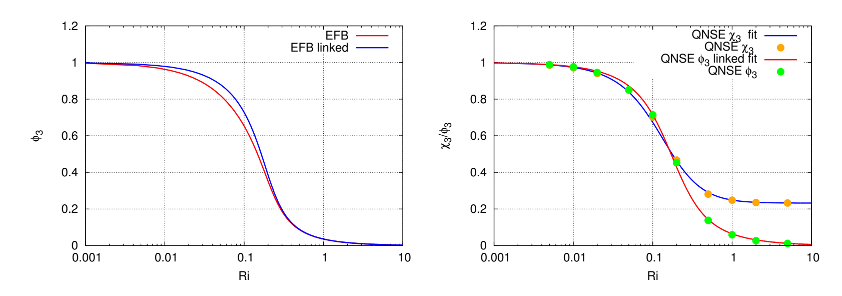
$\phi_3(Ri)$ computed from linking equation
derived in modified CCH02 (no R dependence):

$$C_3 Ri \phi_3(Ri)^2 - \left[\chi_3(Ri) + \frac{C_3 Ri}{Ri_{fc}}\right] \phi_3(Ri) + \chi_3(Ri) = 0$$

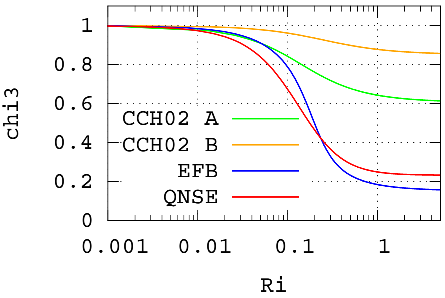
EFB scheme:

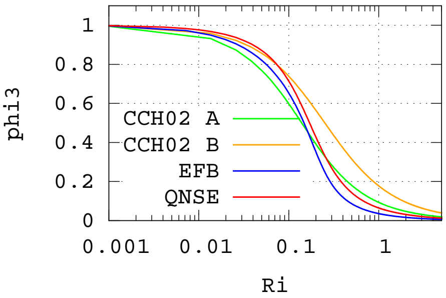
- EFB=Energy- and Flux-Budget
- Zilitinkevich et al. 2012
- based on budget equations for turbulence energy (kinetic and potential) and fluxes
- prognostic equation for time scale (resp. length scale)
- valid for stable stratification ($Ri > 0$)
- no Ri_{cr}

Linking relation:



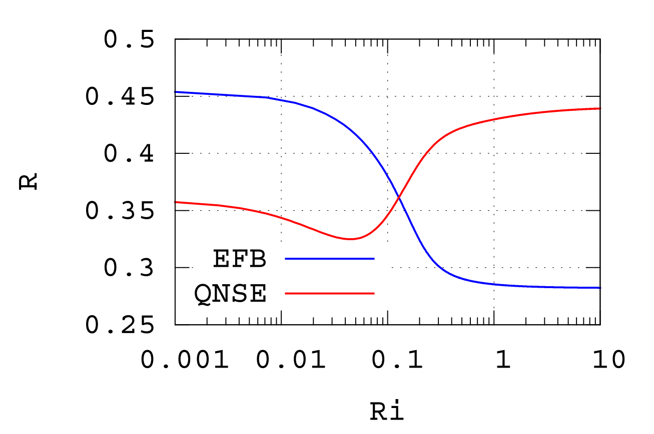
Stability functions comparison:



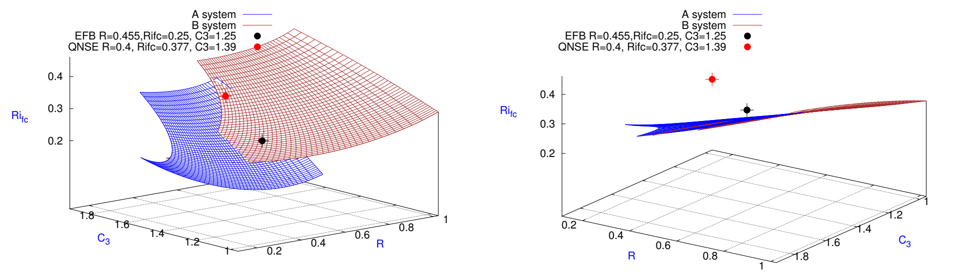


QNSE and EFB

- QNSE fit and EFB(MPM) would have non constant $R = \frac{Ri_f}{1-\chi_3(1-Ri_f)} \Rightarrow \frac{\partial R}{\partial Ri} \neq 0$



R-Ri_{fc}-C_3 space:



Summary

- by modification of CCH02 system we derived a scheme that is:
  - compact - 3 parameter - C_3, Ri_{fc}, R - system
  - has no critical gradient Richardson number Ri_{cr}
  - is valid for whole range of Ri
  - enables extension towards TOMs parametrisation
- 2 ways of CCH02 modifications, which lead to the same χ_3, ϕ_3 model: A and B system
- comparison with QNSE and EFB(MPM):
  - similar linking relation between χ_3 and ϕ_3
  - would have non-constant R

Thank you for your attention!