Linear model for investigation of nonlinear NWP model accuracy

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Introduction

A method for finding numerical solution of non-hydrostatic linear equations of atmospheric dynamics is introduced.

The general solution includes the stationary solution with optional initial conditions.

Horizontally homogeneous, but otherwise arbitrary reference state and arbitrary orography is implemented.

Generally, homogeneous temperature and wind profiles in vertical are used in linear models.
This is a restricting factor in investigation of NWP models, because the distribution of fields of real atmosphere is actually not constant.

The proposed method of ´exact´ numerical solution of linear model makes more complex atmospheric stratifications accessible.

Model description

Initial continuous equations: Linear, non-hydrostatic pressure-coordinate equations with filtered internal sound waves (Miller-Pearce-White model). When using the nondimensional log-pressure coordinate

$$\zeta = \ln(p_0/p), \quad \Rightarrow \quad p = p_0 e^{-\zeta},$$

the linearised version of this model is following:

$$\frac{d\omega}{dt} = -R\frac{p}{H^2}T + \frac{p}{H^2}\frac{\partial\varphi}{\partial\zeta} ,$$

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{U}}{\partial\zeta}\frac{\omega}{p} - \nabla\varphi - \mathbf{f} \times \mathbf{v} ,$$

$$\frac{dT}{dt} = \theta\frac{\omega}{p} ,$$

$$\nabla \cdot \mathbf{v} - \frac{\partial\omega}{p\partial\zeta} = 0 .$$

Dynamic fields are the omega-velocity ω = Dp / Dt, temperature fluctuation T, geopotential fluctuation φ, and horizontal velocity fluctuation **v.** Background state of the atmosphere is presented by stationary, horizontally homogeneous wind vector U, background temperature $\bar{T}$, gas constant of dry air R, and Coriolis parameter f.

Discretization and spectral presentation

For solution:

✔ Miller-Pearce-White model equations are spatially (Arakawa C-grid) and temporally discretised. 3D staggering with constant horizontal grid-step $\Delta x = \Delta y$ and variable vertical step Δp.
✔ Two time level, semi-implicit, semi-Lagrangian time scheme.

✔ The discrete equations are embedded into three-dimensional (x, y, t) discrete Fourier space:

$$\Psi^{n}_{ijk} = \sum_{qrs} \hat{\Psi}^{s}_{qrk} \mathrm{e}^{\mathbf{i}(\eta^{x}_{q} i + \eta^{y}_{r} j - d^{s}_{qrk} n)},$$

where

$$\Psi = \{T', u, v, \omega, \varphi\}$$

✔ Spectral presentations for averaging and difference operators on staggered grids are formulated, arriving finally at the discrete orthogonal mode equations for wind, temperature and nonhydrostatic geopotential fluctuations:

$$\mathrm{i}\nu^s_{qrk+1/2}\hat{\omega}^s_{qr,k+1/2} = -R\left(\frac{p}{H^2}\right)_{k+1/2}\hat{T}^s_{qrk+1/2} + \left(\frac{p}{H^2\Delta\zeta}\right)_{k+1/2}\Delta\hat{\varphi}^s_{qrk+1/2} ,$$

$$\mathrm{i}\nu^s_{qrk}\hat{u}^s_{qrk} = \frac{U'_k}{p_k}a^x_q\overline{\hat{\omega}^s}^{\zeta}_{qrk} - \mathrm{i}\mu^x_q\hat{\varphi}^s_{qrk} + fa^x_q(a^y_r)^*\hat{v}^s_{qrk},$$

$$\mathrm{i}\nu^s_{qrk}\hat{v}^s_{qrk} = \frac{V'_k}{p_k}a^y_r\overline{\hat{\omega}^s}^{\zeta}_{qrk} - \mathrm{i}\mu^y_r\hat{\varphi}^s_{qrk} - f(a^x_q)^*a^y_r\hat{u}^s_{qrk},$$

$$\mathrm{i}\nu^s_{qrk+1/2}\hat{T}^s_{qrk+1/2} = \frac{\theta_{k+1/2}}{p_{k+1/2}}\hat{\omega}^s_{qrk+1/2},$$

$$\hat{D}^s_{qrk} - \frac{\Delta\hat{\omega}^s_{qrk}}{(p\Delta\zeta)_k} = 0 ,$$

✔ The discrete spectral wave equation for omega velocity is derived:

$$\left[\frac{p}{\Delta\zeta}\Delta\left(\frac{\tilde{\nu}\Delta\hat{\omega}}{p\Delta\zeta}\right)\right]^s_{qrk+1/2} - \left(\frac{p}{\Delta\zeta}\overline{\frac{b}{p}\Delta\hat{\omega}}^{\zeta}\right)^s_{qrk+1/2} + (\tilde{\nu}\lambda)^s_{qrk+1/2}\hat{\omega}^s_{qrk+1/2} = 0 .$$

✔ Boundary conditions are: free-slip condition on the surface and radiative boundary condition on the top.

✔ **The algorithm for stationary solution** computation is based on calculation of the decrease factors of the omega-velocity spectral amplitudes from a recurrence formula, with initialization from the radiative boundary condition on the top. Solution is looked for in the form:

$$\omega_{1/2} = 1, \quad \omega_{k+1/2} = \prod_{j=1}^{k} c_j \,.$$

Subtitution into wave equation yields a two-point recurrence for c_k:

$$L^{+}_{k+1/2}(c_{k+1} - 1) + L^{-}_{k+1/2}(1/c_k - 1) + \Delta\zeta^{2}_{k+1/2}\lambda_{k+1/2} = 0 \,.$$

The modulus of a complex decrease factor c_j presents the actual decrease of the wave amplitude per single layer of discrete model, whereas its argument is the phase angle increment in this layer:

$$\frac{\omega_{k+1/2}}{\omega_{k-1/2}} = c_k \,.$$

Recurrence is started from the top because recurrence is stable for moving from top to bottom, in direction of decreasing k and increasing ω.

After the initial value on the top is specified, and all c_k are evaluated, solution of wave equation is designed as a cumulative product of decrease factors:

$$\omega_{k+1/2} = \omega_{1/2} \prod_{j=1}^{k} c_j,$$

where the bottom value $\omega_{1/2}$ is specified from bottom boundary condition.

The developed model is 4D-discrete (x,y,z,t), spectral, semi-implicit, semi-Lagrangian (SISL) scheme for stationary case.

Solution examples

✔ The following examples present three-dimensional flow regimes over a model orography.

$$h(x, y) = \frac{h_0}{1 + (x - x_0)^2 / a_x^2 + (y - y_0)^2 / a_y^2} \quad ,$$

where h_0 is the maximum height; x_0, y_0 – center location and a_x, a_y – half-width of the hill.

✔ In the following simulations, the non-varied constants in vertical cross-sections are:

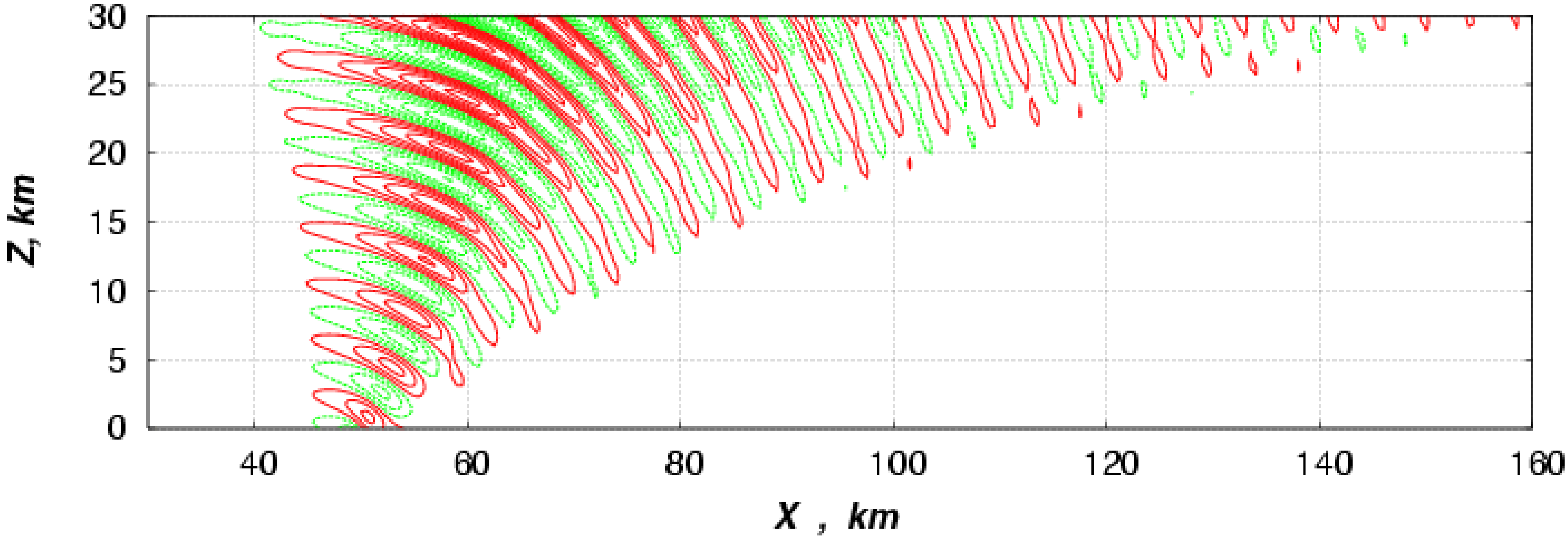
Δx = 500 m, Δz = 200 m, N_z = 155, h_0 = 100 m,
a_x = 2 km, $a_y \to \infty$, x_0 = 50 km, Δw = 0,1 m/s.

✔ The reference temperature T(z) and velocity U(z) profiles are shown in each particular case.

The modelled field is the log-pressure vertical velocity close coinciding with the ordinary vertical velocity dz / dt:

$$w = -\frac{H}{p}\omega \, .$$

1. Stationary orographic waves in homogeneous atmosphere conditions. Vertical cross-section.



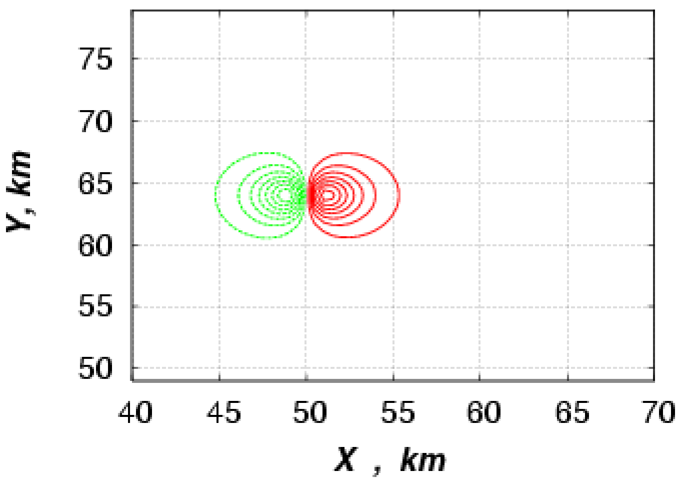
Waves of vertical velocity for

constant wind U = 12 m/s and temperature T = 280 K.

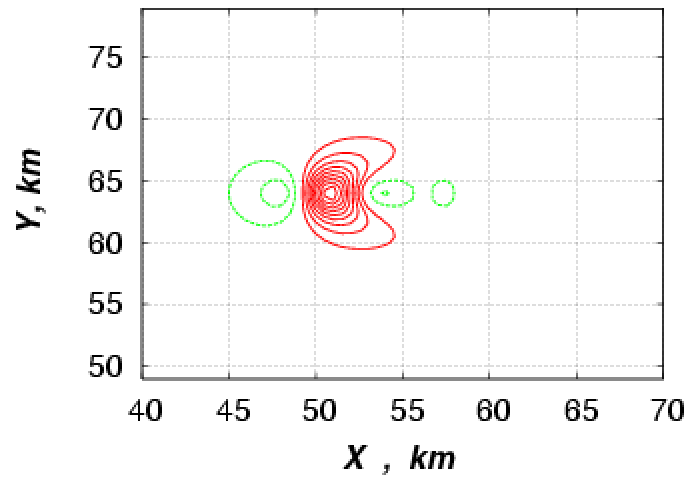
Mid-latitude case: $f = 10^{-4}\ s^{-1}$. a_x = 2 km, h_0 = 100 m, Δw = 0,1 m/s.

Green – positive velocity, red – negative velocity.

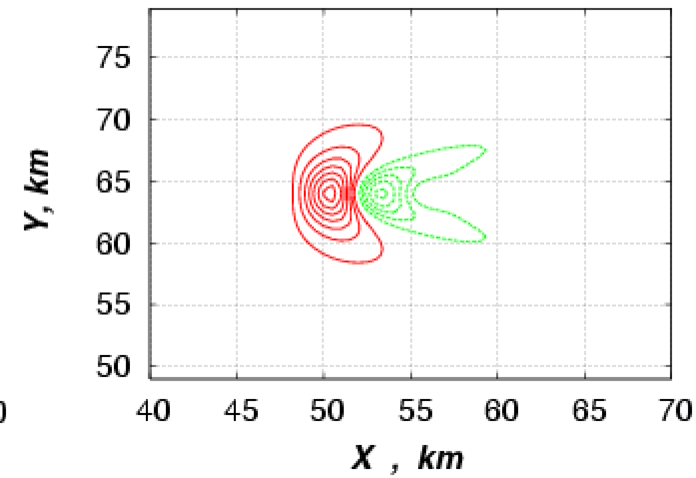
2. Stationary orographic waves in homogeneous atmosphere conditions. Horizontal cross-section.



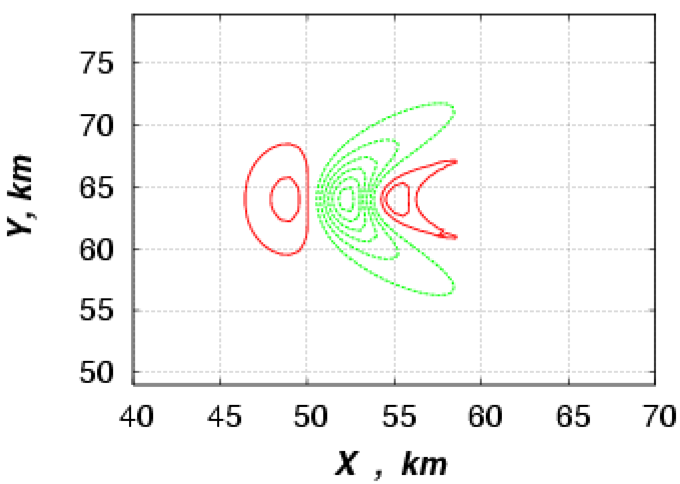
surface



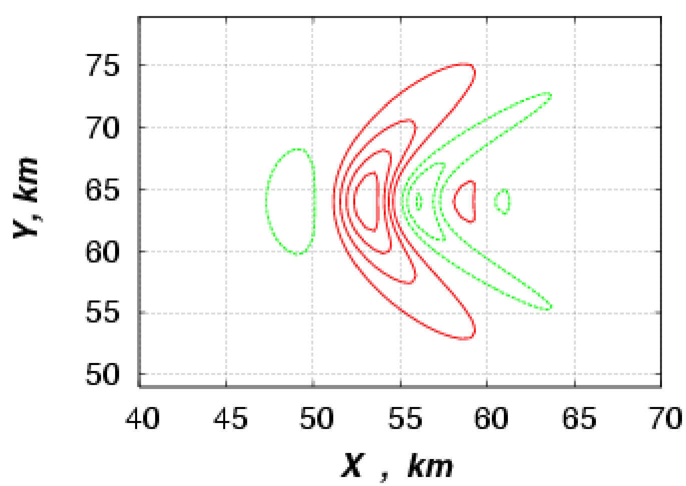
z = 0.5 km



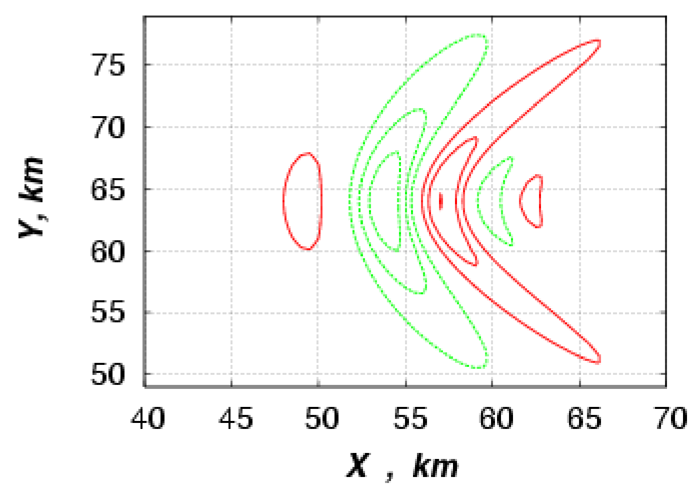
z = 1 km



z = 2 km



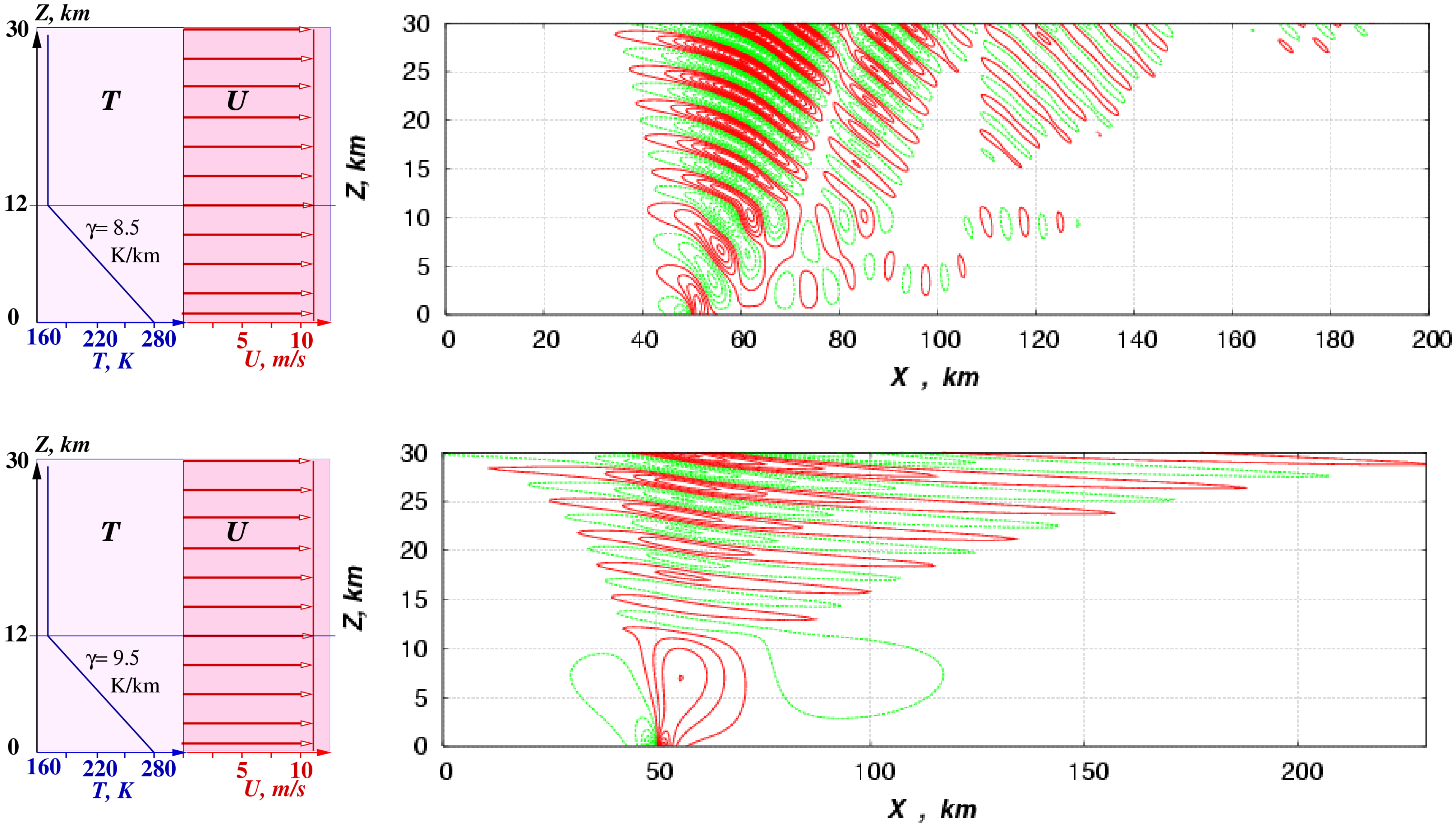
z = 4 km



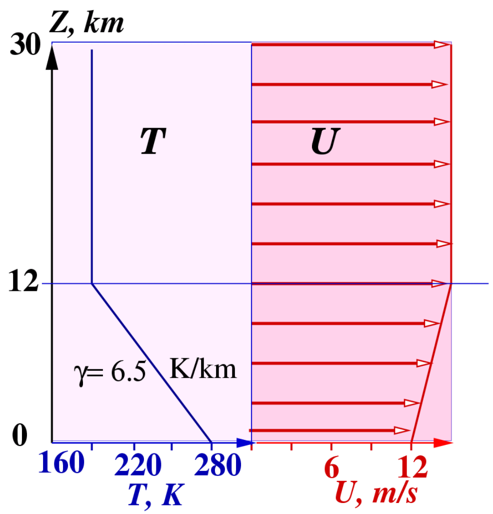
z = 6 km

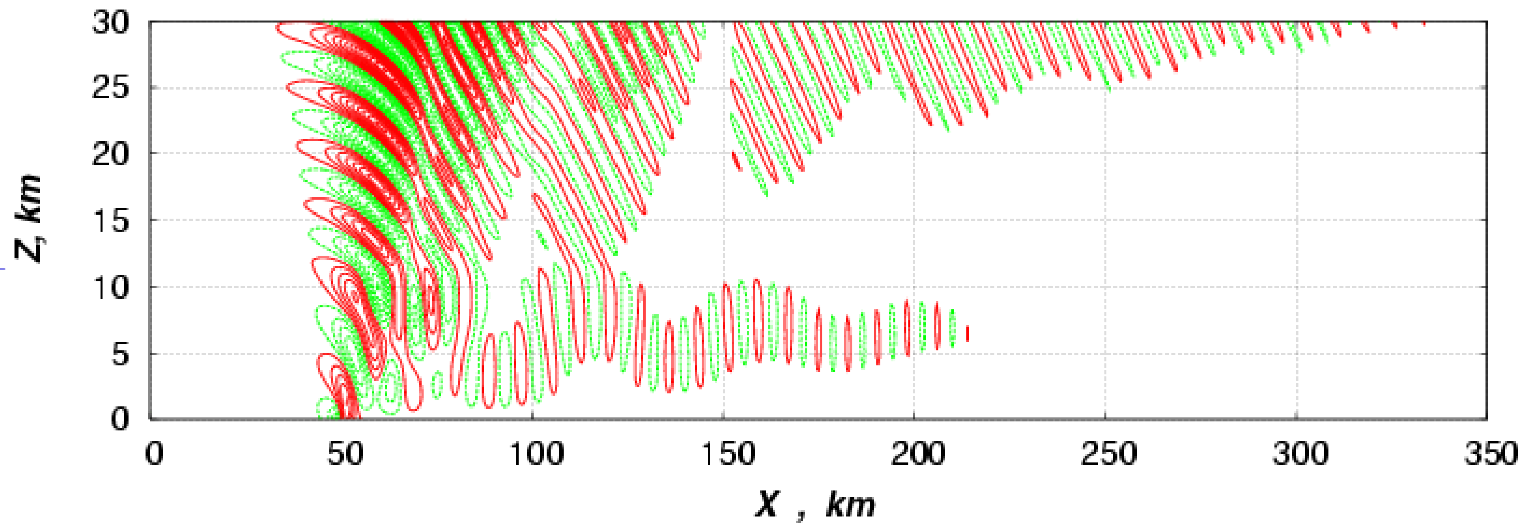
Constant wind U = 12 m/s and temperature T = 280 K, $\Delta x = \Delta y = 500$ m, $\Delta z = 25$ m, $N_z = 300$, $a_x = a_y = 2$ km, $\Delta w = 0{,}05$ m/s.

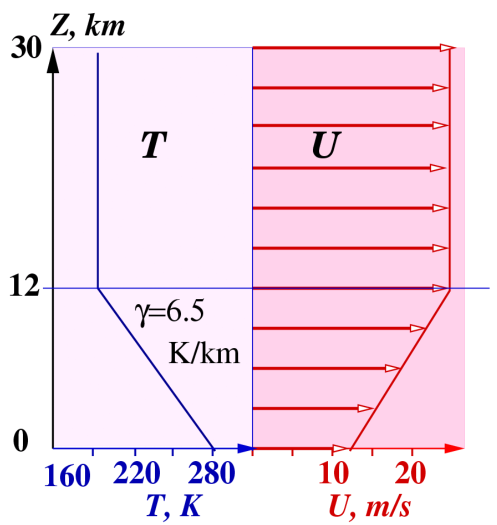
3. Stationary orographic waves in non-homogeneous atmosphere conditions. Refraction and reflection on tropopause.

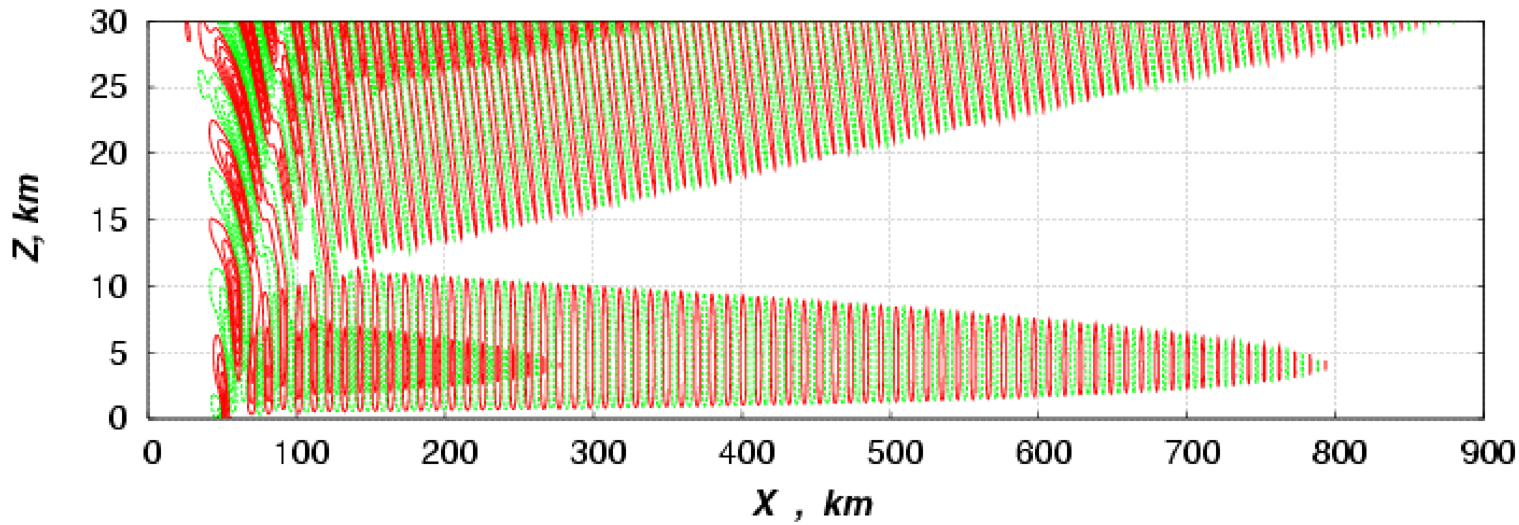


4. Stationary orographic waves in non-homogeneous atmosphere conditions. Refraction and reflection on tropopause in the case of linear wind shear in the troposphere.

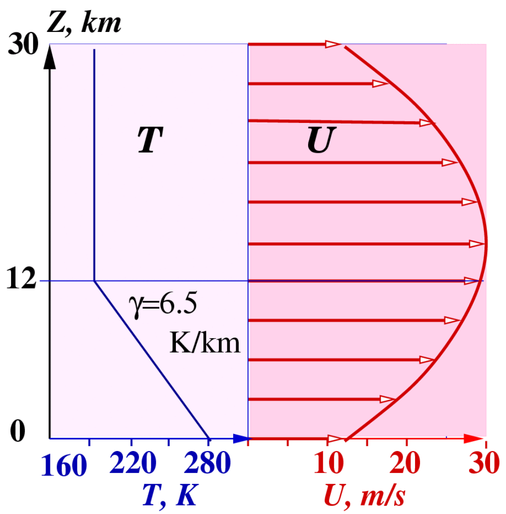


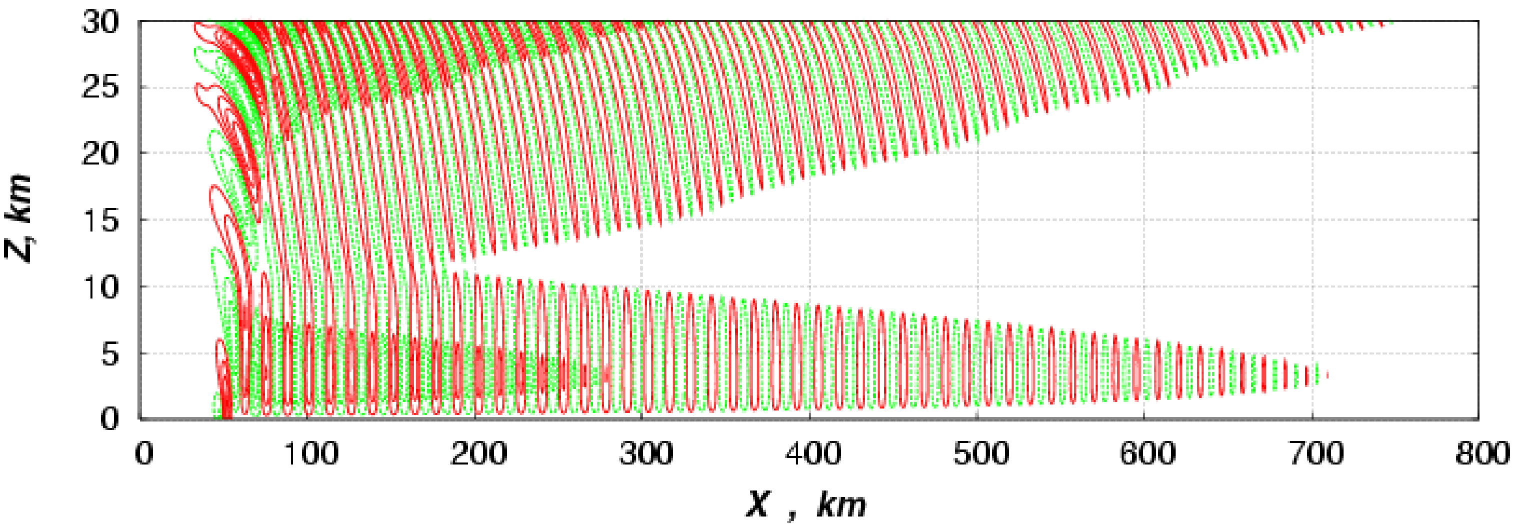


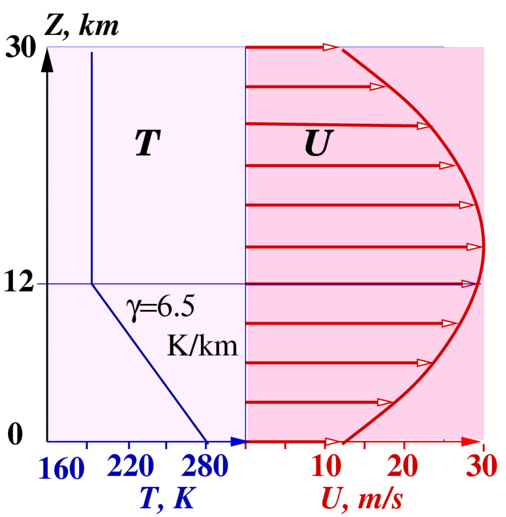


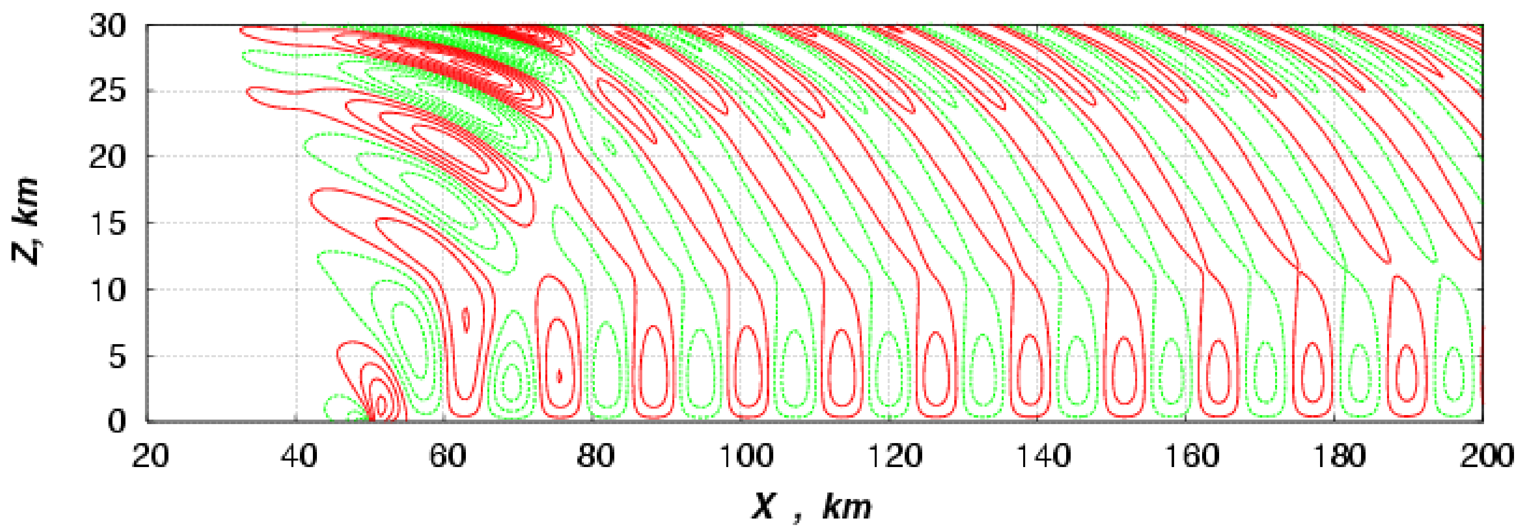


5. Stationary orographic waves in non-homogeneous atmosphere conditions. Refraction and reflection on tropopause in the case of hyperbolic wind shear.









Summary

The derived solution can be used to study the stratification effects on orographic flow pattern.

- ✔ Numerical algorithm is computationally effective and extremely fast.
- ✔ High horizontal and vertical resolution.
- ✔ Sophisticated wind and temperature profiles can be used.

As a test-bed, the solution can be used for testing of adiabatic cores of limited area NWP models.

- ✔ Investigation of the impact of spatial and temporal discretization.
- ✔ Investigation of the time step size impact on the numerical stability.

The approach is also suited for analytical investigation of different numerical effects on solution accuracy.

- ✔ Spectral smoothing

Remaining problems

Some instability may occur in computation cycle when reference wind is approaching to zero – such vertical layers are called critical. To overcome this complicated situation, a computation in fine vertical resolution is needed (sometimes even 1m vertical resolution is required).

References

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