A new spectral theory of anisotropic turbulence and its application in a regional weather prediction system HIRLAM

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Problematics of Turbulence Parameterization

- NWP models involve eddy viscosity K_m and eddy diffusivityand K_h, that account for unresolved turbulent mixing and diffusion.
- The most sophisticated turbulent closure models used today for NWP belong to the family of Reynolds stress models.
- These models are formulated for the physical space variables; they consider a hierarchy of turbulent correlations and employ a rational way of its truncation.
- In the process, unknown correlation are related to the known ones via “closure assumptions” that are based upon preservation of tensorial properties and the principle of invariant modelling according to which the **constants in the closure relationships are universal**
- **However, the physics is different on different scales**

Complicating factors for turbulence modeling

- Geophysical flows are often strongly anisotropic and include various waves
- Gravity force => stratification, gravity waves
- Larger scales: Coriolis force due to planetary rotation ⇨ quasi-2D flows, inertial waves
- Largest planetary scales: Variation of Coriolis force with latitude => β-effect, Rossby waves, flow zonation
- Reynolds averaging does not differentiate between scales;
- Reynolds stress models employ the concept of “invariant modeling” (constants in closure assumptions are assumed invariant and are calibrated in simple flows)

Spectral approach naturally accounts for effects on different scales

The Quasi-Normal Scale Elimination (QNSE) theory of turbulence

- QNSE is a spectral quasi-normal theory
- Utilizes the N-S and temperature equations
- The algorithm – successive elimination (by ensemble averaging) of small shells of modes of the near-dissipation range and calculation of the corresponding corrections to the viscosity and diffusivity
- Partial scale elimination yields SGS parameterization for LES; full scale elimination provides eddy viscosities and eddy diffusivities for RANS models
- The QNSE theory works for neutral, stable (including strong stratification) and moderately unstable BLs without any adjustable parameters
- QNSE is an alternative to the Reynolds stress method in obtaining eddy viscosities and eddy diffusivities

Stable stratification. Basics of the theory

The theory is developed for a fully three-dimensional turbulent flow field with imposed vertical temperature gradient. The flow is governed by the

momentum $$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} - \alpha g T \hat{e}_3 = \nu_0 \nabla^2 \mathbf{u} - \frac{\nabla P}{\rho} + \mathbf{f}_0$$

temperature $$\frac{\partial T}{\partial t} + (\mathbf{u}\nabla)T + \frac{d\Theta}{dz}\mathrm{u}_3 = \kappa_0 \nabla^2 T$$

and **continuity** $$\nabla \mathbf{u} = 0$$ equations in Boussinesq approximation.

Central problem is **treatment of nonlinearity**. Perturbative solution based on expansion parameter Re? **It is strongly divergent!**

Spectral approach is most appropriate for dealing with this problem.

The general idea: Re is small for smallest scales of motion =>

- Derive perturbative solution for these small scales
- Using this solution and assumption of Quasi-Gaussianity perform averaging over infinitesimal band of **small scales**. Compute corrections to "effective" or "eddy" viscosity and heat diffusivity. Viscosity increases; Re for the next band remains small
- Repeat the above procedure for next band of smallest scales.
- **Final result:** coupled system of 4 differential equations for all corrections. Scale-dependent horizontal and vertical eddy viscosities and diffusivities are obtained.

Fourier-transformed velocity and temperature equations

Using continuity equation eliminate pressure from the momentum equation. Write the momentum equation in a self-contained form using formal solution to the temperature equation:

$$u_\beta(\hat{k}) = G_{\alpha\beta}(\hat{k}) f^0_\alpha(\hat{k}) - \frac{i}{2} G_{\alpha\beta}(\hat{k}) P_{\alpha\mu\theta}(\mathbf{k}) \int u_\mu(\hat{q}) u_\theta(\hat{k} - \hat{q}) \frac{d\hat{q}}{(2\pi)^4}$$

$$T(\hat{k}) = G_T(\hat{k}) f_T(\hat{k}) - i G_T(\hat{k}) k_\alpha \int u_\alpha(\hat{q}) T(\hat{k} - \hat{q}) \frac{d\hat{q}}{(2\pi)^4},$$

Velocity Green function acquires tensorial structure

$$G_{\alpha\beta}(\mathbf{k},\omega) = G(\mathbf{k},\omega)[\delta_{\alpha\beta} + A(\mathbf{k},\omega) P_{\alpha 3}(\mathbf{k}) \delta_{\beta 3}]$$

complex poles ⇓ waves!

where $G(\mathbf{k},\omega) = [-i\omega + \nu_h(k) k_h^2 + \nu_z(k) k_3^2]^{-1}$

is the auxiliary Green function, $\nu_{\mathbf{h}}$ and $\nu_{\mathbf{z}}$ are the horizontal and the vertical eddy viscosities, $P_{\alpha\beta}$ – projection operator,

$$G_T(\mathbf{k},\omega) = [-i\omega + \kappa_h(k) k_h^2 + \kappa_z(k) k_3^2]^{-1}$$

is temperature Green function, where κ_h and κ_z are the horizontal and the vertical eddy diffusivities,

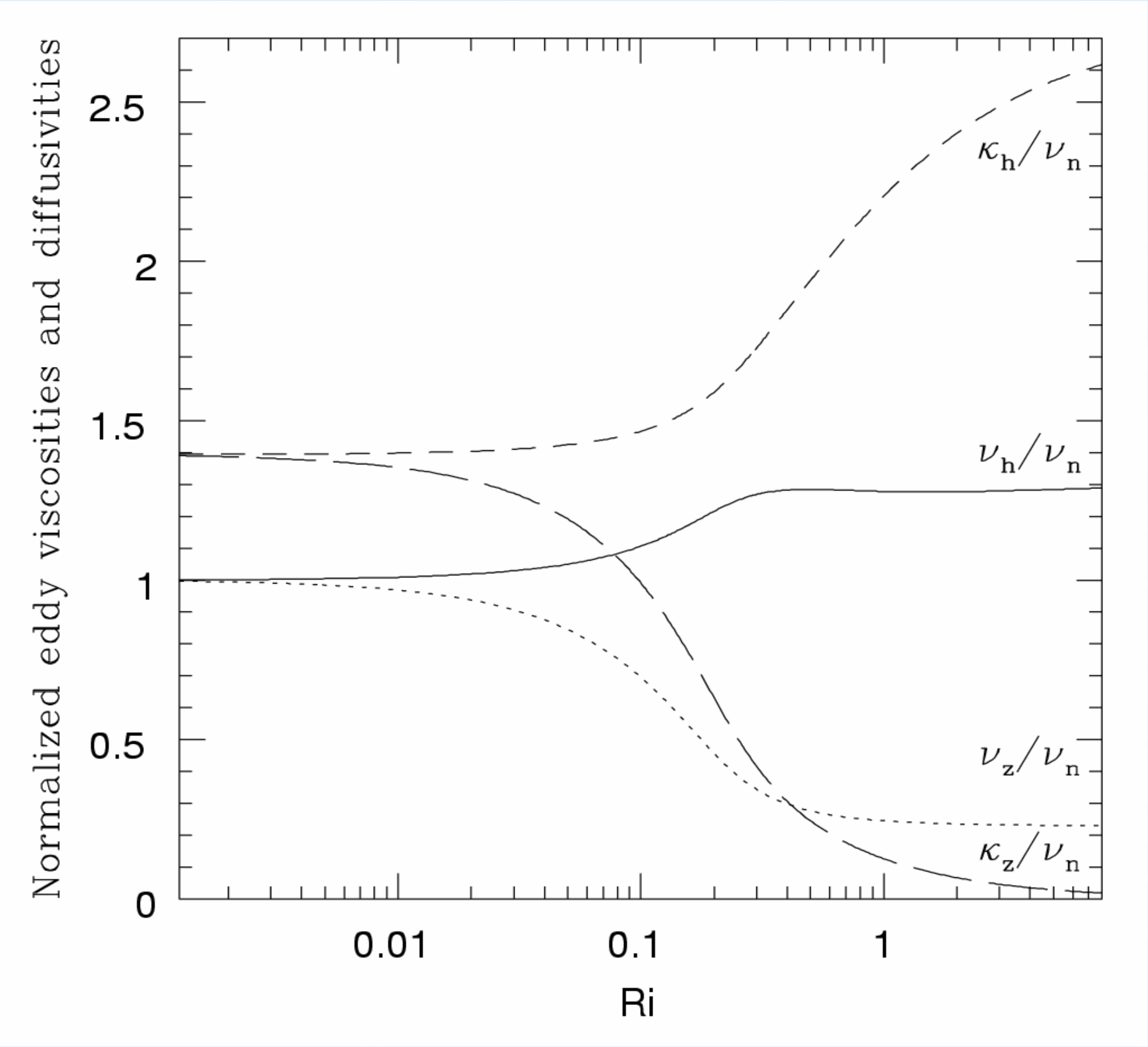
$$A(\mathbf{k},\omega) = -\frac{N^2}{\left(-i\omega + \nu k_h^2 + \nu_z k_3^2\right)\left(-i\omega + \kappa k_h^2 + \kappa_z k_3^2\right) + N^2 \sin^2\phi}$$

φ is an angle between **k** and the vertical, $N \equiv \left(\alpha g \frac{d\Theta}{dz}\right)^{1/2}$ -Brunt-Vaisala frequency

Advantages of the QNSE over Reynolds stress models

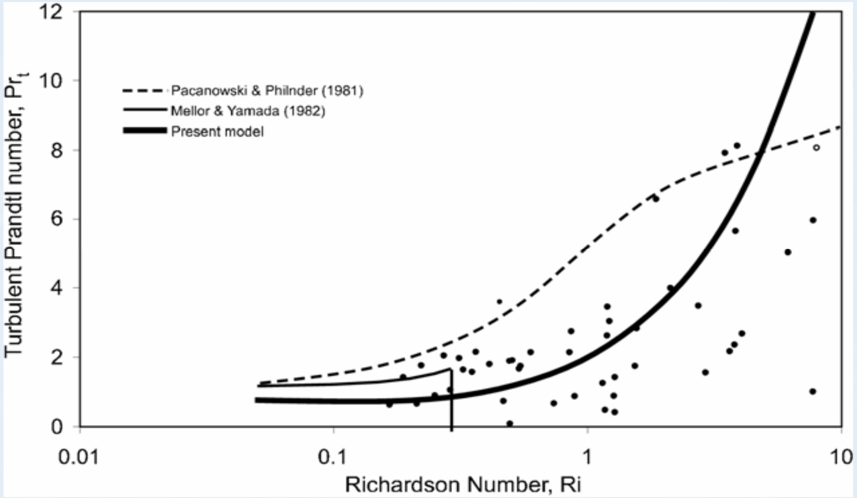
- QNSE explicitly accounts for the processes on the eliminated scales
- The effect of IGW is included; the dispersion relation for IW in the presence of turbulence is derived, and the threshold of IW generation is obtained
- QNSE accounts for modifications of the spectral characteristics; for instance, it captures the transition from the Kolmogorov -5/3 to the steeper, $N^2 k_z^{-3}$, vertical spectrum of the horizontal velocity
- QNSE accounts for the flow anisotropization and so it yields vertical and horizontal eddy viscosities and eddy diffusivities
- These turbulent exchange coefficients are given as analytical functions of the moving dissipation cutoff for LES or of Ri (or Fr) for RANS and are easy to implement in mesoscale or global models
- QNSE provides a realistic prediction of the dependency of Pr_t on Ri

RANS modeling

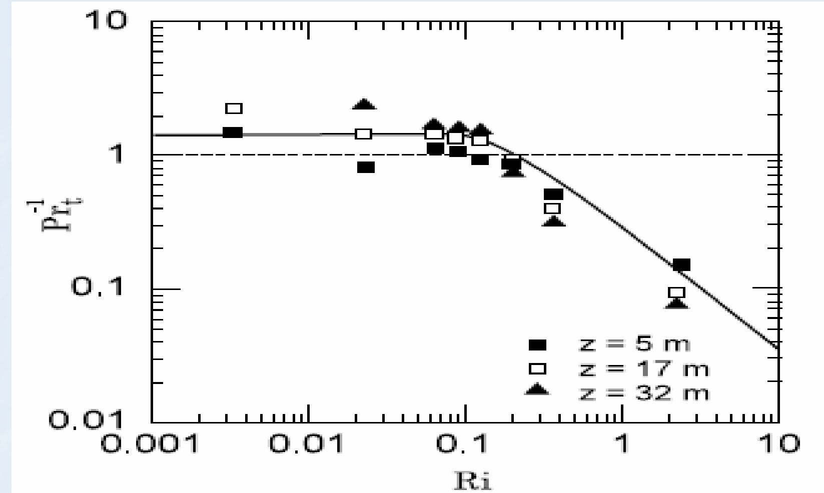


Normalized horizontal and vertical eddy viscosities and eddy diffusivities as functions of *Ri*

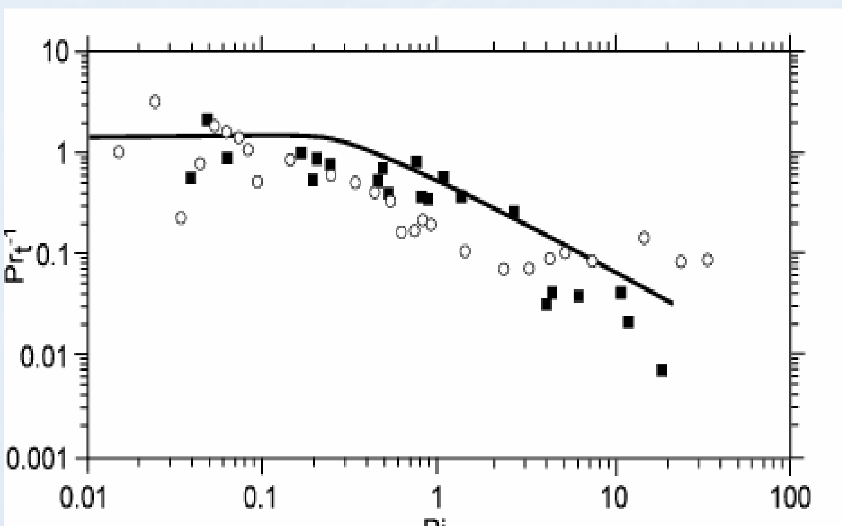
Comparison with data: Pr_t as a function of Ri



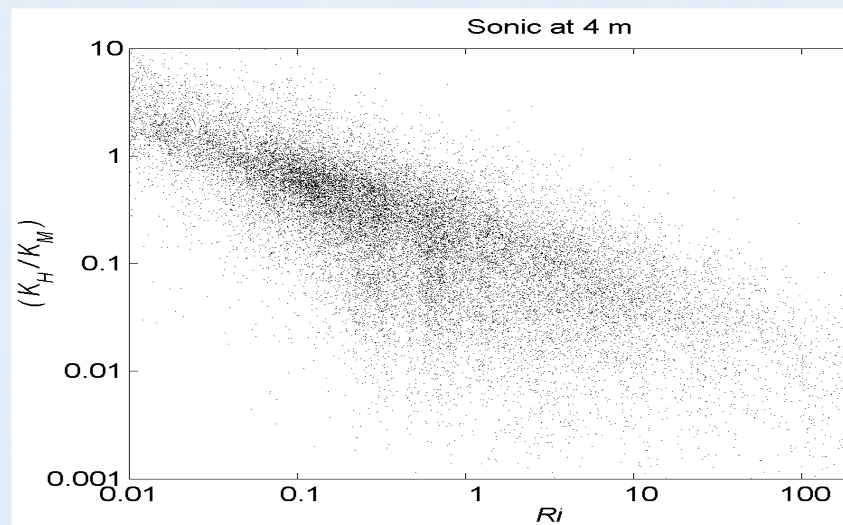
Huq & Stewart (2004)



Yagüe et al. (2001)

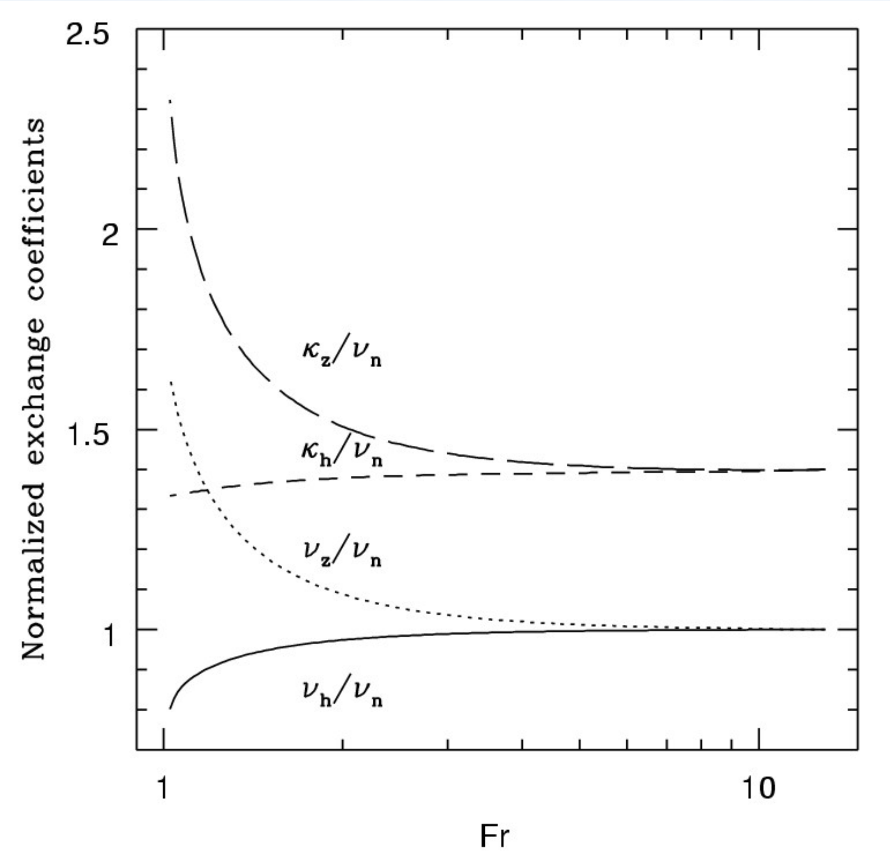


Monti et al. (2002)



King (2003)

Unstable stratification (Convection)



Validation of the QNSE-based RANS models in a single-column formulation

K-ε format

- BASE (GABLS 1)
- SHEBA
- CASES-99 (particularly, IOP-9 event)
- CASES-99 (GABLS 2)

K-ℓ format

- CASES-99 (GABLS 2)
- HIRLAM 7.0 – simulations of January and March, 2005

QNSE-based K-ε model

Vertical eddy viscosity and eddy diffusivity are given as $K_M = C_\mu \alpha_M K^2/\varepsilon$, $K_H = C_\mu \alpha_H K^2/\varepsilon$

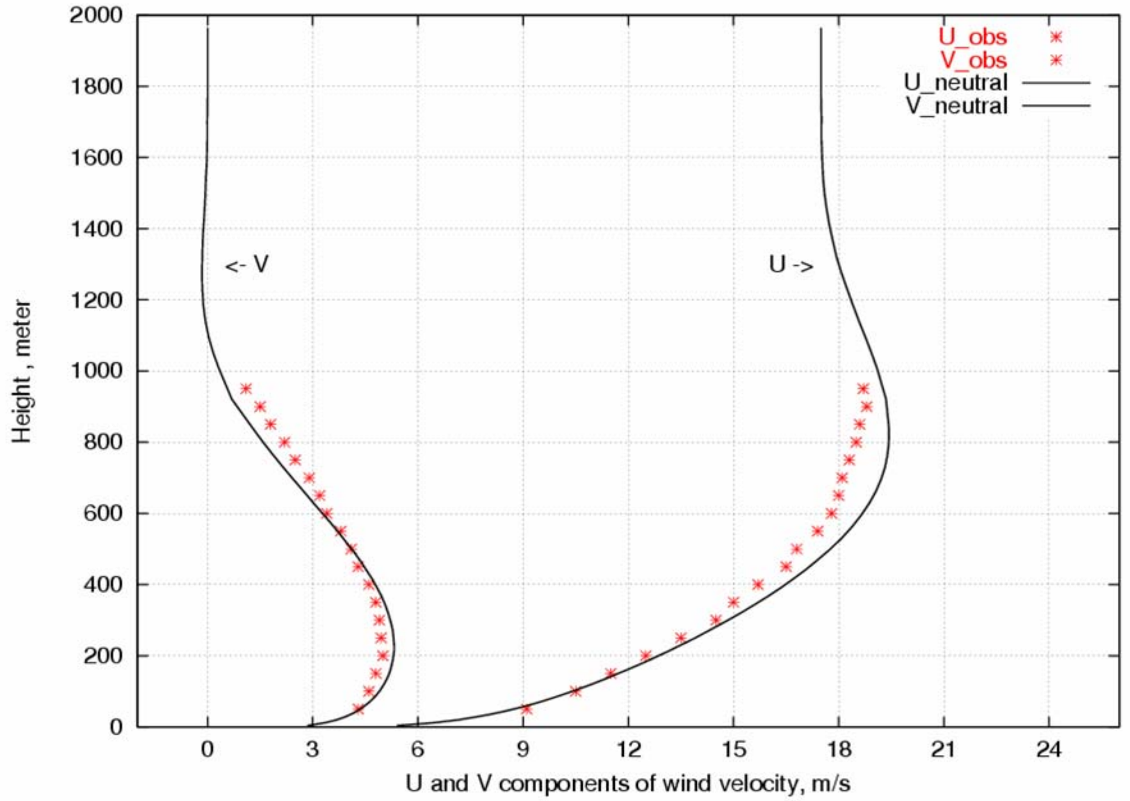
$$\frac{\partial \varepsilon}{\partial t} = \frac{\varepsilon}{K}\left\{C_1 K_M\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - C_3 \frac{g}{\Theta_0} K_H \frac{\partial \theta}{\partial z}\right\} - C_2 \frac{\varepsilon^2}{K} + \frac{\partial}{\partial z}\left(K_\varepsilon \frac{\partial \varepsilon}{\partial z}\right)$$

- C_1 is a linear function of $Ro_* = u_*/|f|L$ and $Fr_* = u_*/NL$ which account for the effects of rotation and stratification
- Stability functions α_M and α_H are given by the QNSE theory

- More details are in:

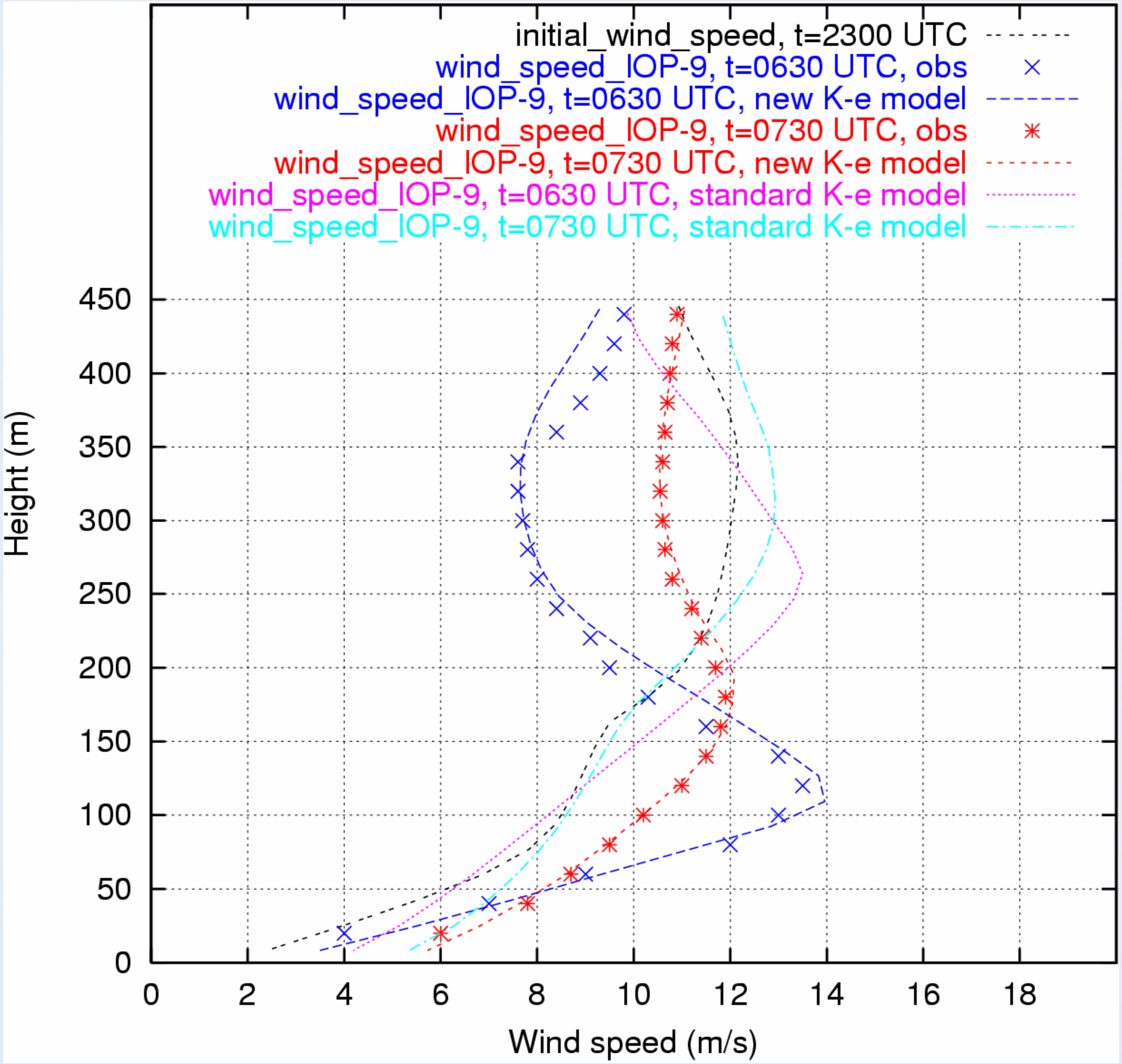
Sukoriansky, S., B. Galperin, and V. Perov, Application of a new spectral theory of stably stratified turbulence to atmospheric boundary layers over sea ice. *Boundary-Layer Meteorology*, **117**, 231–257, 2005.

Neutral ABL

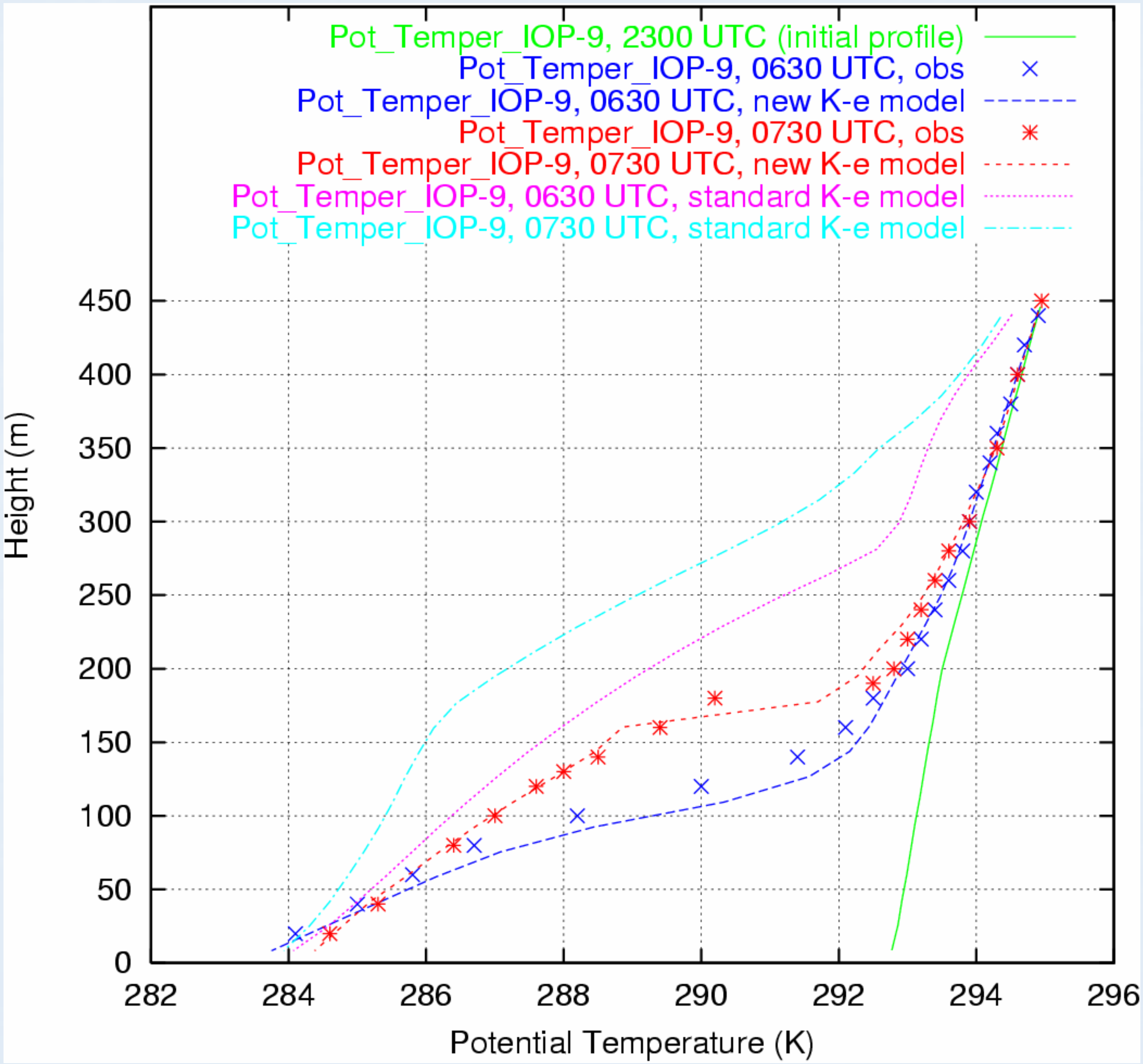


Comparison with Leipzig wind profile

Comparison with CASES-99 – Velocity profiles



Comparison with CASES-99 – Temperature profiles



QNSE-based K-l model

$$\frac{\partial E}{\partial t} = K_M\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\Theta_0} K_H \frac{\partial \theta}{\partial z} - \varepsilon + \frac{\partial}{\partial z}\left(K_q \frac{\partial E}{\partial z}\right)$$

$$\varepsilon = c_\varepsilon \frac{E^{3/2}}{l}; \quad l_B = \frac{kz}{1 + \frac{kz}{\lambda}}, \quad l_N = c_N \frac{K^{1/2}}{N}; c_N = 0.75$$

$$\frac{1}{l} = \frac{1}{l_B} + \frac{1}{l_N}; \quad c_\varepsilon = 0.167, \quad \lambda = Bu_* / f$$

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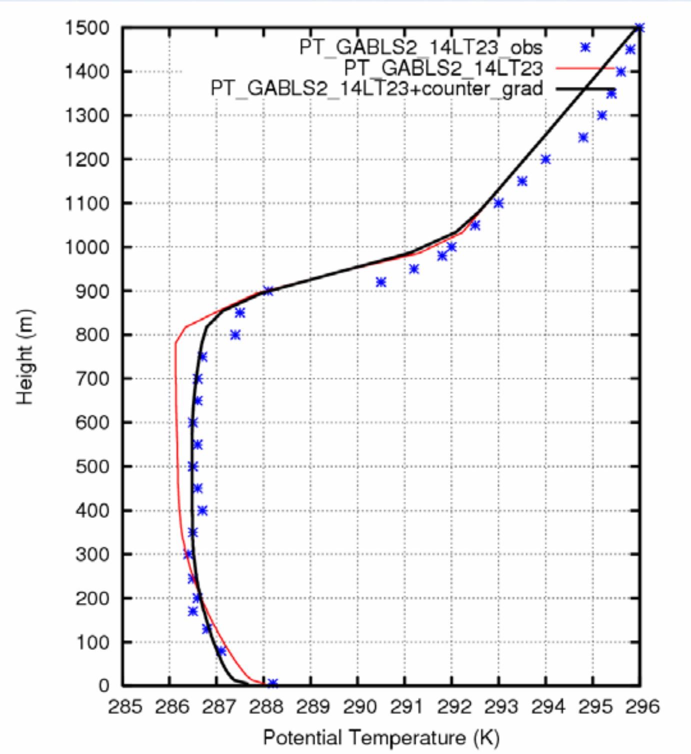
$$K_M = \alpha_M l E^{1/2}, \quad K_H = \alpha_H l E^{1/2}$$

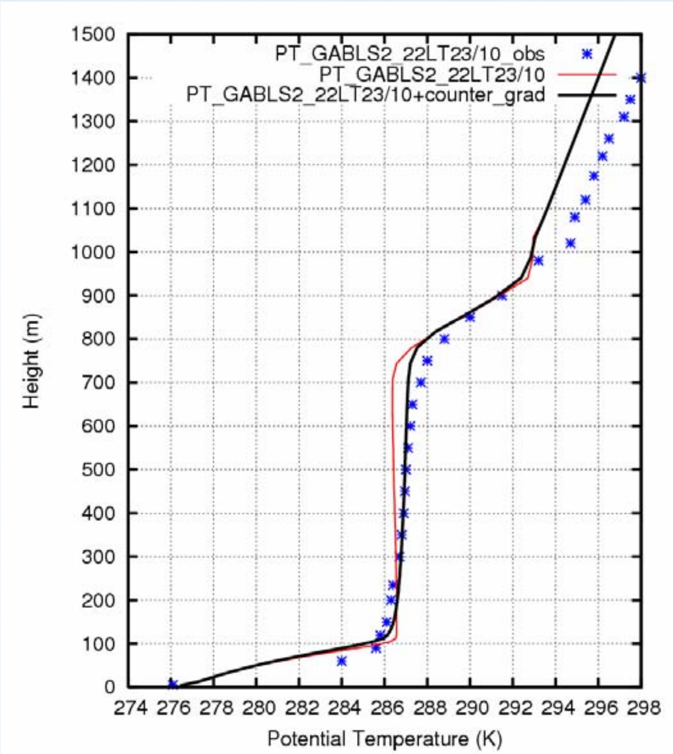
Comparison with CASES-99 – diurnal cycle

The simulation covered a 60-hour period of the CASES-99 experiment, from 14 00 (local time, LT) 22 October 1999 through 02 00 LT 24 October 1999. During the day, the surface temperature was higher than the air temperature hence unstable stratification; the following night was characterized by stable stratification. The diurnal cycle starting on 14 00 LT 23 October and extending for 24 hours was designated by the GABLS community for the model inter-comparison experiment GABLS-2.



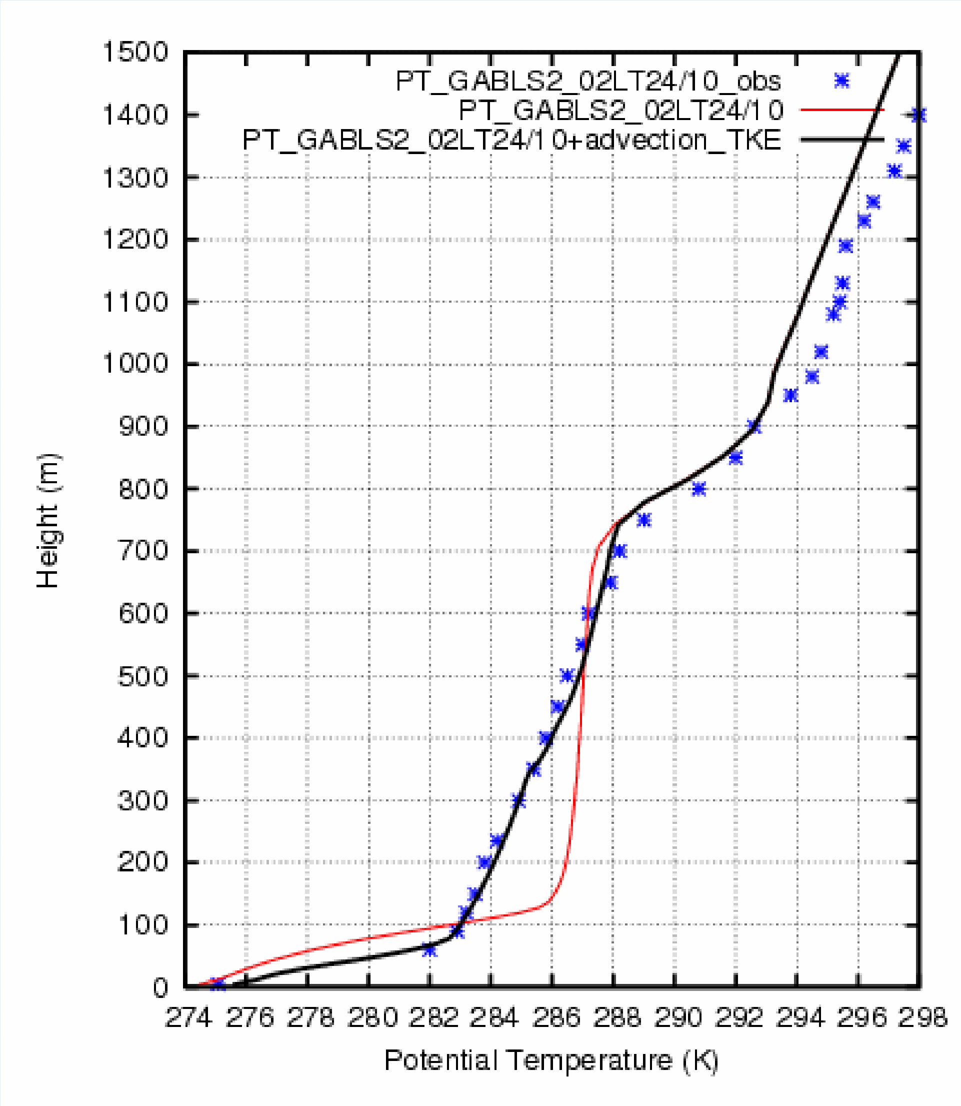
Unstable stratification, 14 00 LT 23 Oct.

Potential temperature



Stable stratification, 22 00 LT, 23 Oct.

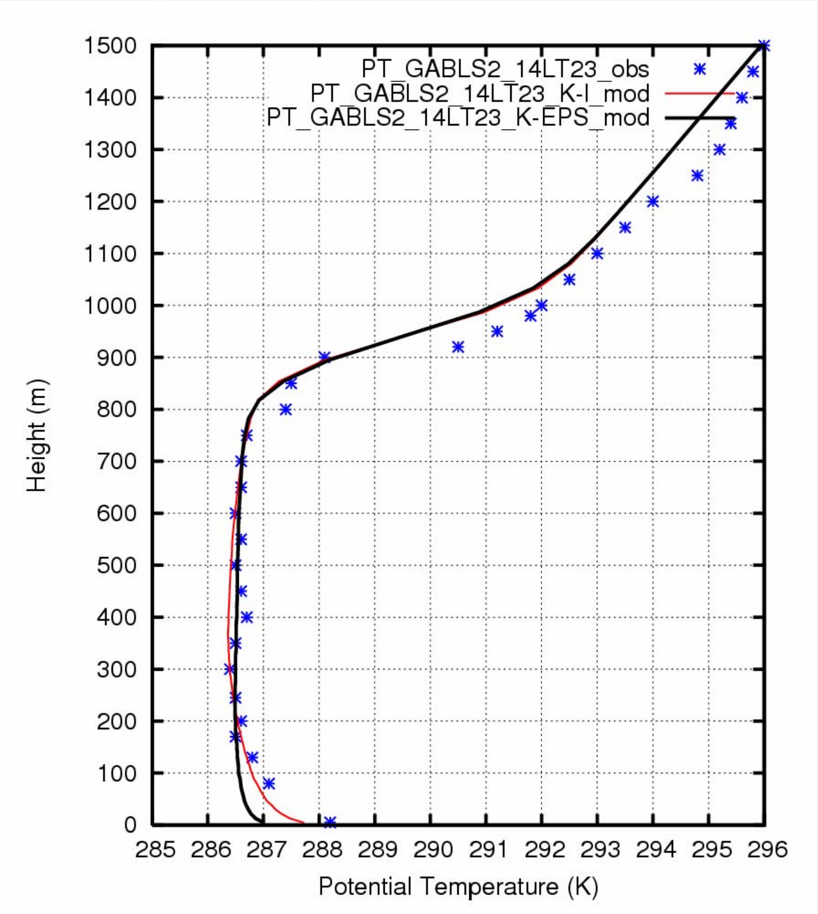
GABLS 2 – Potential temperature

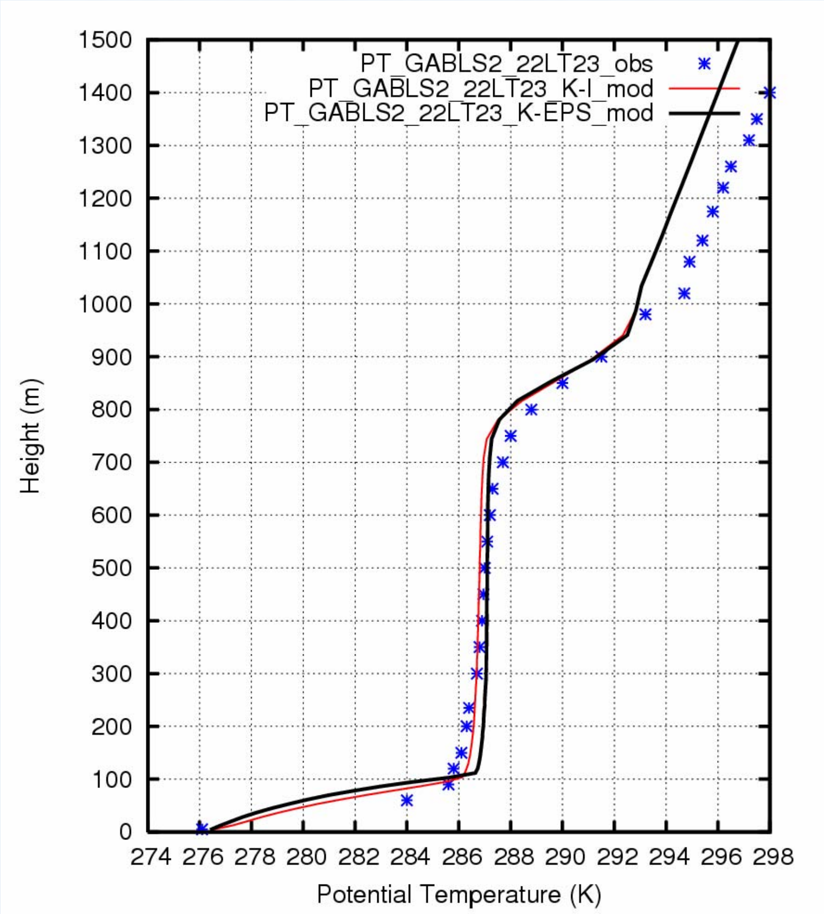


Stable stratification, 02 00 LT, 24 Oct.

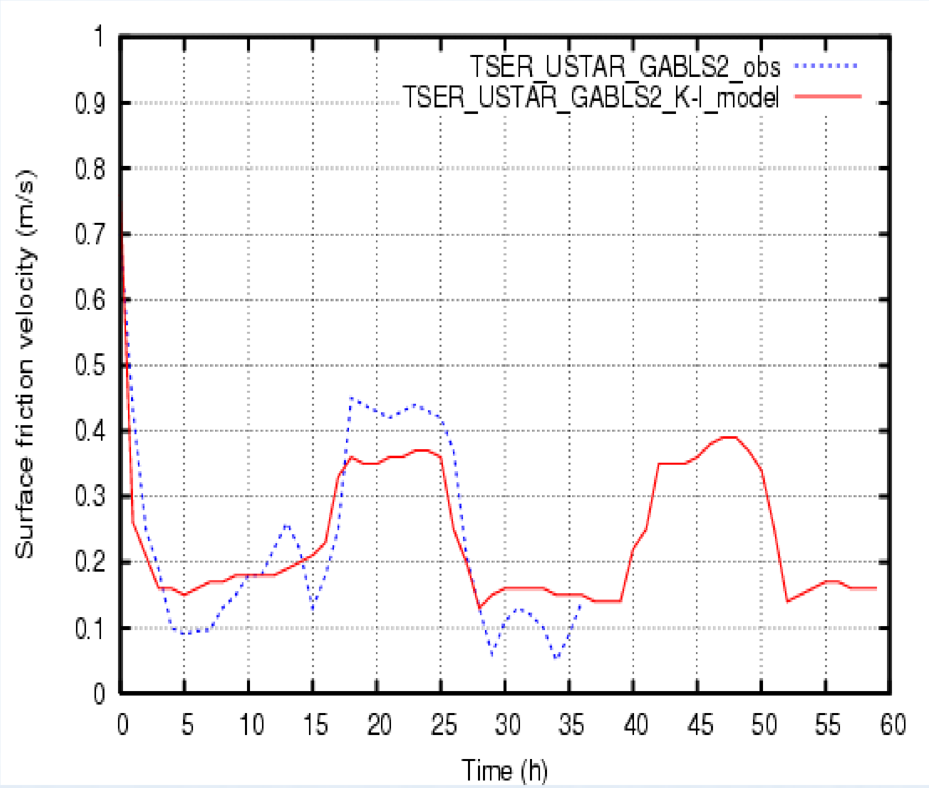
GABLS 2 – Potential temperature

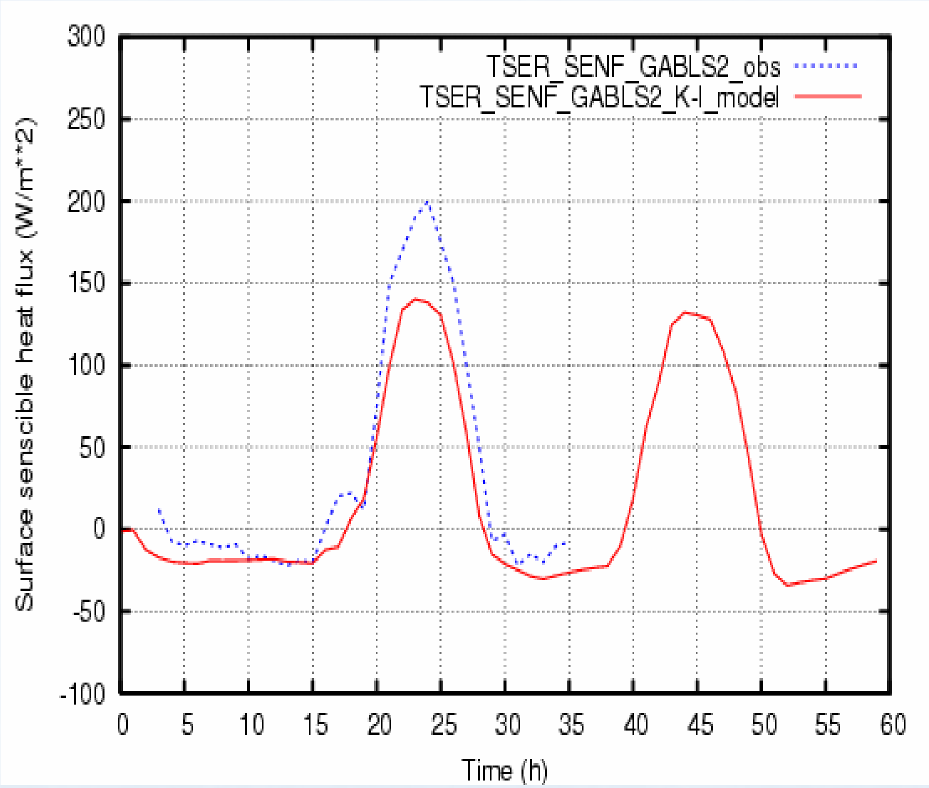
Comparison of the QNSE-based K-ℓ and K-ε models





CASES-99: 60-hr simulation for GABLS2

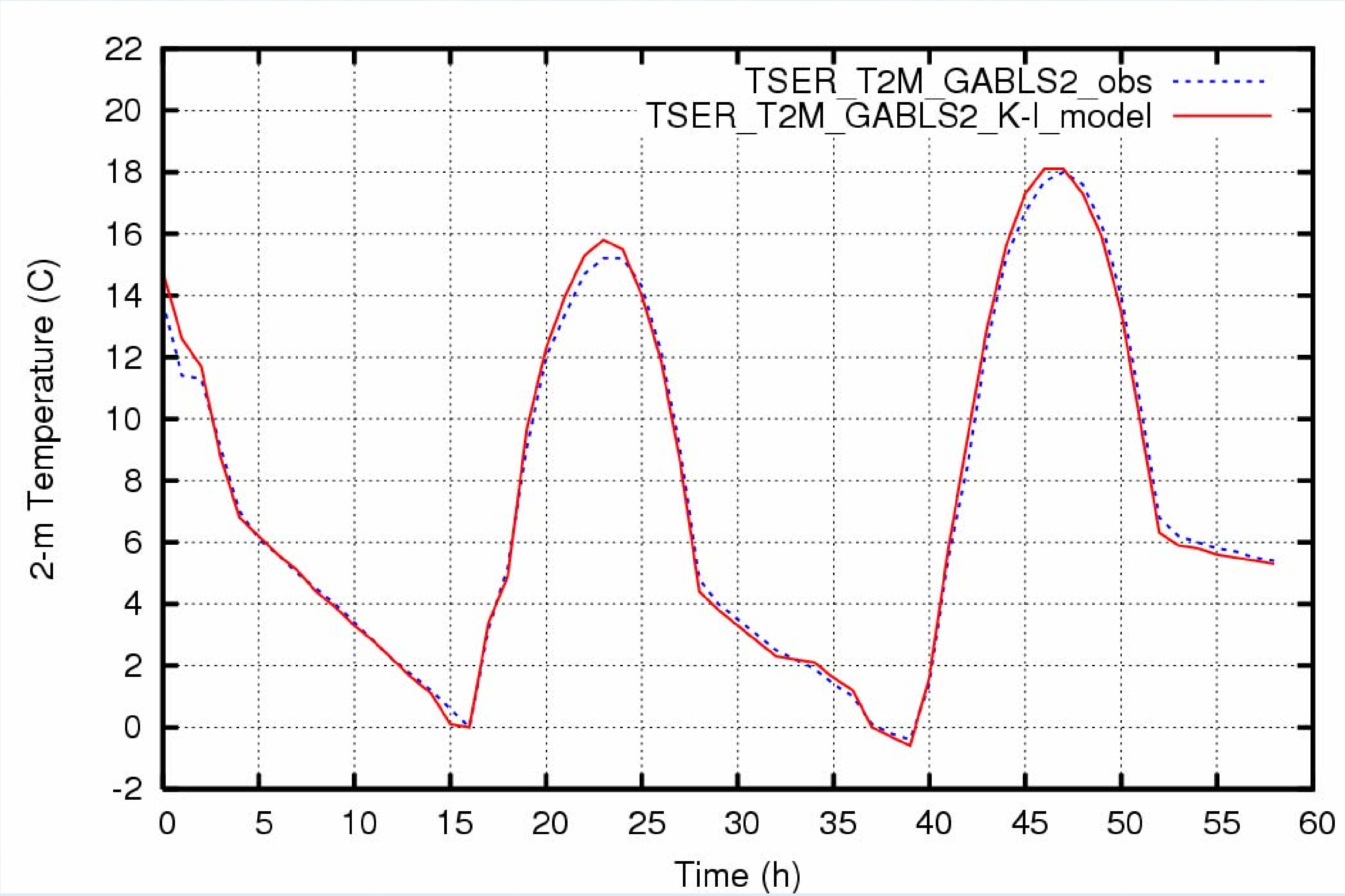




Friction velocity

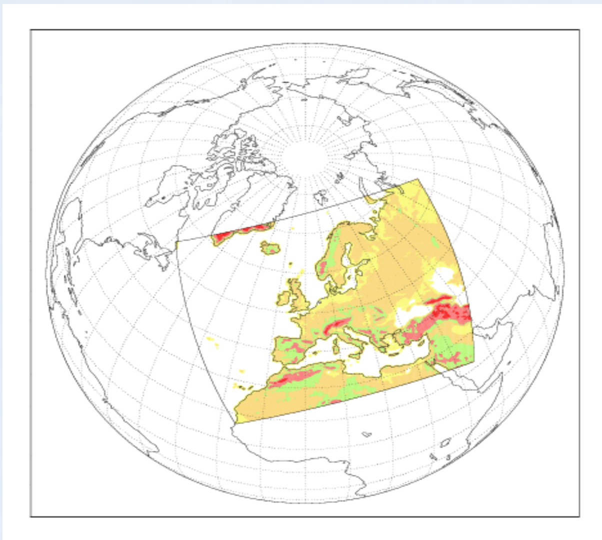
Surface sensible heat flux

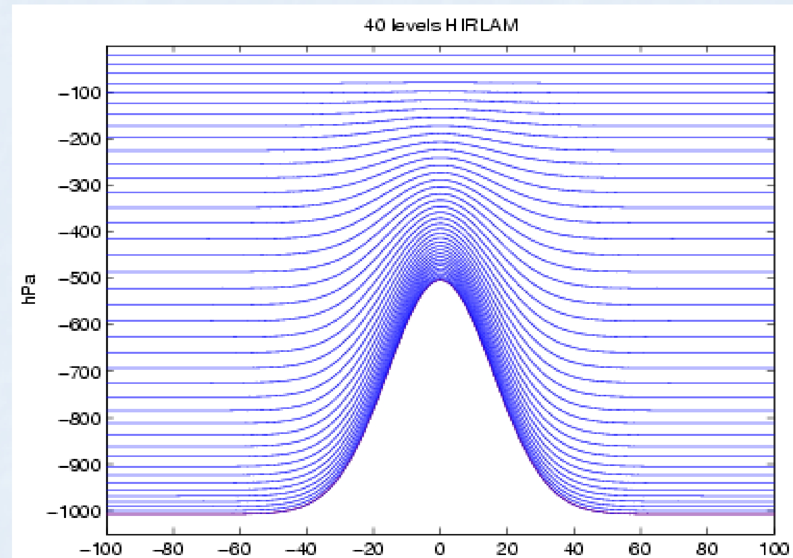
The time series of the temperature at 2m height for GABLS2



Testing of the QNSE-based K-l model in the numerical weather prediction system HIRLAM

- NWP system HIRLAM (version 7.0): High Resolution Limited Area Model
- Covers the North-East Atlantic, Europe, and Greenland





- Hydrostatic model; 438x336 points; 22km x 22km resolution with 40 vertical levels
- Lateral boundary conditions and the first guess field are from ECMWF operations
- Massive data assimilation: over 1000 stations all over Europe
- Data assimilation cycle is 6 hours
- From each 00, 06, 12, 18 UTC, a +48 hours forecast is run
- Total: 120, +48 h forecasts in one month
- January and March 2005 were chosen as cold months
- Region of interest: Scandinavia

The reference and QNSE-based K-ℓ models in HIRLAM

$$\frac{\partial E}{\partial t} = K_M\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right] - \frac{g}{\Theta_0} K_H \frac{\partial \theta}{\partial z} - \varepsilon + \frac{\partial}{\partial z}\left(K_q \frac{\partial E}{\partial z}\right)$$

$$\varepsilon = c_\varepsilon \frac{E^{3/2}}{l}; \quad l_B = \frac{kz}{1 + \frac{kz}{\lambda}}, \quad l_{m,h} = c_{m,h}\frac{E^{1/2}}{N}$$

$$\frac{1}{l} = \frac{1}{l_B} + \frac{1}{l_{m,h}}, \qquad K_{m,h} = l_{m,h} \cdot E^{1/2}$$

$$c_m = c_h * \max(1, \min[(1 + \gamma Ri * \exp(-Ri^2)) * pfunc, 4])$$

$$pfunc = \max\left(0, \frac{P - 100}{P(nlev) - 100}\right), \quad c_h = 0.2$$

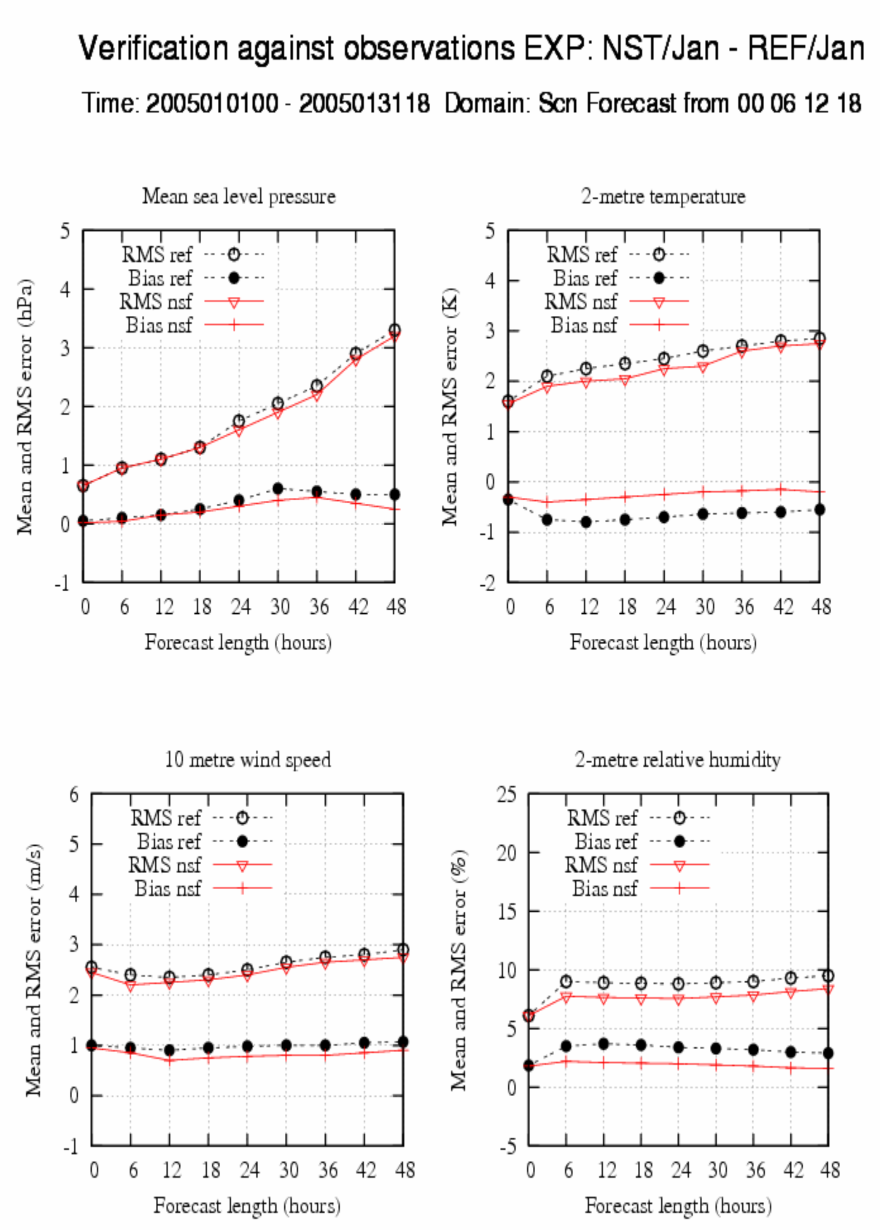
Problems with the reference model

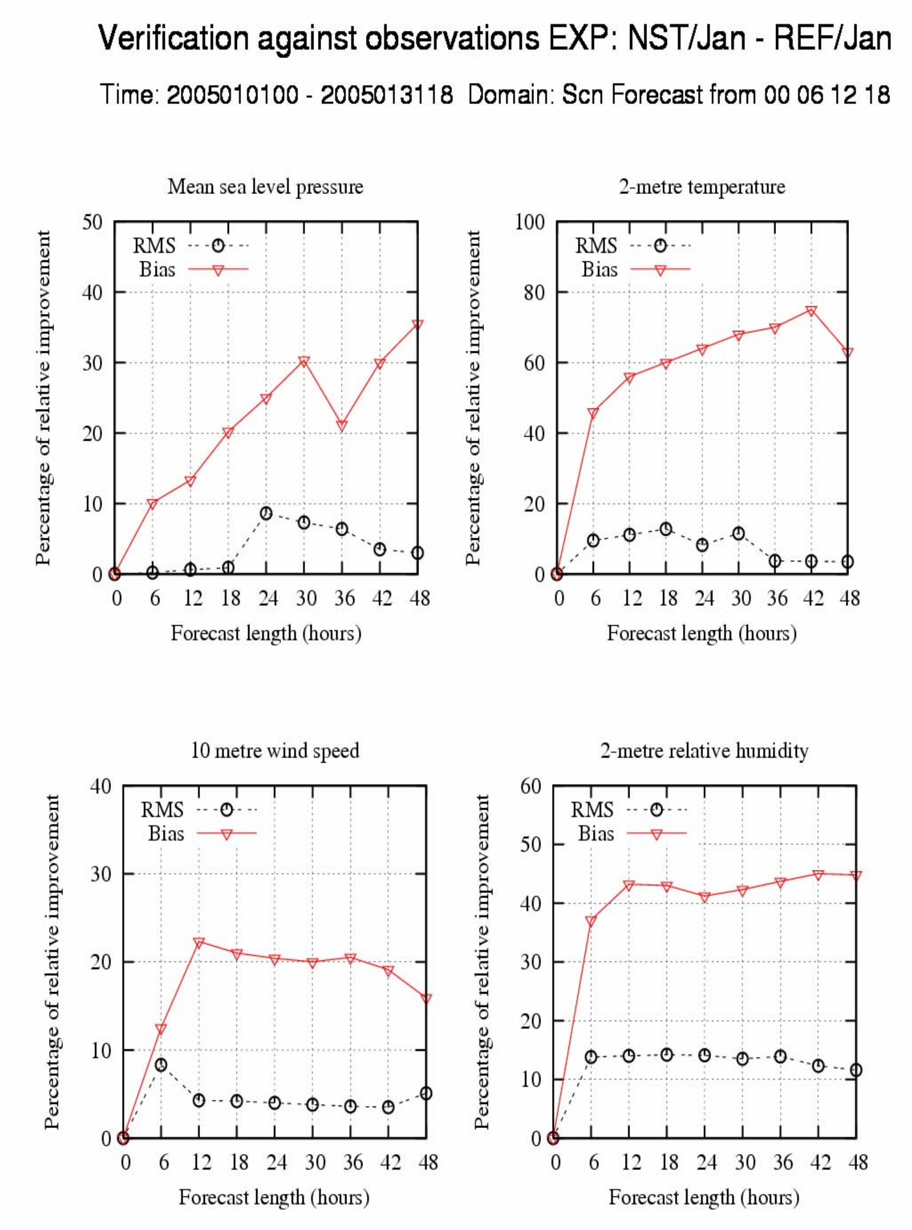
- Positive bias in the wind direction, accompanied by too strong near-surface winds

- Too fast a deepening and too slow filling of cyclones, making HIRLAM too active towards the end of the forecast period

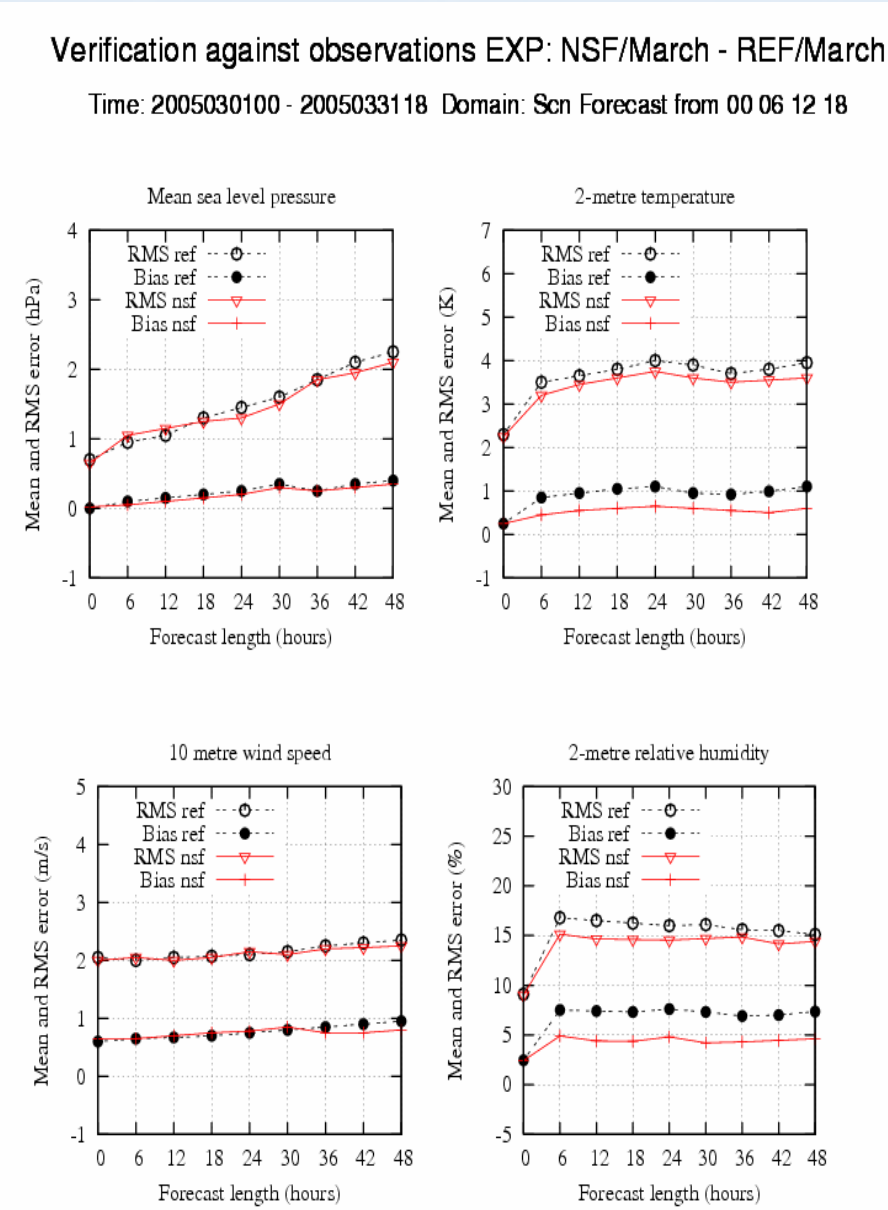
- Modus Operandi: replace K_M and K_H by the QNSE-derived stability functions; run twin experiment; analyze the difference (reference - new)/reference

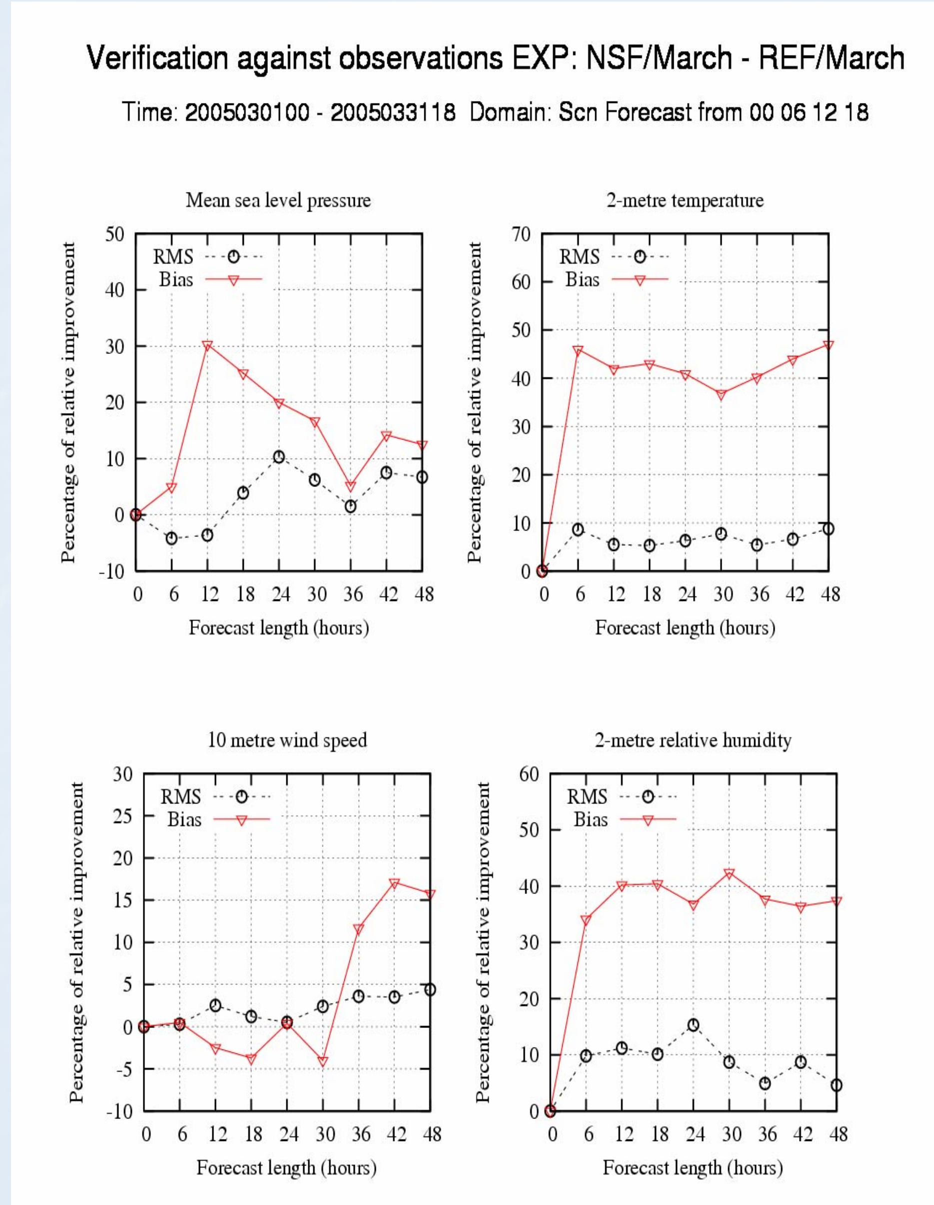
January, 2005





March, 2005


Verification against observations EXP: NSF/March - REF/March
Time: 2005030100 - 2005033118 Domain: Scn Forecast from 00 06 12 18
Mean sea level pressure
Mean and RMS error (hPa)
2-metre temperature
Mean and RMS error (K)
10 metre wind speed
Mean and RMS error (m/s)
2-metre relative humidity
Mean and RMS error (%)
RMS ref
Bias ref
RMS nsf
Bias nsf
Forecast length (hours)


Verification against observations EXP: NSF/March - REF/March
Time: 2005030100 - 2005033118 Domain: Scn Forecast from 00 06 12 18
Mean sea level pressure
2-metre temperature
10 metre wind speed
2-metre relative humidity
Percentage of relative improvement
RMS
Bias
Forecast length (hours)

Conclusions

- Derivation of the QNSE model of turbulence is maximally proximate to first principles
- Theory explicitly resolves horizontal-vertical anisotropy
- Accounts for the combined effect of turbulence and waves
- Predicts correct behavior of Pr_t as a function of Ri
- Anticipates the absence of the critical Ri
- Yields modification of the classical dispersion relation for internal waves that accounts for turbulence
- Provides subgridscale closures for both LES and RANS
- The QNSE theory has been implemented in K-ε and K-ℓ models of stratified ABL
- Good agreement with CASES-99 data sets has been found for cases selected for the GABLS 2 experiment
- In the case of unstable stratification, a counter-gradient modification is necessary for realistic prediction of temperature
- The new stability functions improve predictive skills of HIRLAM in +24h and +48h weather forecasts
- QNSE-based models are a viable alternative to Reynolds stress models

Acknowledgments

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References

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Sukoriansky, S., B. Galperin, and V. Perov, A quasi-normal scale elimination model of turbulence and its application to stably stratified flows. *Nonlinear Processes in Geophysics,* **13***,* 9–22, 2006.