Simulation of analysis errors

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Basic concepts

The representation of the analysis effect in three error simulation techniques
(Berre, Stefanescu and Belo Pereira, 2006)

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Basic concepts
(Bouttier and Courtier, 1999)

Modelling of errors

Error covariances

Observation error variances and correlations

Background error variances and correlations

Modelling of errors

✔ the correct specification of observation and background error covariances is crucial for the quality of the analysis

✔ they determine the way how the background field will be corrected to match the observations

✔ uncertainty in the background, observations and analysis: some model of the errors between these vectors and the true state

✔ assume some probability density function (pdf) for each kind of error

✔ the averages of the errors (biases) represent the systematic problem in the assimilation system; it is important to subtract them from the errors => unbiased errors

Error covariances

- ✔ **background errors:** $\varepsilon_b = x_b - x_t$ **with mean** $\overline{\varepsilon}_b$

  and covariances $\mathbf{B} = \overline{(\varepsilon_b - \overline{\varepsilon}_b)(\varepsilon_b - \overline{\varepsilon}_b)^T}$

- ✔ **observation errors:** $\varepsilon_o = y - H(x_t)$ **with mean** $\overline{\varepsilon}_o$

  and covariances $\mathbf{R} = \overline{(\varepsilon_o - \overline{\varepsilon}_o)(\varepsilon_o - \overline{\varepsilon}_o)^T}$

- ✔ **analysis errors:** $\varepsilon_a = x_a - x_t$ **with mean** $\overline{\varepsilon}_a$

  and covariances $\mathbf{A} = \overline{(\varepsilon_a - \overline{\varepsilon}_a)(\varepsilon_a - \overline{\varepsilon}_a)^T}$

✔ in a scalar system we have the variance: $\mathbf{B} = var(\varepsilon_b) = \overline{(\varepsilon_b - \overline{\varepsilon}_b)^2}$

✔ in a multidimensional system we have a square symmetric matrix (positive definite) of dimension *n* x *n*:

- the diagonal terms contain the variances for each variable
- the off-diagonal terms contains the cross-covariances between pairs of variables

✔ if the model state is a three-dimensional: (e_1, e_2, e_3)

$$\mathbf{B} = \begin{pmatrix} var(e_1) & cov(e_1, e_2) & cov(e_1, e_3) \\ cov(e_1, e_2) & var(e_2) & cov(e_2, e_3) \\ cov(e_1, e_3) & cov(e_2, e_3) & var(e_3) \end{pmatrix}$$

✔ the off-diagonal terms can be transformed into error correlations:

$$\rho(e_i, e_j) = \frac{cov(e_i, e_j)}{\sqrt{var(e_i)var(e_j)}}$$

Observation error variances and correlations

✔ observation variances are mainly specified according to the knowledge of instrumental characteristics

✔ they should include also the variance of the representativeness errors

✔ the observation biases should be removed from the observation error variances because they will produce biases in the analysis increments

✔ observation error correlations are often assumed to be zero; this is not true for sets of observations performed by the same platform (radiosonde, aircraft or satellite measurements)

✔ the observation pre-processing can also generate artificial correlations between the transformed observations

✔ if the background is used in the observation pre-processing, this will introduce artificial correlations between observations and background errors

Background error variances and correlations

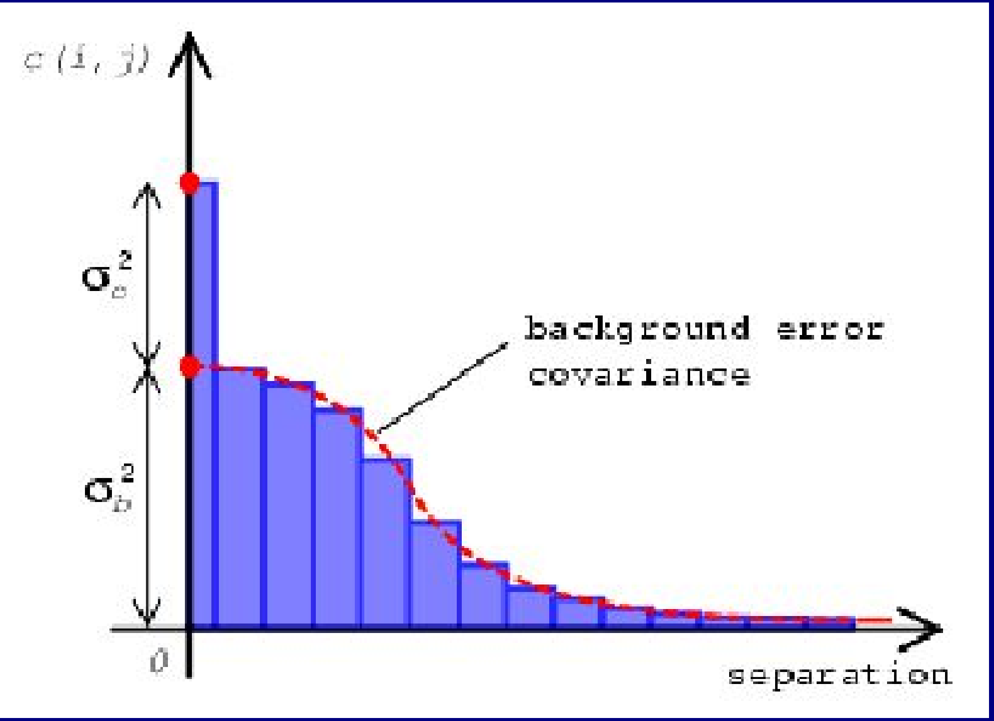
✔ the background covariances are difficult to estimate, because they are never observed directly

✔ the best source of information: study of the background departures

$$y - H(x_b)$$

✔ ***observational method***
(*Hollingworth-Lonnberg method*)
(Hollingsworth and Lonnberg, 1986, and Lonnberg and Hollingsworth, 1986):

relies on the use of background departures in an observing network that is dense and large enough to provide information on many scales, and that can be assumed to consist of uncorrelated and discrete observations



✔ *the NMC method* **(Parrish and Derber,1992): computes background error statistics from differences between two forecasts for different ranges (for ex.** ***36h - 12 h*** **or** ***48h - 24h*****), which are valid at the same time**

- **it is easy to implement in the operational schemes**
- **can not describe well the correlations in data sparse areas**
- **overestimate the correlation length scales because of the 24h time lag**

✔ *the lagged NMC method* **(Siroka et al., 2003): a variant of the NMC method for limited area models -> same coupling files for the two forecast and the initial condition of the short forecast is the same with the coupling file used for the long forecast at the corresponding time**

✔ *the analysis ensemble method* **(Houtekamer et al., 1996, and Fisher, 2003): computes background error statistics from differences between short-range forecasts (for ex.** ***6h*****) coming from different members of the ensemble**

✔ **the ensemble approach has been compared with the two variants of the NMC method (Berre, Stefanescu and Belo Pereira, 2006)**

✔ **the full covariance matrix is usually too big to be specified explicitly**

✔ **additionally, B needs to have some physical properties which are required to be reflected in the analysis:**

- **the correlations must be smooth in physical space**
- **the correlations should go to zero for very large separations**
- **the correlations should not exhibit physically unjustifiable variations according to direction or location**
- **the most fundamental balance properties, like geostrophy, must be reasonably well enforced**
- **the correlations should not lead to unreasonable background error variances for any parameter that is observed**

- ✔ **the specification of background error covariances is a complex problem**
- ✔ **some of the more popular techniques to do this are:**

- correlation models can be specified independently from variance fields

- vertical autocorrelation matrices for each parameter can be specified explicitly

- horizontal autocorrelations can be reduced to sparse matrices by assuming that they are homogeneous and isotropic to some extent

- three-dimensional multivariate correlation models can be built by carefully combining separability, homogeneity and independency hypotheses

- balance constraints can be enforced by transforming the model variables into suitably defined complementary spaces of balanced and unbalanced variables

- the geostrophic balance constraint can be enforced using the classical *f-plane* or β*-plane* balance equations

- more general kinds of balance properties can be expressed using linear regression operators calibrated on actual background error fields (ex. Berre, 2000, for Aladin model: non-separable multivariate scheme)

The representation of the analysis effect in three error simulation techniques
(Berre, Stefanescu and Belo Pereira, 2006)

Introduction

The evolution of the model state errors

The ensemble simulation method

The standard NMC method

Experiments: ensemble against standard NMC

The lagged NMC method

Comparison between the statistics of the three techniques

Conclusions and perspectives

Introduction

✔ **Three error simulation techniques are compared formally, in particular regarding their representation of the analysis step:**

- **ensemble method (Houtekamer et al., 1996, and Fisher, 2003)**
- **(standard) NMC method (Parrish and Derber, 1992)**
- **lagged NMC method (Siroka et al., 2003)**

✔ **The associated results are examined for the Aladin-France limited area model, which is coupled with the Arpege global model.**

The evolution of the model state errors

The forecast step

t_i x_*^i x_a^i $e_a^i = x_a^i - x_*^i$

$t_{i+1} = t_i + 6h$ $x_b^{i+1} = M x_a^i$

$$x_*^{i+1} = M_* x_*^i$$

$$e_b^{i+1} = x_b^{i+1} - x_*^{i+1}$$

$$e_b^{i+1} = M e_a^i + e_m^{i+1} \quad \textbf{(1)}$$

$$e_m^{i+1} = (M - M_*)\, x_*^i$$

The analysis step

$$t_{i+1} = t_i + 6h$$

$$x_a^{i+1} = x_b^{i+1} + \mathbf{K}(y^{i+1} - Hx_b^{i+1})$$ **(2)**

$$\mathbf{K} = \mathbf{B}H^T(H\mathbf{B}H^T + \mathbf{R})^{-1}$$

Apply the analysis equation to x_*^{i+1} **and** $y_{*,H}^{i+1} = Hx_*^{i+1}$

$$x_*^{i+1} = x_*^{i+1} + \mathbf{K}(y_{*,H}^{i+1} - Hx_*^{i+1})$$ **(3)**

by definition $y_{*,H}^{i+1} - Hx_*^{i+1} = 0$ **=>** $\mathbf{K}(0) = 0$

(2) – (3) => $e_a^{i+1} = e_b^{i+1} + \mathbf{K}(e_o^{i+1} - He_b^{i+1})$ **(4)**

with $e_o^{i+1} = y^{i+1} - y_{*,H}^{i+1} = y^{i+1} - Hx_*^{i+1}$

$e_{om}^{i+1} = y^{i+1} - y_*^{i+1}$ $e_{or}^{i+1} = y_*^{i+1} - y_{*,H}^{i+1} = y_*^{i+1} - Hx_*^{i+1}$

The ensemble simulation method

The forecast step

t_i $x_{a,k}^i, x_{a,l}^i$ $\delta_{o,k}, \delta_{o,l}$ $\varepsilon_a^i = x_{a,k}^i - x_{a,l}^i$

$t_{i+1} = t_i + 6h$

$$x_{b,k}^{i+1} = Mx_{a,k}^i + \delta_{m,k}^{i+1}$$

$$x_{b,l}^{i+1} = Mx_{a,l}^i + \delta_{m,l}^{i+1}$$

$\delta_{m,k}^{i+1}, \delta_{m,l}^{i+1}$

$$\varepsilon_b^{i+1} = x_{b,k}^{i+1} - x_{b,l}^{i+1} \qquad \varepsilon_m^{i+1} = \delta_{m,k}^{i+1} - \delta_{m,l}^{i+1}$$

$$\varepsilon_b^{i+1} = M\varepsilon_a^i + \varepsilon_m^{i+1} \quad (5)$$

The analysis step

$$t_{i+1} = t_i + 6h$$

$$y_k^{i+1} = y^{i+1} + \delta_{o,k}^{i+1} \qquad x_{a,k}^{i+1} = x_{b,k}^{i+1} + \mathbf{K}(y_k^{i+1} - Hx_{b,k}^{i+1})$$

$$y_l^{i+1} = y^{i+1} + \delta_{o,l}^{i+1} \qquad x_{a,l}^{i+1} = x_{b,l}^{i+1} + \mathbf{K}(y_l^{i+1} - Hx_{b,l}^{i+1})$$

$$\varepsilon_o^{i+1} = y_k^{i+1} - y_l^{i+1}$$

$$\varepsilon_a^{i+1} = \varepsilon_b^{i+1} + \mathbf{K}(\varepsilon_o^{i+1} - H\varepsilon_b^{i+1}) \quad \textbf{(6)}$$

The comparison between the model state errors and ensemble simulation errors

$$e_b^{i+1} = Me_a^i + e_m^{i+1} \quad (1)$$

$$e_a^{i+1} = e_b^{i+1} + \mathbf{K}(e_o^{i+1} - He_b^{i+1}) \quad (4)$$

$$\varepsilon_b^{i+1} = M\varepsilon_a^i + \varepsilon_m^{i+1} \quad (5)$$

$$\varepsilon_a^{i+1} = \varepsilon_b^{i+1} + \mathbf{K}(\varepsilon_o^{i+1} - H\varepsilon_b^{i+1}) \quad (6)$$

The evolution processes and equations that affect the ensemble difference fields are the same as those of the true error fields.

Some other requirements for the perturbation covariances of the observations and of the model are involved, in order to achieve a realistic error covariance estimation.

$$e_a = (\mathbf{I} - \mathbf{K}H)e_b + \mathbf{K}e_o$$

$$\mathbf{A}_* = (\mathbf{I} - \mathbf{K}H)\mathbf{B}_*(\mathbf{I} - \mathbf{K}H)^T + \mathbf{K}\mathbf{R}_*\mathbf{K}^T$$

$$\varepsilon_a = (\mathbf{I} - \mathbf{K}H)\varepsilon_b + \mathbf{K}\varepsilon_o$$

$$\mathbf{A}_\varepsilon = (\mathbf{I} - \mathbf{K}H)\mathbf{B}_\varepsilon(\mathbf{I} - \mathbf{K}H)^T + \mathbf{K}\mathbf{R}_\varepsilon\mathbf{K}^T$$

$$\delta_{o,k}, \delta_{o,l} \sim \mathcal{N}(0, \mathbf{R}) \quad \mathbf{R}_\varepsilon = 2\ \mathbf{R}_*$$

$$\delta_{b,k}, \delta_{b,l} \sim \mathcal{N}(0, \mathbf{B}) \quad \mathbf{B}_\varepsilon = 2\ \mathbf{B}_*$$

=> $\mathbf{A}_\varepsilon = 2\ \mathbf{A}_*$

$$\begin{aligned} \mathbf{R}_\varepsilon &= \overline{\varepsilon_o(\varepsilon_o)^T} \qquad\qquad \mathbf{R} \approx \mathbf{R}_* \\ &= \overline{\delta_{o,k}(\delta_{o,k})^T} + \overline{\delta_{o,l}(\delta_{o,l})^T} - \overline{\delta_{o,k}(\delta_{o,l})^T} - \overline{\delta_{o,l}(\delta_{o,k})^T} = 2\ \mathbf{R} \end{aligned}$$

The standard NMC method

The first assimilation cycle

t_i **(30h before the verification time)**

$$dx^i = x_a^i - x_b^i$$ **the first analysis increment**

$$\begin{aligned}
\varepsilon_a^i &= dx^i \\
&= \mathbf{K}(y^i - Hx_b^i) \\
&= \mathbf{K}(e_o^i - He_b^i)
\end{aligned}$$

$$\varepsilon_a^i = -\mathbf{K}H\ e_b^i + \mathbf{K}\ e_o^i$$

✔ NMC equation $\varepsilon_a^i = -\mathbf{K}H\ e_b^i + \mathbf{K}\ e_o^i$ can be compared to:

- exact analysis error equation $e_a^i = (I - \mathbf{K}H)\ e_b^i + \mathbf{K}\ e_o^i$

- ensemble analysis dispersion equation $\varepsilon_a^i = (I - \mathbf{K}H)\ \varepsilon_b^i + \mathbf{K}\ \varepsilon_o^i$

✔ rezonable approximation if observations are very dense (Bouttier, 1994):

$H \sim I$ $\mathbf{K}H \sim I/2$ $\mathbf{R} \sim H\mathbf{B}H^T \sim \mathbf{B}$

✔ if the observation density is poor:

$\mathbf{K}H$ acts as a low-pass filter

$I - \mathbf{K}H$ acts as a high-pass filter

The observation error spectrum tends to be white, while the background error spectrum tends to be red (Daley, 1991).

The next three assimilation cycles

$$t_{i+1} = t_i + 6h$$

$$\begin{aligned} \varepsilon_b^{i+1} &= M\ \varepsilon_a^i \\ &= M\ dx^i \end{aligned}$$

Second analysis step: 24h before the verification time

$$\begin{aligned} \varepsilon_a^{i+1} &= M dx^i + dx^{i+1} \\ &= \varepsilon_b^{i+1} + \mathbf{K}(e_o^{i+1} - He_b^{i+1}) \end{aligned}$$

Different representation of the analysis effect:

- NMC: ***adding the analysis increment to some earlier increments***

- ensemble method: ***applying the analysis equation to the perturbations***

After the fourth analysis, the final NMC perturbation is:

$$\begin{aligned}\varepsilon_a^{i+3} &= \varepsilon_b^{i+3} + dx^{i+3} \\ &= M^3 dx^i + M^2 dx^{i+1} + M dx^{i+2} + dx^{i+3}\end{aligned}$$

M^j **is the forecast evolution operator during a period of** ***j x 6h***

The final 12-h evolution of the fourth analysis perturbation is:

$$\varepsilon_b^{i+5} = M^2 \varepsilon_a^{i+3}$$

ε_a^{i+3} **corresponds to the** ***00h-24h*** **differences**

ε_b^{i+5} **corresponds to the 12*h-36h* differences**

The three specific components of the NMC method

✔ **ensemble method:** $$\varepsilon_b^{i+5} = M\ \varepsilon_a^{i+4}$$

✔ **NMC method:** $$\varepsilon_b^{i+5} = M^2\ \left(\sum_{j=1}^{4} M^{4-j}\ dx^{i+j-1} \right)$$

✔ **The three specific components of the NMC method (compared to the ensemble method) are:**

- **the involvement of longer forecast ranges (see the occurrence of the matrices M^2, M^{4-j} instead of the 6-h matrix M)**

- **the accumulation of several increments (see the occurrence of the operator Σ)**

- **the involvement of analysis increments dx, instead of analysis differences ε_a**

Experiments: ensemble against standard NMC

The Arpege/Aladin experiments

✔ time averaged statistics: computed over a 48 days period (4 February 2002 – 23 March 2002), using fields valid at 12 UTC

✔ Aladin ensemble: generated in 2 steps

➔ the Arpege global ensemble (perfect model framework) (Belo Pereira and Berre, 2006)

✗ Arpege 4D-Var experiments:

- stretched grid (factor = 3.5)
- 41 vertical levels
- spectral truncation T298

➔ the ensemble of Aladin 6-h forecast: integrate the Aladin model with IC and LBCs provided by the Arpege ensemble (Stefanescu et al., 2006)

✗ Aladin experiments:

- Aladin/France integration domain
- 41 vertical levels
- square domain with L_x=L_y=2850 km
- number of grid points J=K=300
- δx=δy=9.5 km
- M=N=149

Comparison between the two final statistics

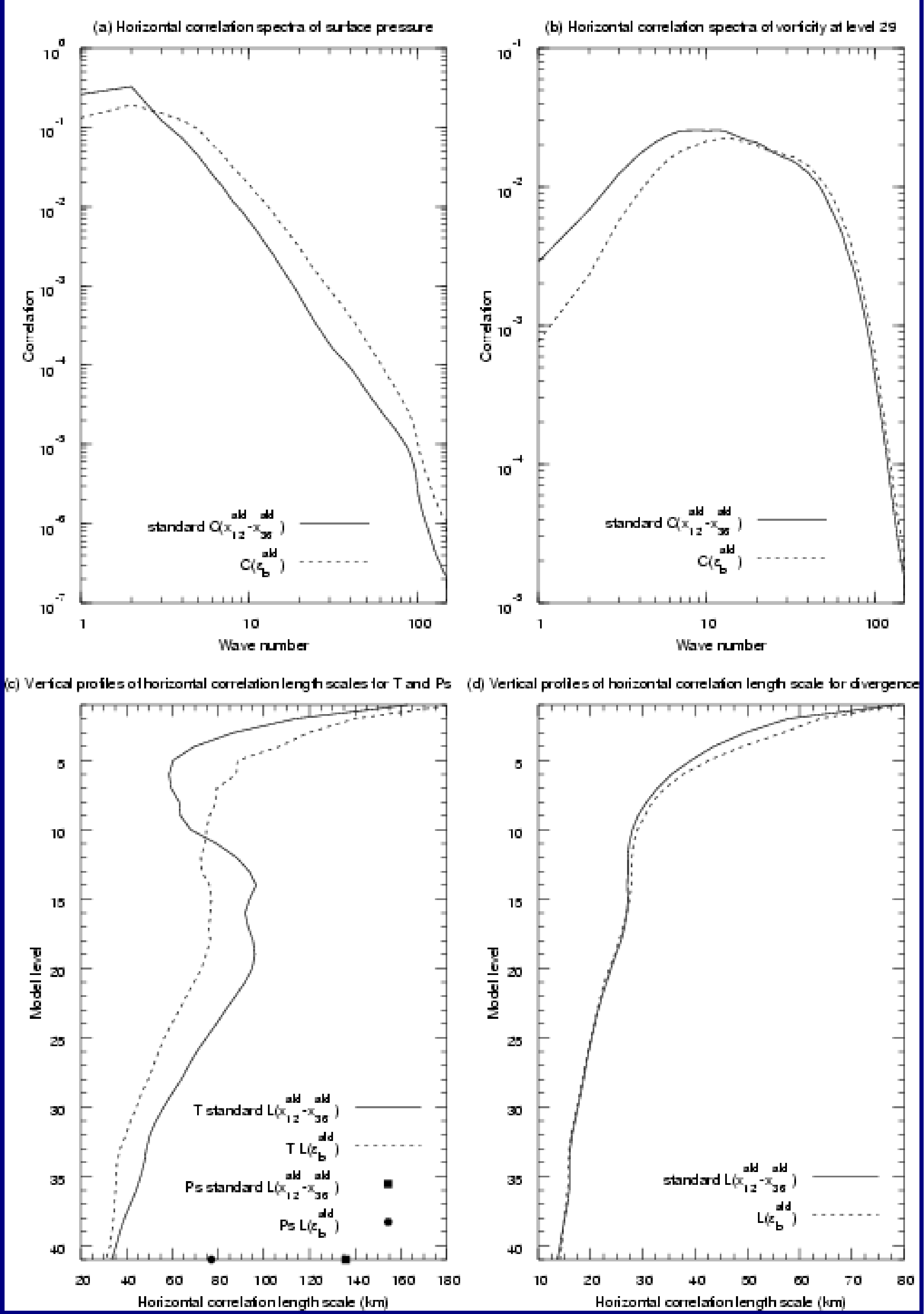
✔ ensemble approach: error spectra are shifted towards the small scales => smaller length scales

✔ the situation is opposite for highest 10-15 levels

✔ length scale differences are more pronounced for a large scale variable

✔ Fisher (2003): the NMC overestimation of the large scale contributions was attributed to the long forecast ranges

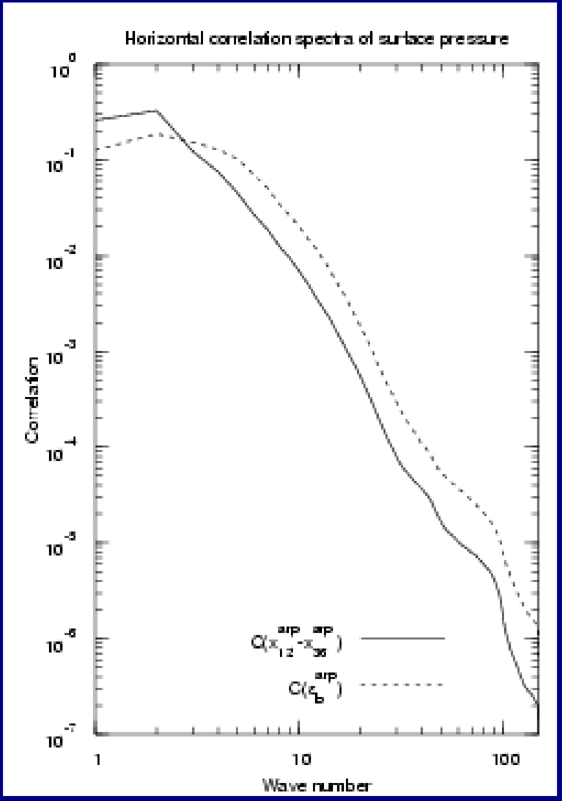
✔ another explanation could be the representation of the analysis effect



Diagnosis of the three contributions in the NMC method

✔ **examination of Arpege uninitialized spectra**

✔ **the NMC/ensemble contrast for Aladin spectra matches well with the contrast for the Arpege spectra**

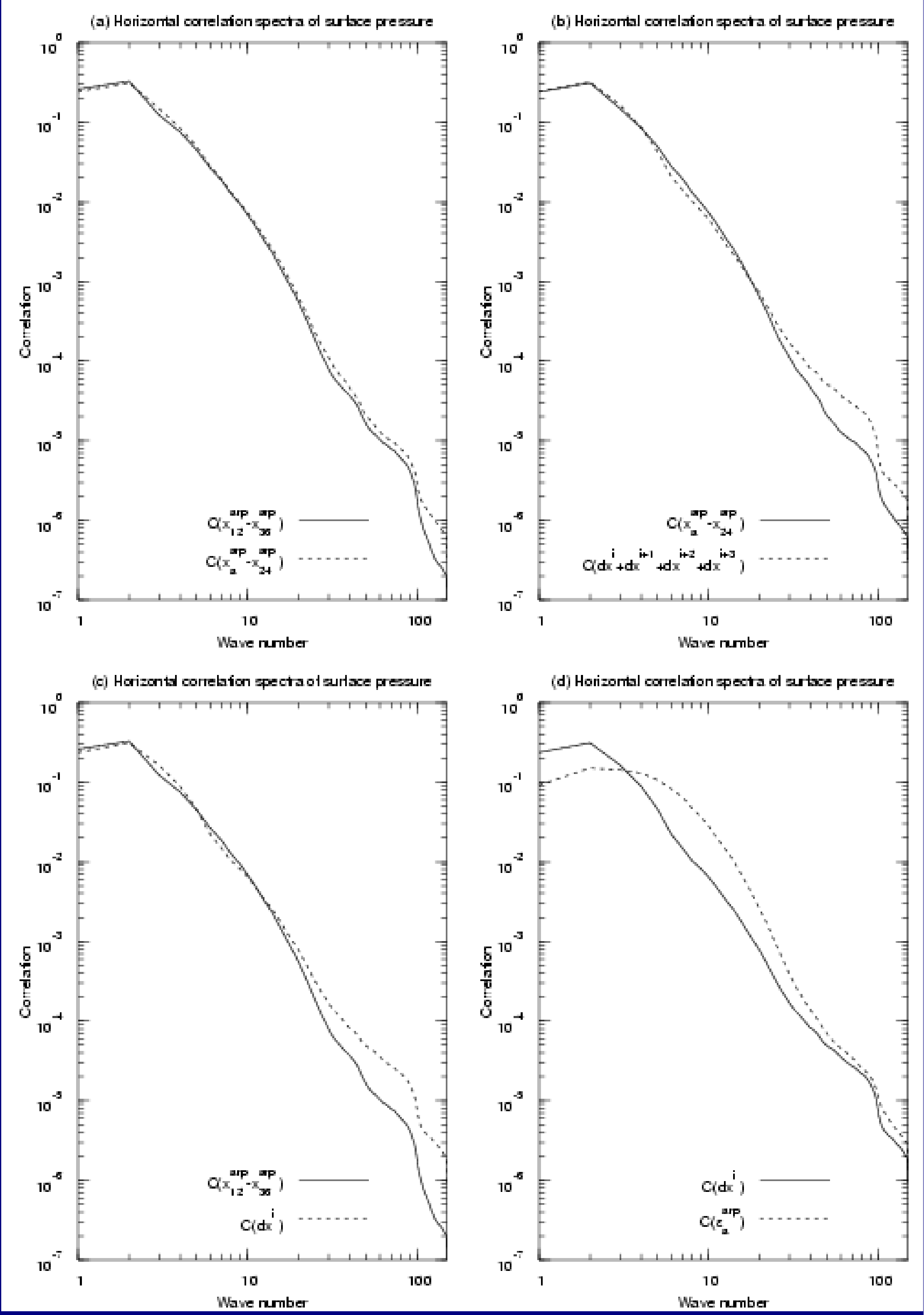


✔ the 12-h evolution: slight increase of the relative contribution of the larger scale

✔ the evolution of the first three increments contribute mostly to a slight enhancement of some intermediate larges scales (wn 4-20)

✔ the forecast evolutions and increment accumulation imply an increase of the large scale contributions

✔ the increment spectrum appear to be of a much larger scale than the analysis ensemble spectrum



✔ **00h-24h differences:**

$$
\begin{aligned}
\varepsilon_a^{i+3} &= \sum_{j=1}^{4} M^{4-j} dx^{i+j-1} \\
&= M^3 dx^i + M^2 dx^{i+1} + M dx^{i+2} + dx^{i+3}
\end{aligned}
$$

✔ **diagnose the effect of successive forecast evolutions of the first three analysis increments**

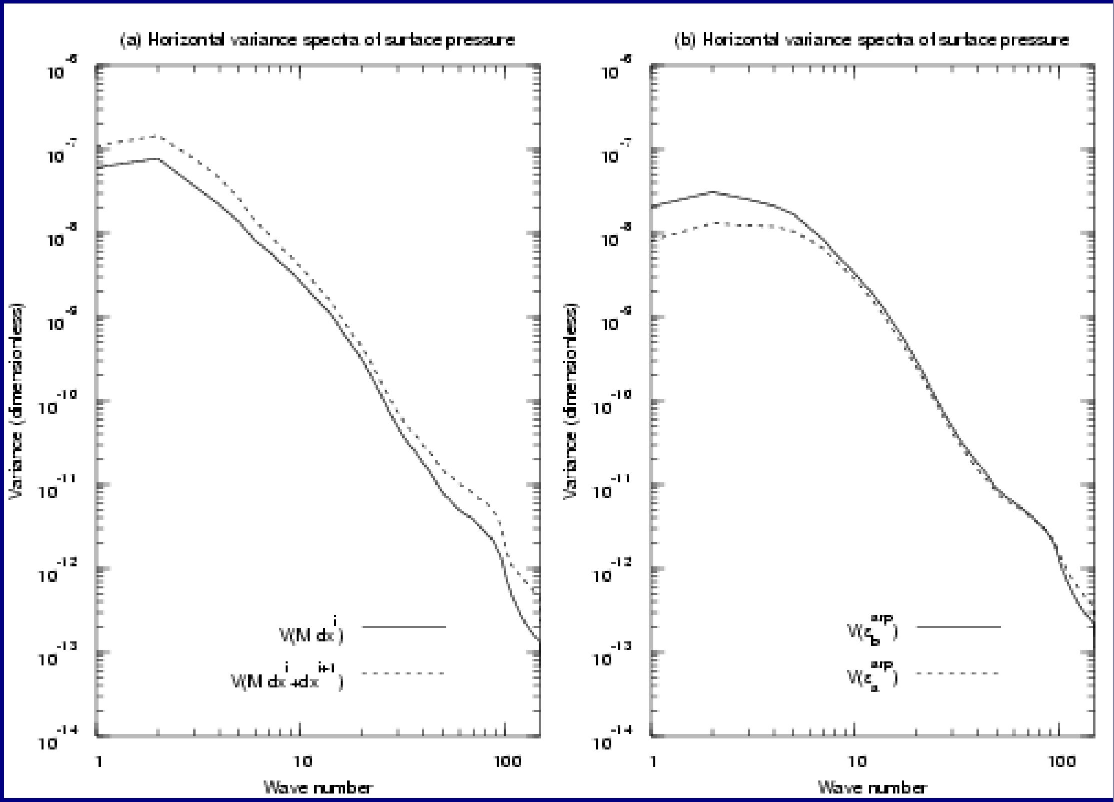
$$
\sum_{j=1}^{4} dx^{i+j-1} = dx^i + dx^{i+1} + dx^{i+2} + dx^{i+3}
$$

The evolution of the spectra during an analysis step

✔ **addition of the second increment implies a general increase of the variances**

✔ **in the ensemble method, the analysis contribute to the decrease of the large scale dispersion**

✔ **the NMC method is strongly based on the *analysis increments and on their accumulation and evolution,* while the ensemble method is rather *simulating the error reduction during the analysis step***

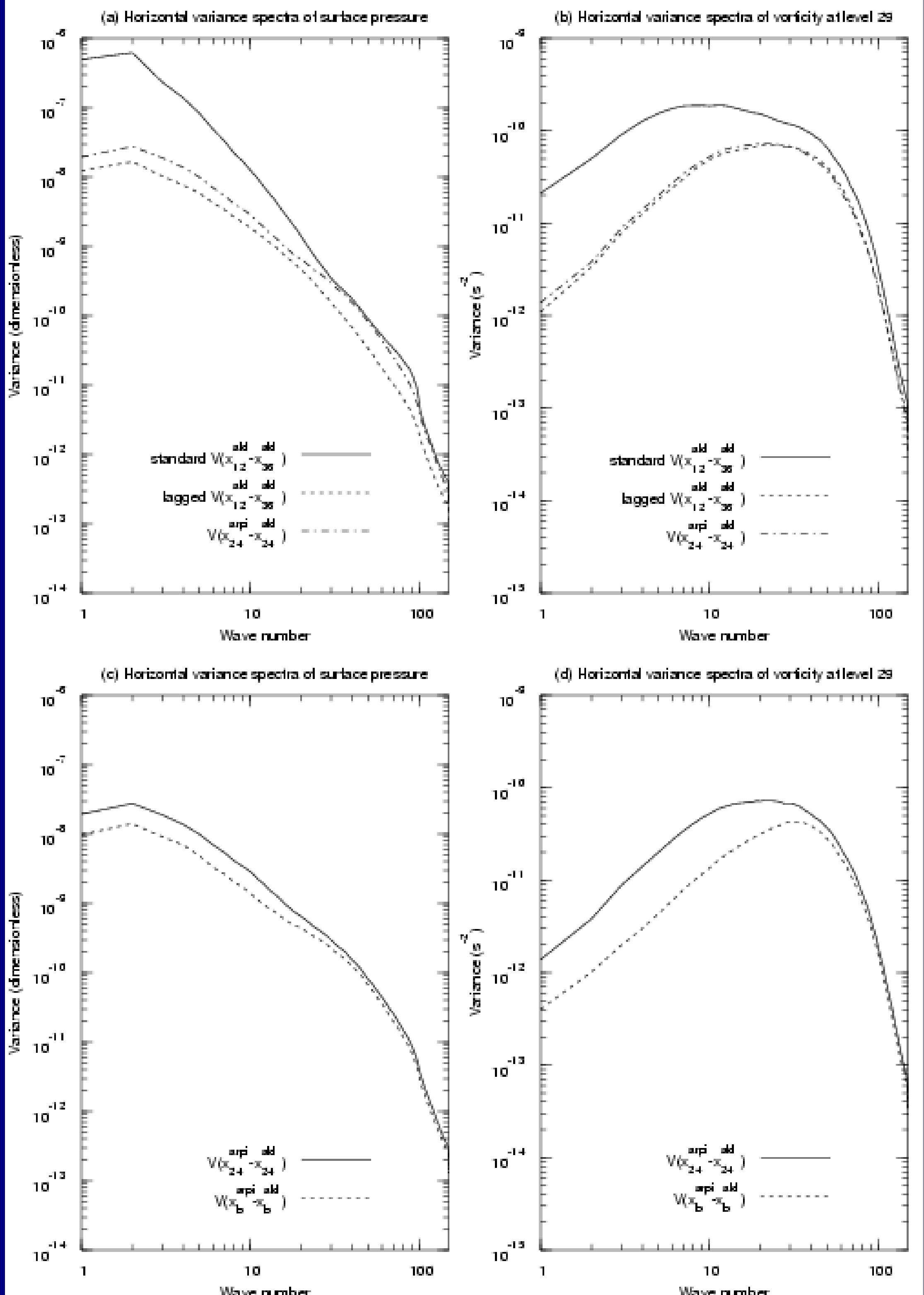


The lagged NMC method

Description of the method

✔ Siroka et al. (2003): instead of the usual operational ("fresh") fields, the LBCs and the IC of the 12h forecast correspond now to some fields that are derived from the "old" 36h forecast integration

✔ compared to standard NMC method: similar variances in the small scales, but much smaller variances in the large scales



The link with the Arpege/Aladin model differences

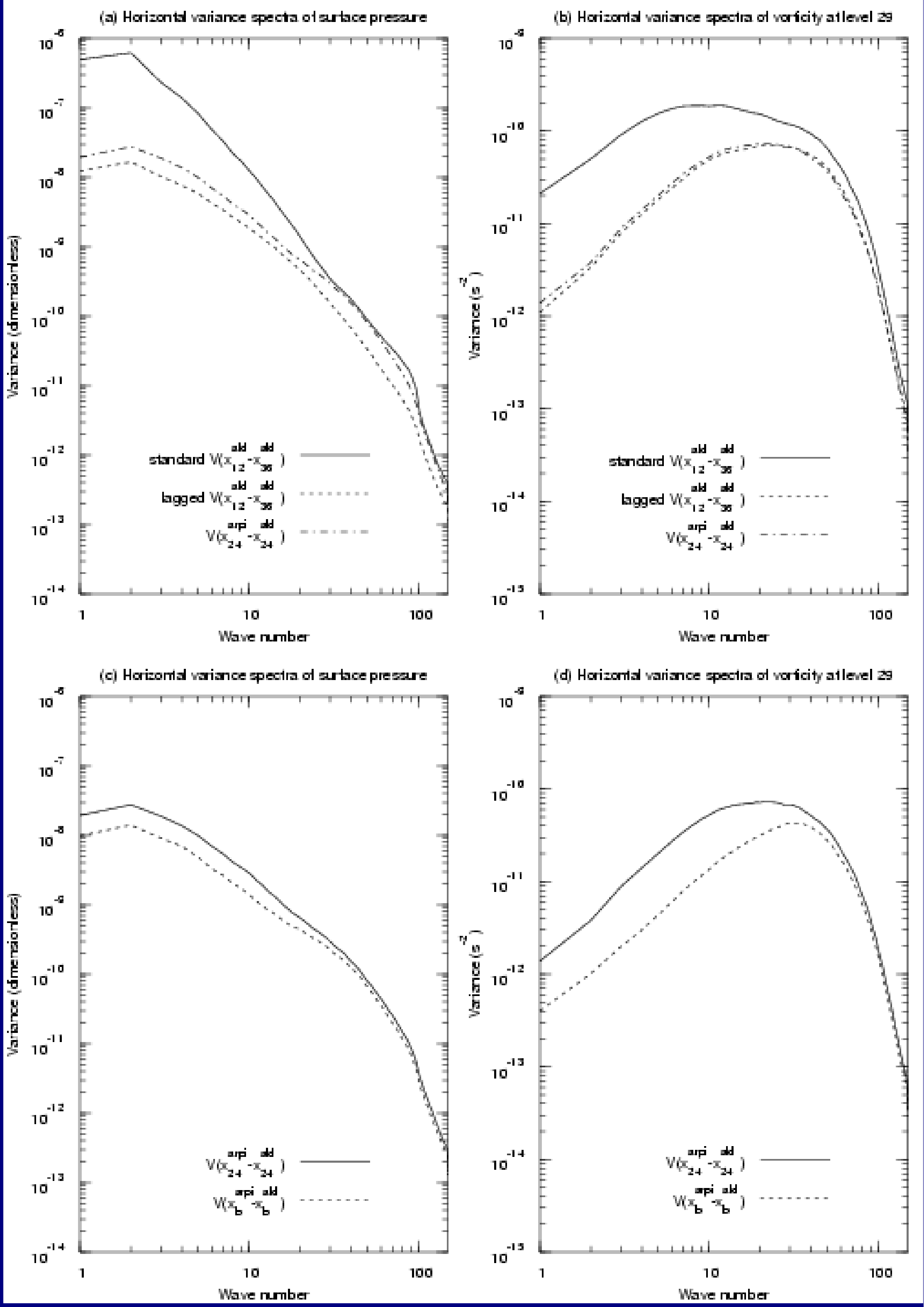
$$x_{12}^{ald} = \tilde{M}^2 D\tilde{H}\; x_{24}^{arp}$$

$$x_{36}^{ald} = \tilde{M}^2\; x_{24}^{ald}$$

$$x_{12}^{ald} - x_{36}^{ald} = \\ = \tilde{M}^2\; (D\tilde{H}\; x_{24}^{arp} - x_{24}^{ald})$$

✔ initial differences are caused by the differences between Arpege and Aladin models

$$D\tilde{H}\; x_{24}^{arp} - x_{24}^{ald} = \\ = (D\tilde{H} M^4 - \tilde{M}^4 D\tilde{H})\; x_a^{arp}$$

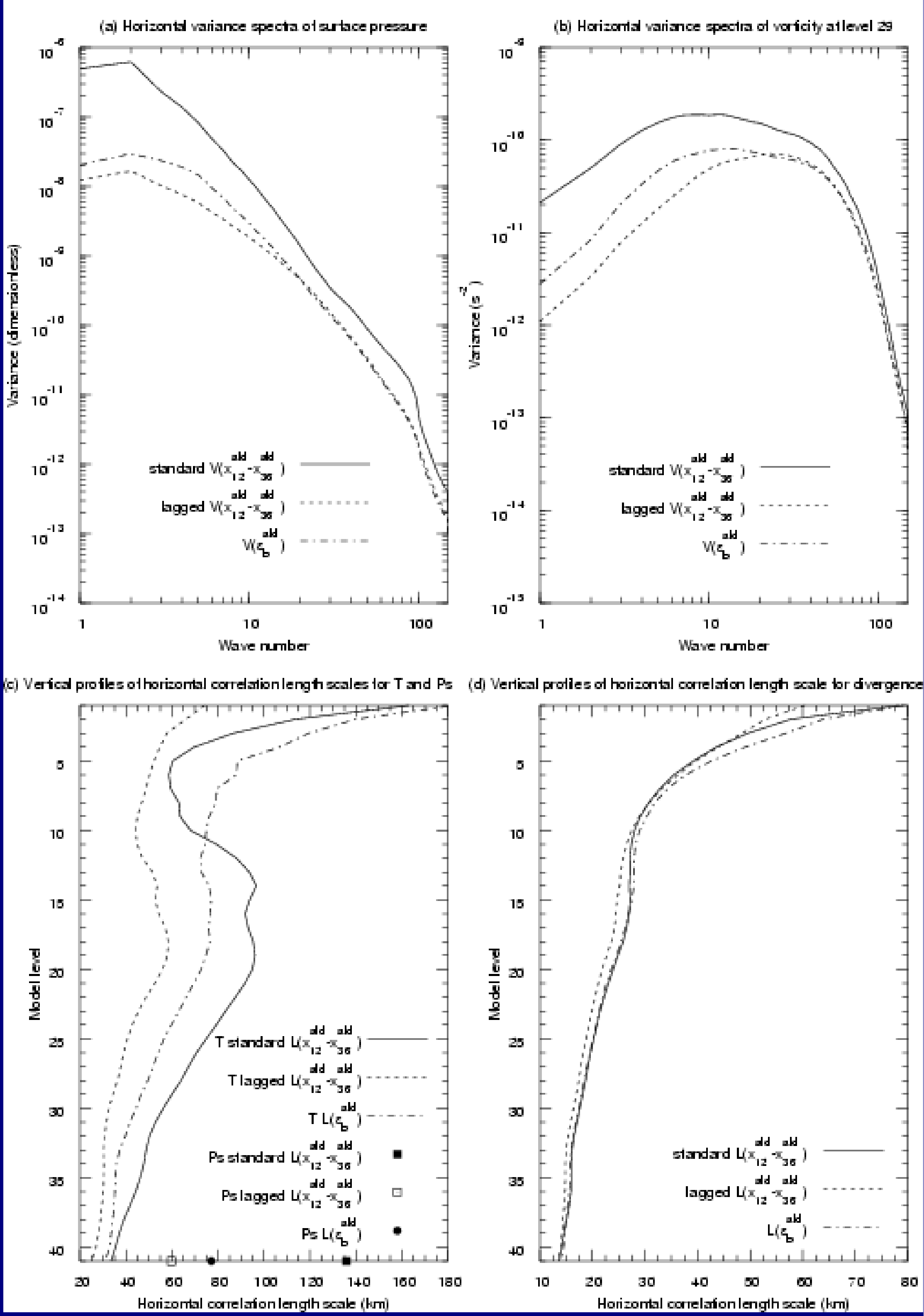


Comparison between the statistics of the three techniques

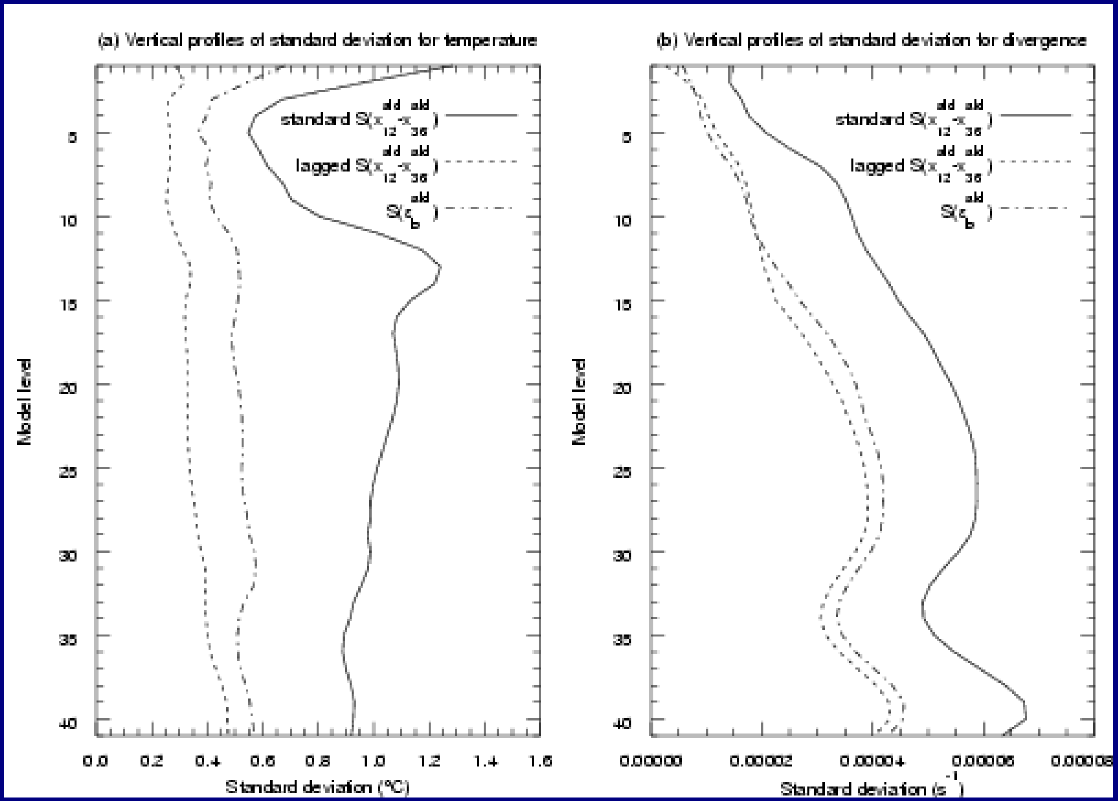
✔ the ensemble spectra are intermediate between those of standard and lagged NMC spectra

✔ the smaller amplitude of the ensemble large scale variances, compared to the standard NMC method: a more accurate representation of the influence of the analysis step and of the short forecast ranges

✔ the larger amplitude of the ensemble large scale variances, compared to the lagged NMC method: the representation of the IC and LBC uncertainties



✔ **the standard deviation results are consistent with the variance spectra: ensemble results are intermediate between those of the two NMC methods**



Conclusions

✔ the analysis equation is the equation that transforms the background and observation errors into the analysis errors

?

✔ ensemble method: the analysis equation is the equation that transforms the background and observation perturbations into the analysis perturbations

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✔ the representation of the analysis step is inaccurate in the standard NMC method: it relies essentially on the accumulation and the evolution of some analysis increments

✔ lagged NMC method: the representation of the initial errors and of the analysis effect is even more poor; it is closely related to the Arpege/Aladin model differences

✔ the ensemble approach provides results that are intermediate between those of the standard and lagged NMC method variants

✔ ensemble method: the influences of the analysis equation, of the implied IC and LBCs uncertainties, and of the involved short forecast ranges, are represented in a more accurate way than in the two NMC method variants

Perspectives

✔ the ensemble method is used to specify the corresponding error statistics of the Aladin 3D-Var (Fischer et al., 2006); the latter may include a term which will control the distance to the Arpege analysis (Bouttier, 2002)

✔ a posteriori diagnostics can be carried out: this will allow to compare the ensemble-based statistics with the estimates retrieved from observations (Desroziers and Ivanov, 2001, Sadiki and Fischer, 2005)

✔ the introduction of model perturbations may help to increase further the realism of the ensemble simulation (Stefanescu et al, 2006)

✔ time dependent features may be explored, e.g. by running a larger number of ensemble members and/or e.g. by relating the error structures to the background field structures (poster Stefanescu et al, 2006)

✔ use wavelets to diagnose and to specify the local error structures (on the globe: Fisher, 2003; for LAM: Deckmyn and Berre, 2005)

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Thank you for your attention !