Flow over resolved and unresolved orography

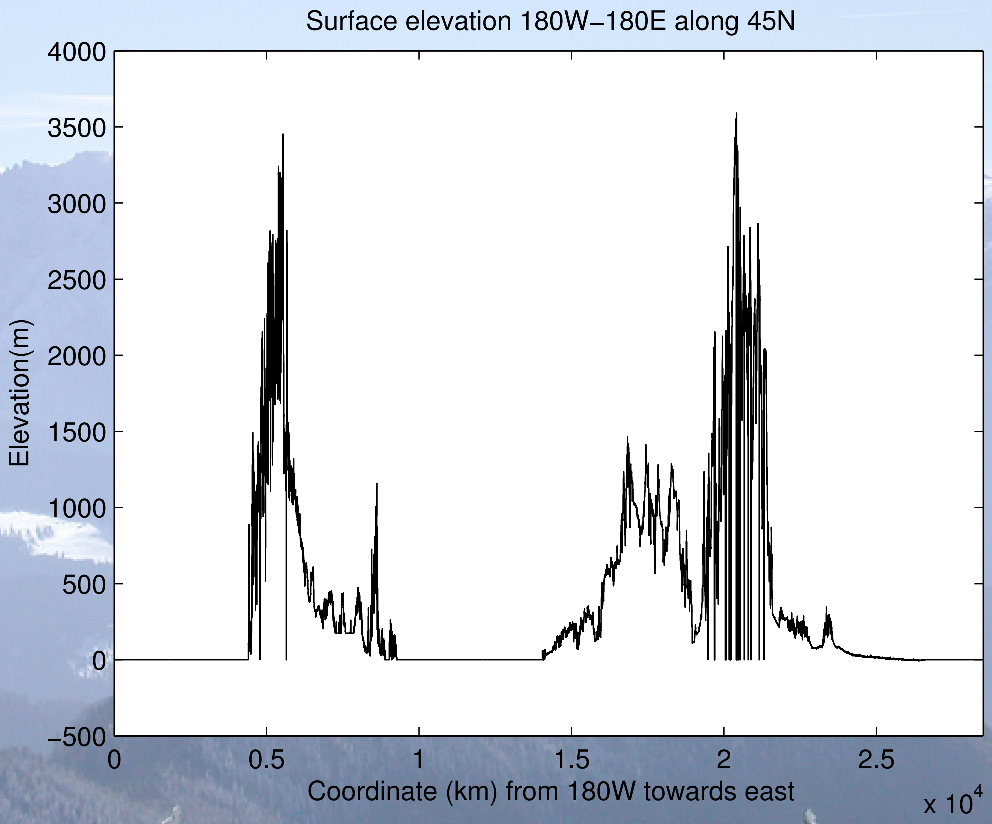
Laura Rontu, FMI
laura rontu@fmi.fi

June 3, 2006

FINNISH METEOROLOGICAL INSTITUTE

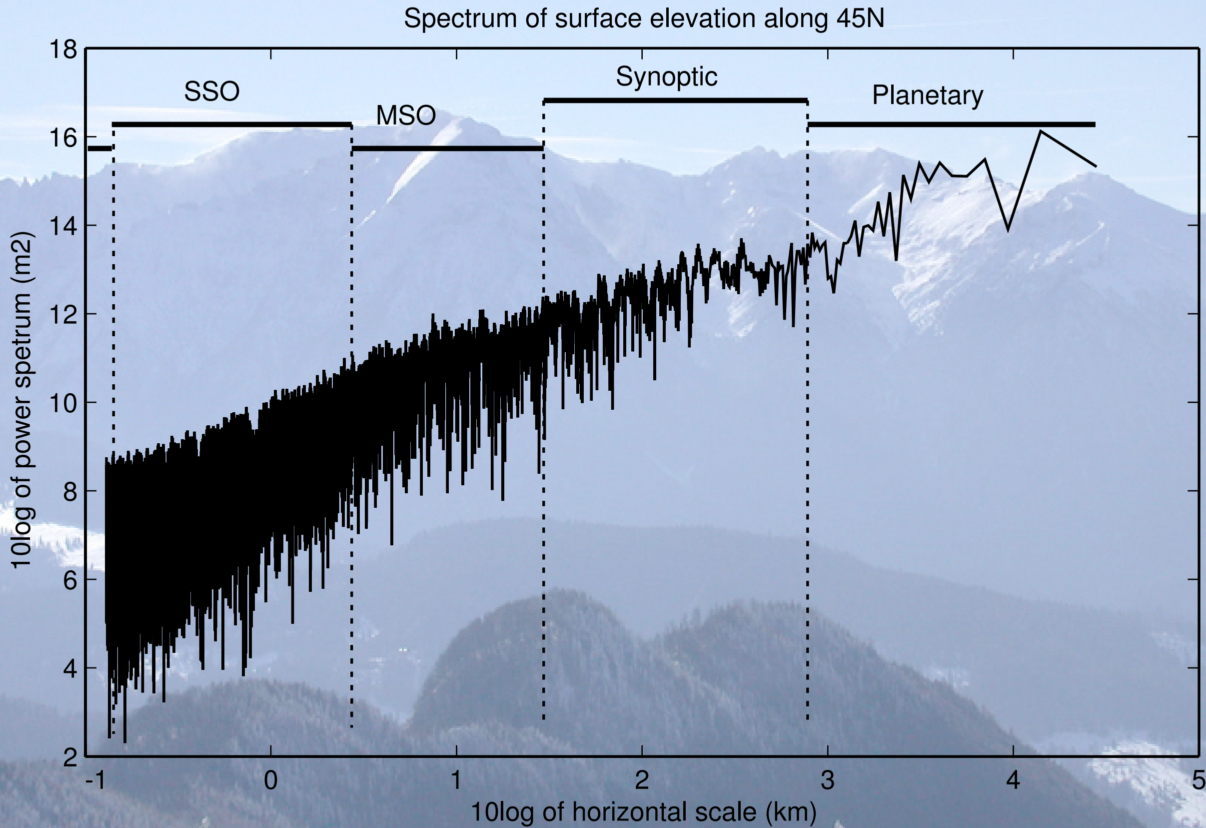
Contents

Global scales



Shuttle radar topography mission 3 arc second data: resolution 65 m

Home Page
Title Page
Page 4 of 30
Go Back
Full Screen
Close
Quit

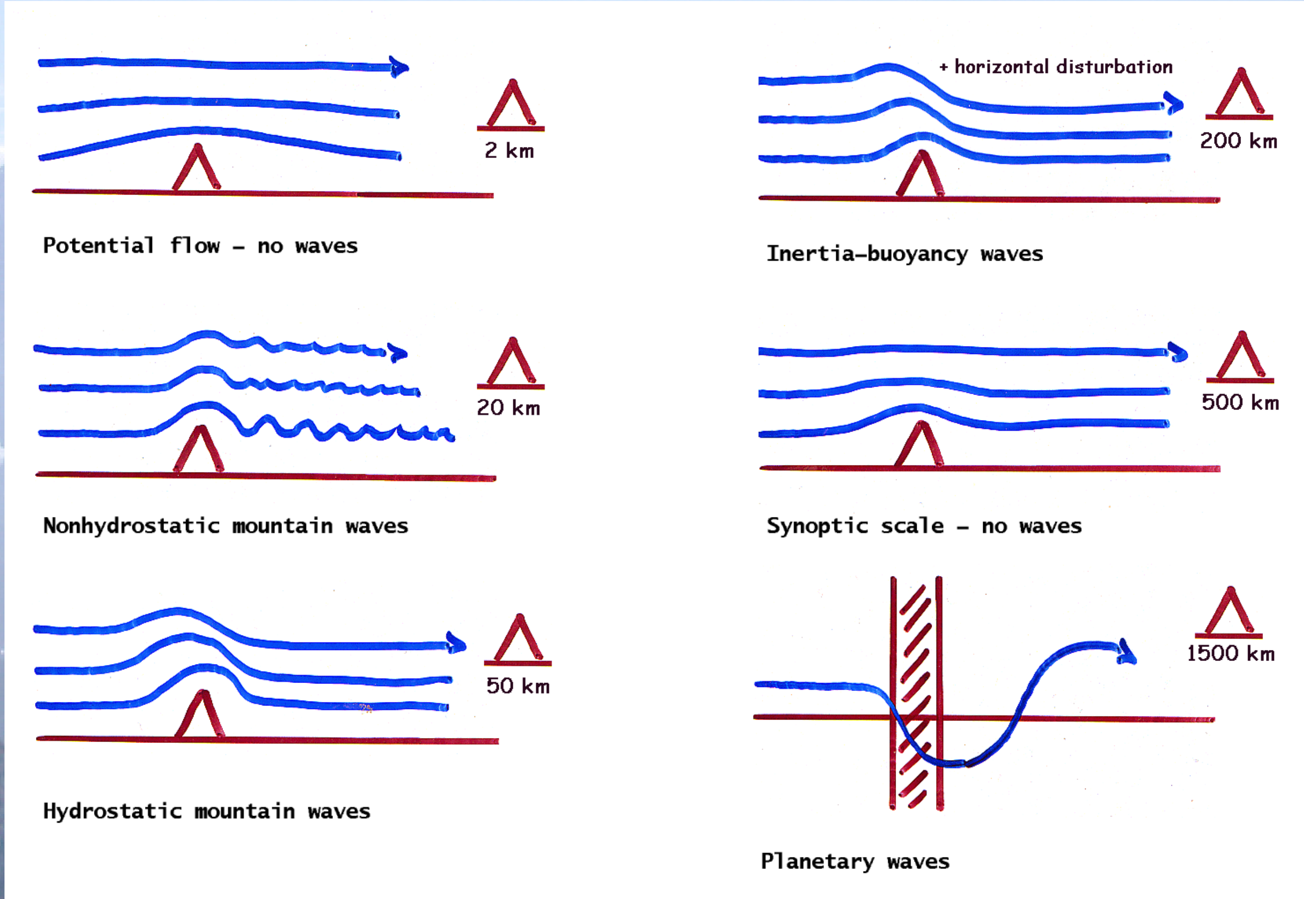


Some mountain-related atmospheric processes

Scale	Orographic phenomena	Time scale	Horizontal scale	Essential dynamics
Planetary	Planetary waves	Weeks	1000 - 10000 km	barotropic, hydrostatic conservation of absolute vorticity
Synoptic	Cyclo- and frontogenesis Large-scale precipitation Orographic lift	Days	100 - 1000 km	baroclinic quasi-geostrophic, hydrostatic conservation of potential vorticity
Meso	Buoyancy waves and blocking[1] Local (thermal) circulations Orographic convection Fog and low clouds	Hours - Day	1 - 100 km	[1]stable stratification hydrostatic $\longrightarrow$ nonhydrostatic rotating $\longrightarrow$ nonrotating directional effects
Small Micro	Turbulent eddies	Minutes - Hours	100 m - 1 km 10 m	non-hydrostatic non-rotating isotropic

Home Page
Title Page
Page 6 of 30
Go Back
Full Screen
Close
Quit

Orography-related waves in atmosphere



Momentum equation

The equation of horizontal hydrostatic motion in a pressure-based, terrain-following hybrid vertical ζ-coordinate system (Simmons and Burridge, 1981) is written following Kasahara, 1974:

$$\frac{\mathrm{d}\vec{\mathrm{v}}}{\mathrm{dt}} = -\frac{1}{\rho}\nabla_\zeta p - \nabla_\zeta \Phi - f\vec{k}\times\vec{v} - \frac{g}{p_s}\frac{\partial\vec{\tau}}{\partial\zeta}, \tag{1}$$

where $\vec{v}$ is the horizontal wind, ∇_ζ is the gradient operator applied along the constant ζ-surface, $\vec{k}$ is the unit vector in direction of $\vec{g}$, g is acceleration due to gravity, $\Phi = gz$ is geopotential, ρ is density of air, p is pressure and p_s surface pressure. $\vec{\tau} = -\rho\overline{\vec{v}' w'}$ is the stress vector related to the subgrid-scale vertical momentum fluxes; w is the vertical velocity, an overline denotes gridbox average and a prime $'$ subgrid-scale deviation.

Orography-related issues in a NWP

Home Page

Title Page

Page 8 of 30

Go Back

Full Screen

Close

Quit

Orography-related issues in a NWP

Boundary conditions

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Numerical problems

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Numerical problems

Pressure gradient term

Semi-lagragian trajectories

Spectral formulations

Noise and smoothing

Orography-related issues in a NWP

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Numerical problems

Pressure gradient term

Semi-lagragian trajectories

Spectral formulations

Noise and smoothing

Resolved and parametrized processes

Different subgrid scales

Interaction between resolved and parametrized processes

Column physics 1D - 2D - 3D approach

Home Page

Title Page

Page 8 of 30

Go Back

Full Screen

Close

Quit

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Numerical problems

Pressure gradient term

Semi-lagragian trajectories

Spectral formulations

Noise and smoothing

Resolved and parametrized processes

Different subgrid scales

Interaction between resolved and parametrized processes

Column physics 1D - 2D - 3D approach

Problems of data assimilation

Orography-related issues in a NWP

Boundary conditions

Lower boundary condition

Upper boundary condition

Lateral boundary conditions over mountains

Numerical problems

Pressure gradient term

Semi-lagragian trajectories

Spectral formulations

Noise and smoothing

Resolved and parametrized processes

Different subgrid scales

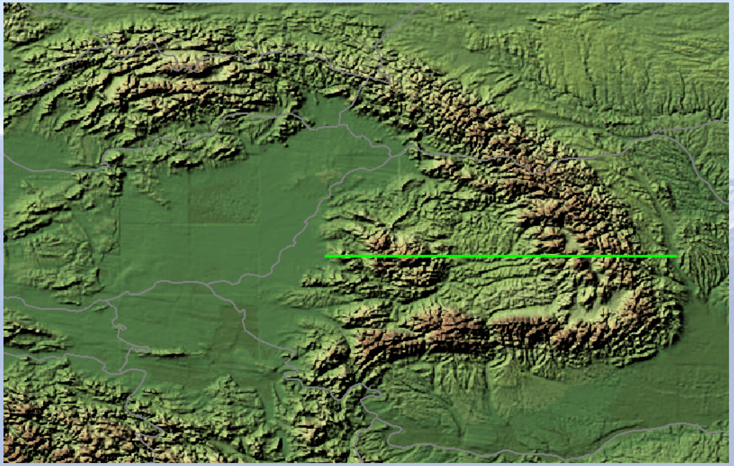
Interaction between resolved and parametrized processes

Column physics 1D - 2D - 3D approach

Problems of data assimilation

Elevation of observation sites and model orography

A Carpathian example



Home Page

Title Page

Page 9 of 30

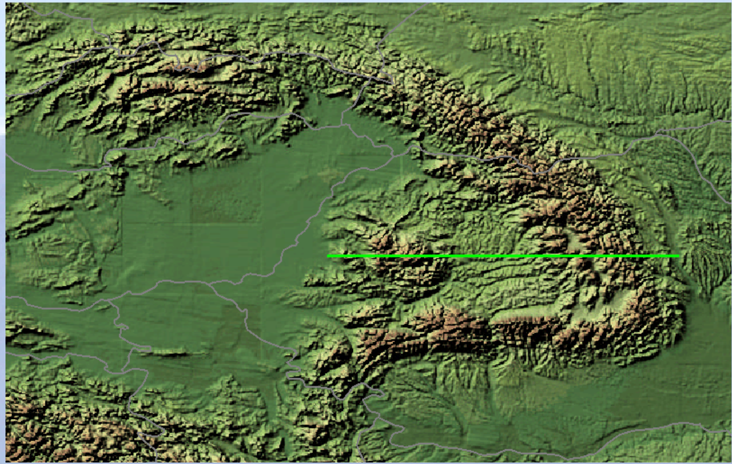
Go Back

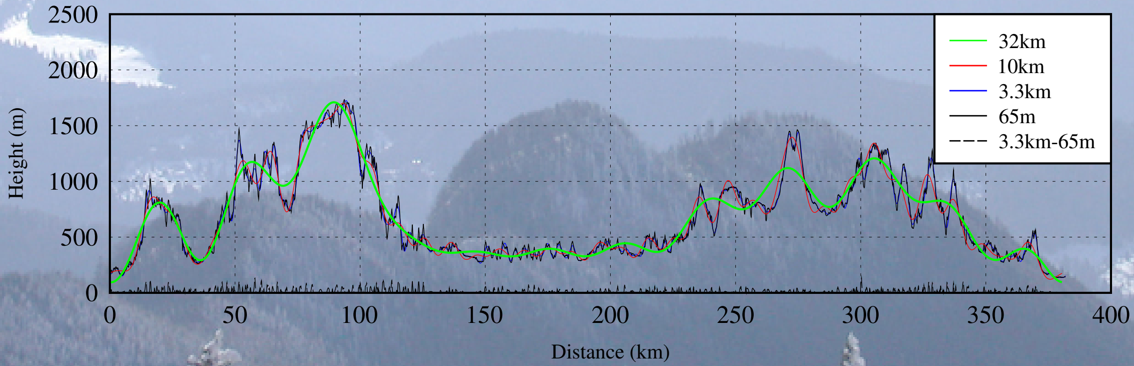
Full Screen

Close

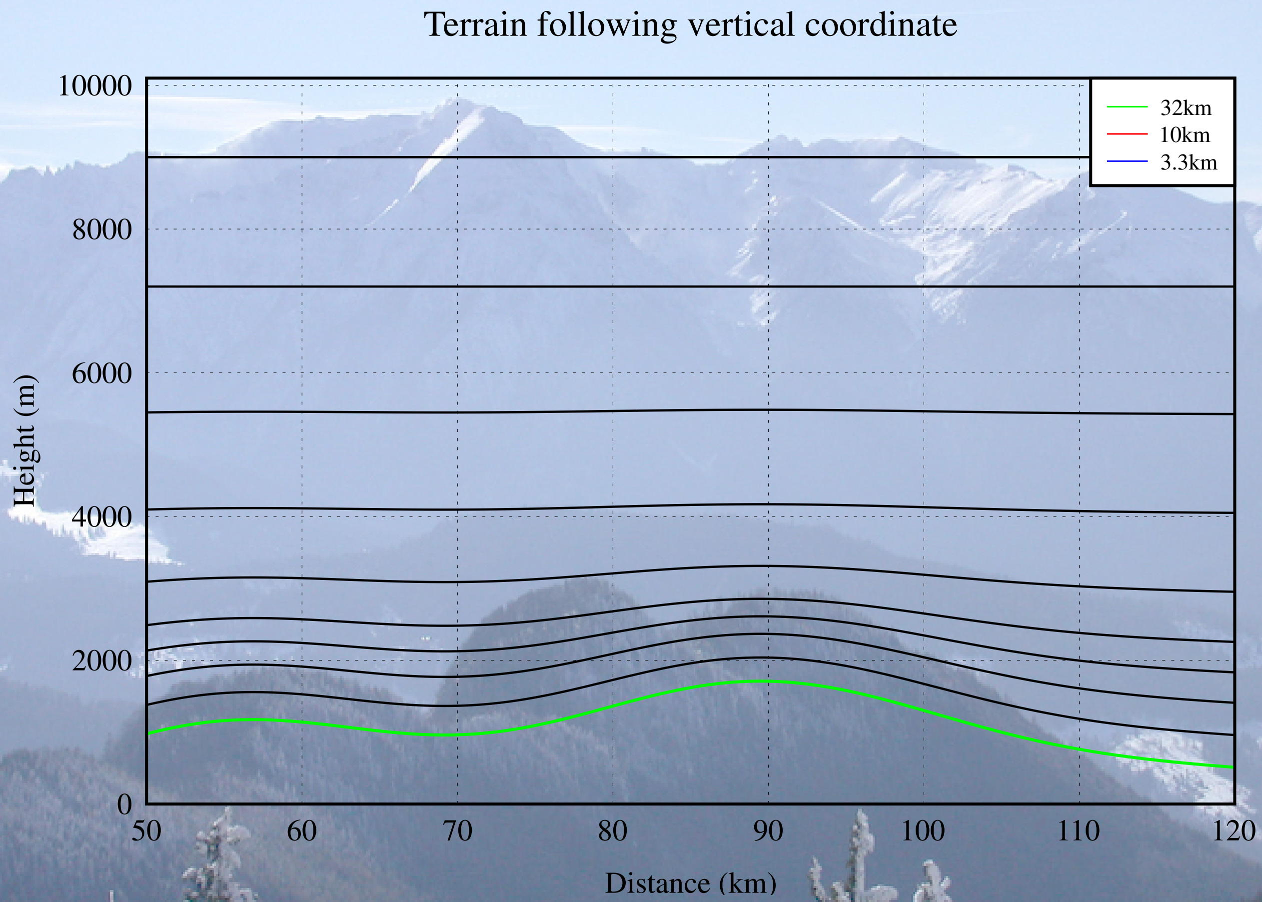
Quit

A Carpathian example



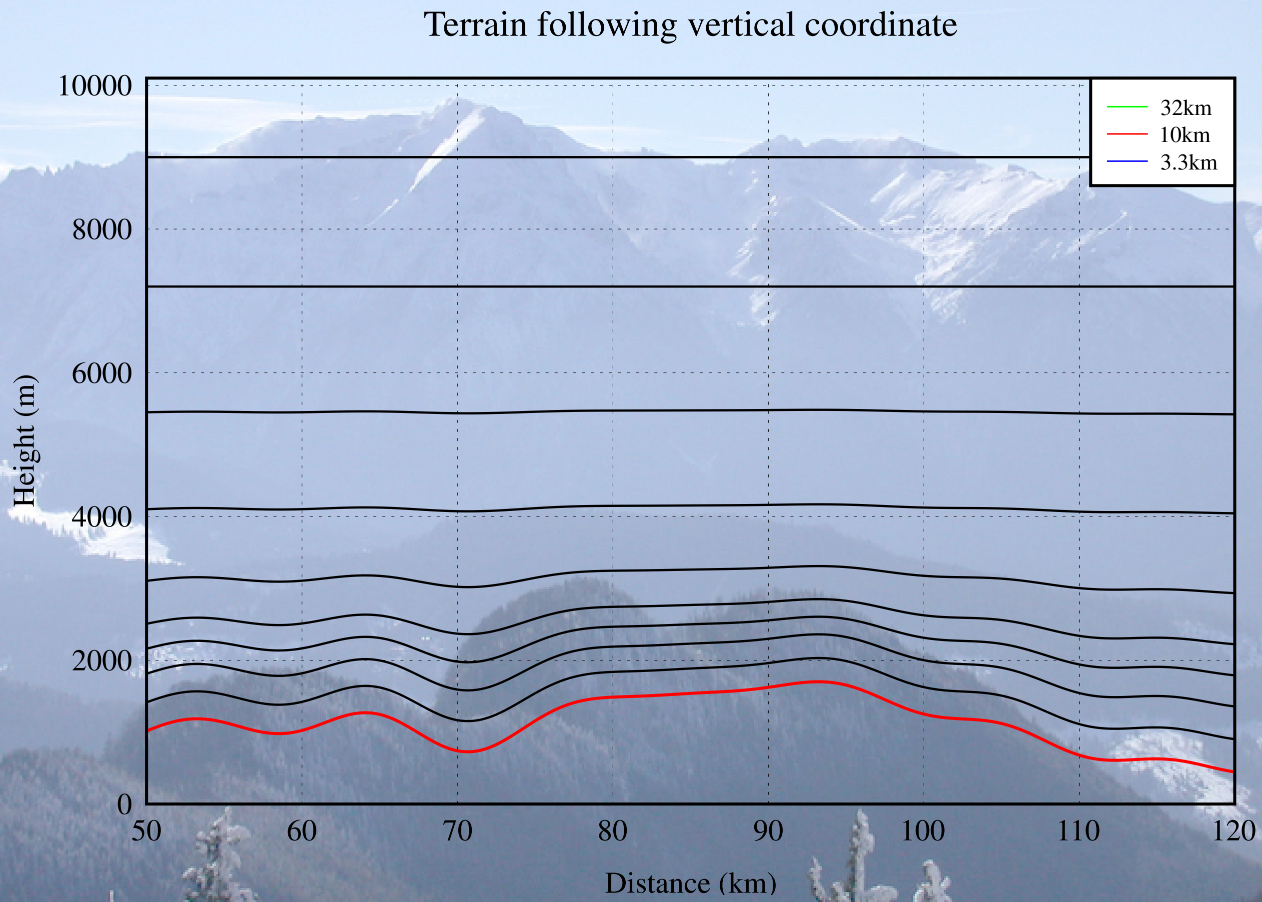


Illustrating terrain-following vertical coordinate - 1

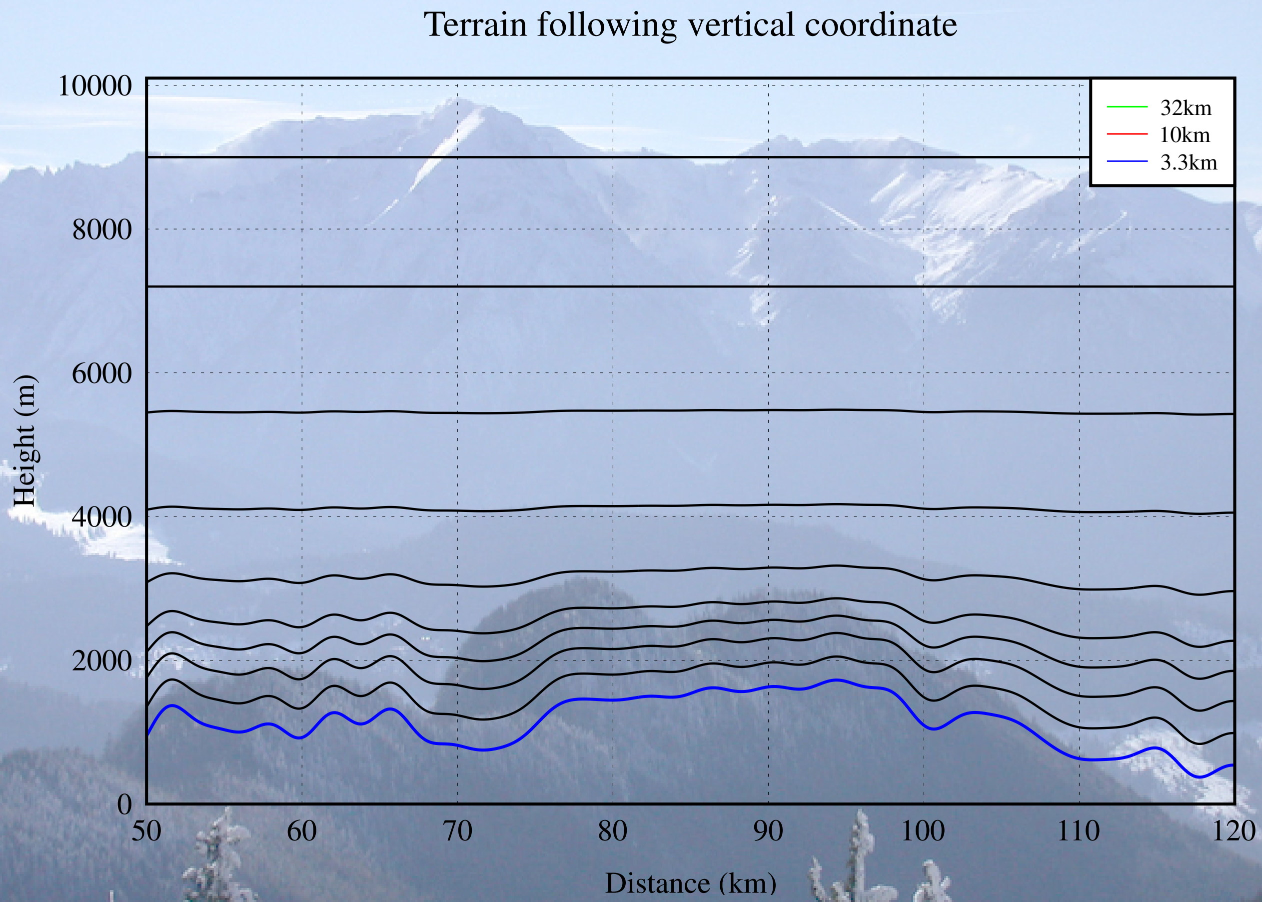


Home Page
Title Page
Page 10 of 30
Go Back
Full Screen
Close
Quit

Illustrating terrain-following vertical coordinate - 2



Illustrating terrain-following vertical coordinate - 3



Home Page

Title Page

Page 13 of 30

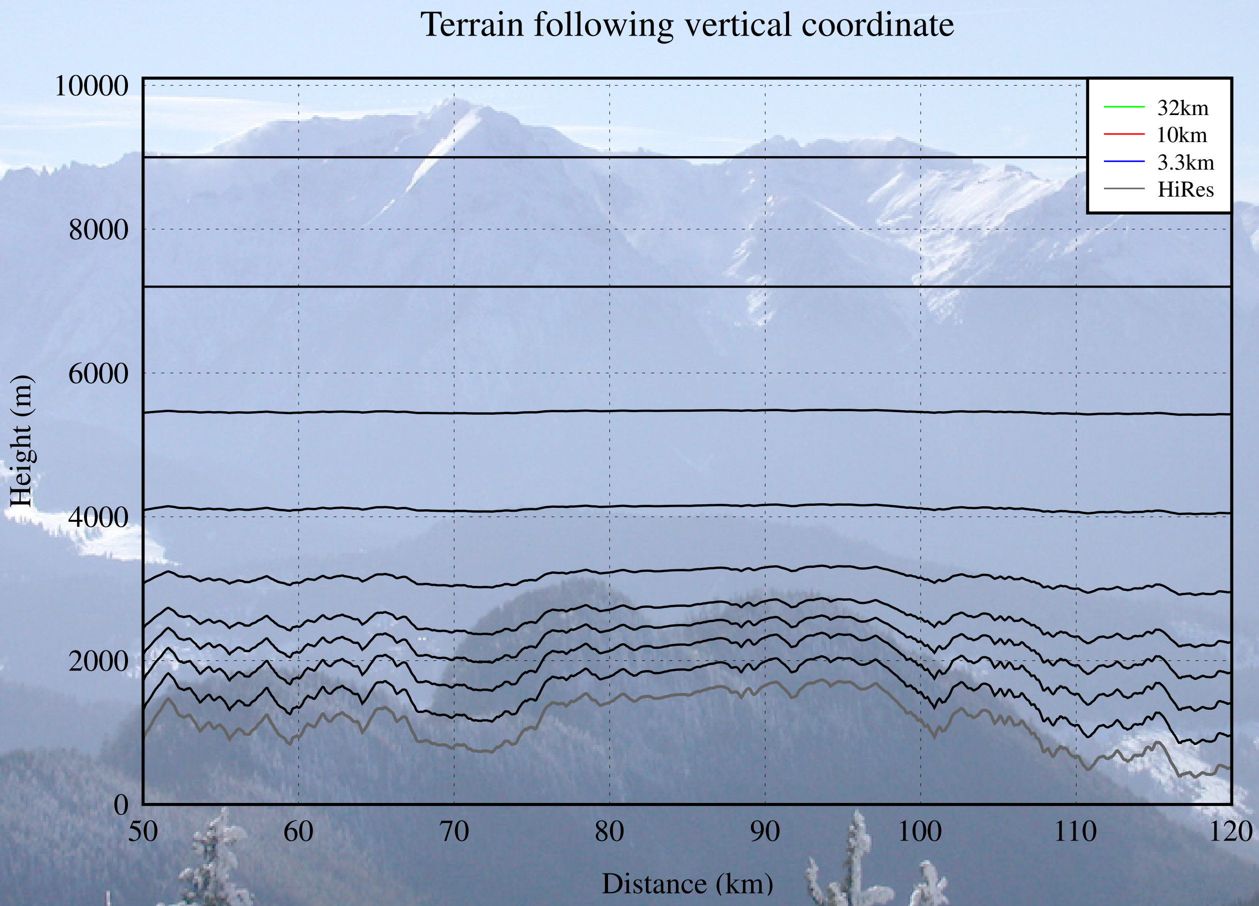
Go Back

Full Screen

Close

Quit

Illustrating terrain-following vertical coordinate - 4



Home Page

Title Page

Page 14 of 30

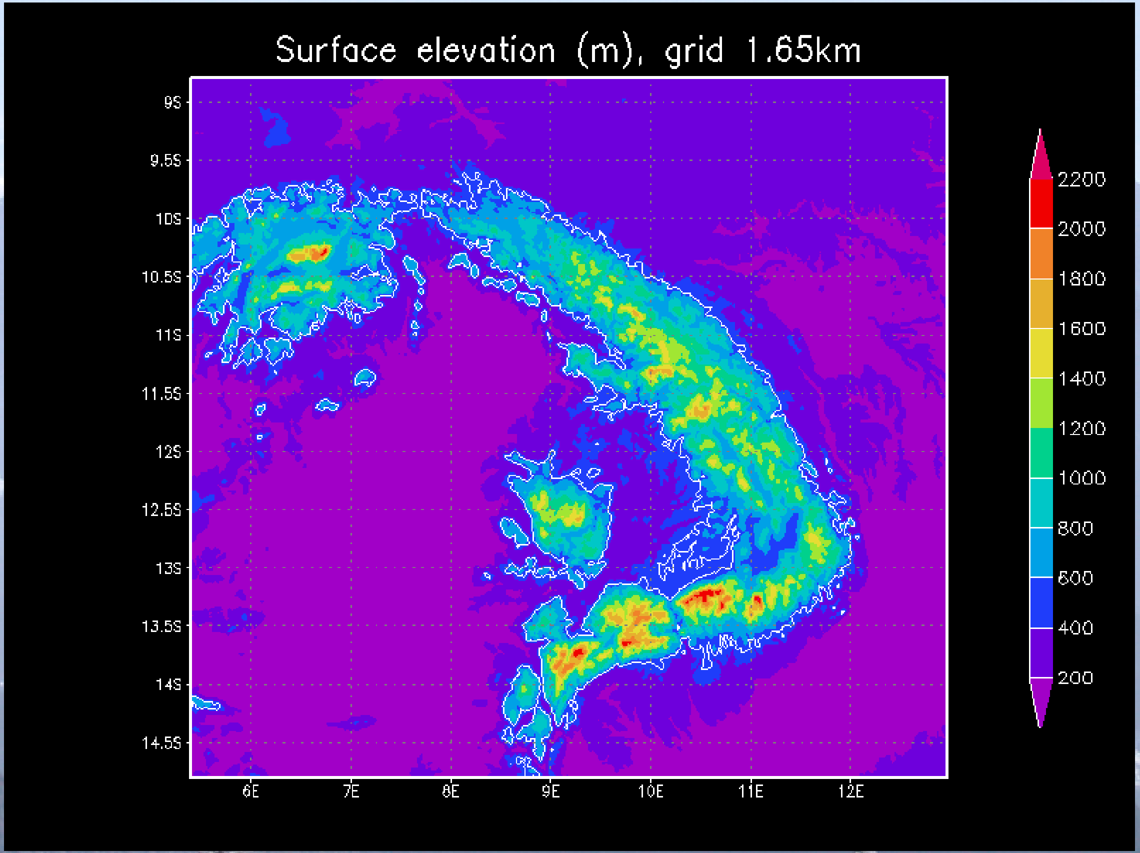
Go Back

Full Screen

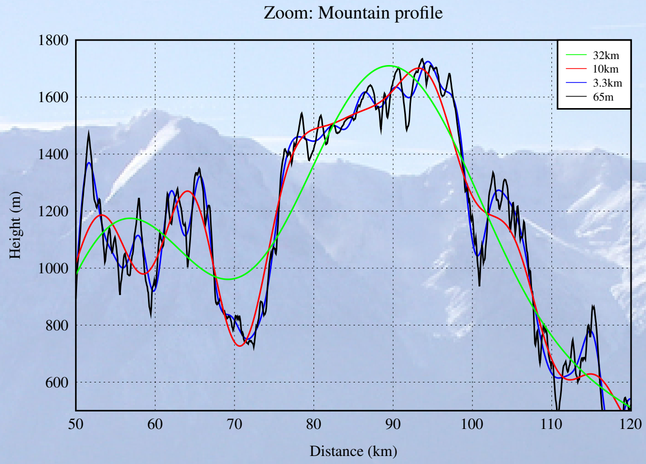
Close

Quit

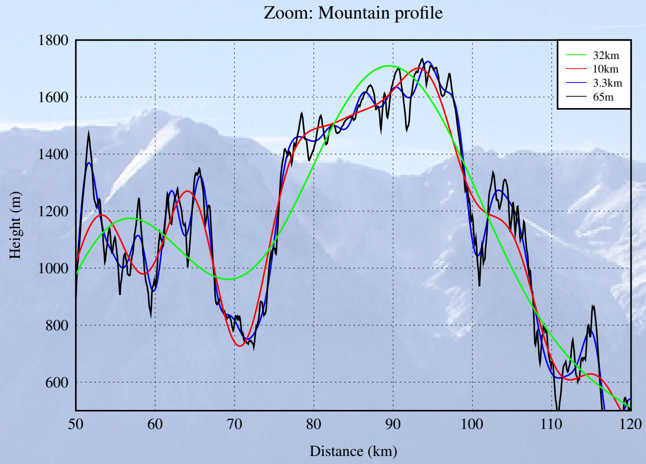
Smoothed mean orography

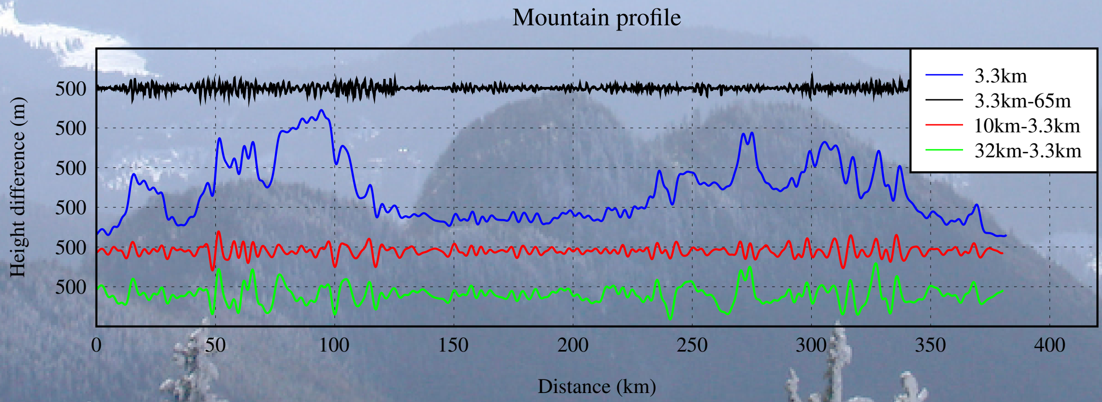


Resolved and parametrized forcing



Resolved and parametrized forcing





Parametrization of subgrid-scale momentum fluxes

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

Home Page

Title Page

Go Back

Full Screen

Close

Quit

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = \left(\frac{\partial \vec{v}}{\partial t}\right)_d + \left(\frac{\partial \vec{v}}{\partial t}\right)_p$$

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = \left(\frac{\partial \vec{v}}{\partial t}\right)_d + \left(\frac{\partial \vec{v}}{\partial t}\right)_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = \left(\frac{\partial \vec{v}}{\partial t}\right)_d + \left(\frac{\partial \vec{v}}{\partial t}\right)_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$\left(\frac{\partial \vec{v}}{\partial t}\right)_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = (\frac{\partial \vec{v}}{\partial t})_d + (\frac{\partial \vec{v}}{\partial t})_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$(\frac{\partial \vec{v}}{\partial t})_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Stress tensor τ_{ij} consists of

friction, form drag and wave drag contributions:

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = (\frac{\partial \vec{v}}{\partial t})_d + (\frac{\partial \vec{v}}{\partial t})_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$(\frac{\partial \vec{v}}{\partial t})_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Stress tensor τ_{ij} consists of

friction, form drag and wave drag contributions:

$$\vec{\tau}_{total} = \vec{\tau}_{friction} + \vec{\tau}_{formdrag} + \vec{\tau}_{wave}$$

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = \left(\frac{\partial \vec{v}}{\partial t}\right)_d + \left(\frac{\partial \vec{v}}{\partial t}\right)_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$\left(\frac{\partial \vec{v}}{\partial t}\right)_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Stress tensor τ_{ij} consists of

friction, form drag and wave drag contributions:

$$\vec{\tau}_{total} = \vec{\tau}_{friction} + \vec{\tau}_{formdrag} + \vec{\tau}_{wave}$$

horizontal scale $\Rightarrow$

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = \left(\frac{\partial \vec{v}}{\partial t}\right)_d + \left(\frac{\partial \vec{v}}{\partial t}\right)_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$\left(\frac{\partial \vec{v}}{\partial t}\right)_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Stress tensor τ_{ij} consists of

friction, form drag and wave drag contributions:

$$\vec{\tau}_{total} = \vec{\tau}_{friction} + \vec{\tau}_{formdrag} + \vec{\tau}_{wave}$$

horizontal scale $\Rightarrow$

stability $\Rightarrow$

Parametrization of subgrid-scale momentum fluxes

Tendencies of the horizontal wind $\vec{v}(x, y, z)$

- explicitly resolved and parametrized:

$$\frac{\partial \vec{v}}{\partial t} = (\frac{\partial \vec{v}}{\partial t})_d + (\frac{\partial \vec{v}}{\partial t})_p$$

Parametrized tendency is due to

divergence of the stress tensor τ_{ij}:

$$(\frac{\partial \vec{v}}{\partial t})_p = \frac{1}{\rho}\frac{\partial \vec{\tau}}{\partial z}, \vec{\tau} = -\sum_{j=1}^{n} \rho(\overline{\vec{v}' w'})$$

Stress tensor τ_{ij} consists of

friction, form drag and wave drag contributions:

$$\vec{\tau}_{total} = \vec{\tau}_{friction} + \vec{\tau}_{formdrag} + \vec{\tau}_{wave}$$

horizontal scale $\Rightarrow$

stability $\Rightarrow$

non-dimensional mountain width $G_L = NL/U$ $\Rightarrow$

Components of subgrid-scale drag

drag	related to	momentum sink	scheme
$\vec{\tau}_{ts}$	turbulent drag due to surface roughness	2D	ISBA $\Rightarrow$ CBR
$\vec{\tau}_{o}$	drag due to unresolved small-scale orography	2-3D	SSO $\Rightarrow$ CBR
$\vec{\tau}_{m}$	blocked flow drag due to mesoscale orography	3D	MSO
$\vec{\tau}_{w}$	drag due to breaking buoyancy waves	3D	MSO
$\vec{\tau}_{t}$	turbulence above surface layer	3D	CBR

Orography-related parameters

param	description	unit	usage	scale (km)	filtering
s_t	mean maximum small-scale slope	rad	SSO	$<$ 3 km	high-pass
σ_t	mean small scale standard deviation	J/kg	SSO	$<$ 3 km	high-pass
σ_m	mean meso-scale standard deviation	J/kg	MSO	3 km $\ldots 3\Delta x$	band-pass
α	coefficient of anisotropy	-	MSO	3 km $\ldots 3\Delta x$	band-pass
Θ	x-angle of orography gradient	rad	MSO	3 km $\ldots 3\Delta x$	band-pass
H	mean surface elevation	m	dynamics	$> 3\Delta x$	low-pass

Parametrization of mesoscale orography (MSO) effects

The buoyancy wave drag is estimated by a formula based on the linear two-dimensional theory,

$$\vec{\tau}_{ws} = K_g \cdot \rho_s \cdot N_s \cdot \vec{v}_{fs} \cdot h_m^2, \tag{2}$$

where the index s refers to mean near-surface values, K_g is a tuning parameter depending on the model resolution (here $K_g = 3.5 \cdot 10^{-06} m^{-1}$), N is the buoyancy (Brunt-Väisalä) frequency, $N^2 = \frac{g}{\theta}\frac{\partial\theta}{\partial z}$, h_m is the subgrid-scale mountain height based on the standard deviation of mesoscale orography and $\vec{v}_{fs}$ is a (fictive) surface wind representing the layer between surface and h_m and parallel to the stress vector $\vec{\tau}_{ws}$.

As long as there is no wave dissipation, the wave momentum flux is constant with height. The momentum sink is realized when the waves break. The parametrization of wave breaking processes follows Lindzen's saturation theory. In addition, (nonlinear) wave reflection from a breaking level is taken into account. Wave breaking and reflection modify the surface value $\vec{\tau}_{ws}$ and the profile of the wave drag $\vec{\tau}_w(z)$.

Low level flow blocking is assumed if a non-dimensional mountain height G, depending on stability, mountain height and upstream wind, exceeds a critical value.

$$G = N_s \frac{h_m}{U_p} \tag{3}$$

where U_p is the velocity of the upstream wind component perpendicular to the ridge below h_m. The blocked flow stress $\vec{\tau}_m$ at each (low troposphere) model level is calculated according to Lott and Miller (1997). Finally, it is combined with the wave drag vector $\vec{\tau}_w$.

Home Page

Title Page

Page 19 of 30

Go Back

Full Screen

Close

Quit

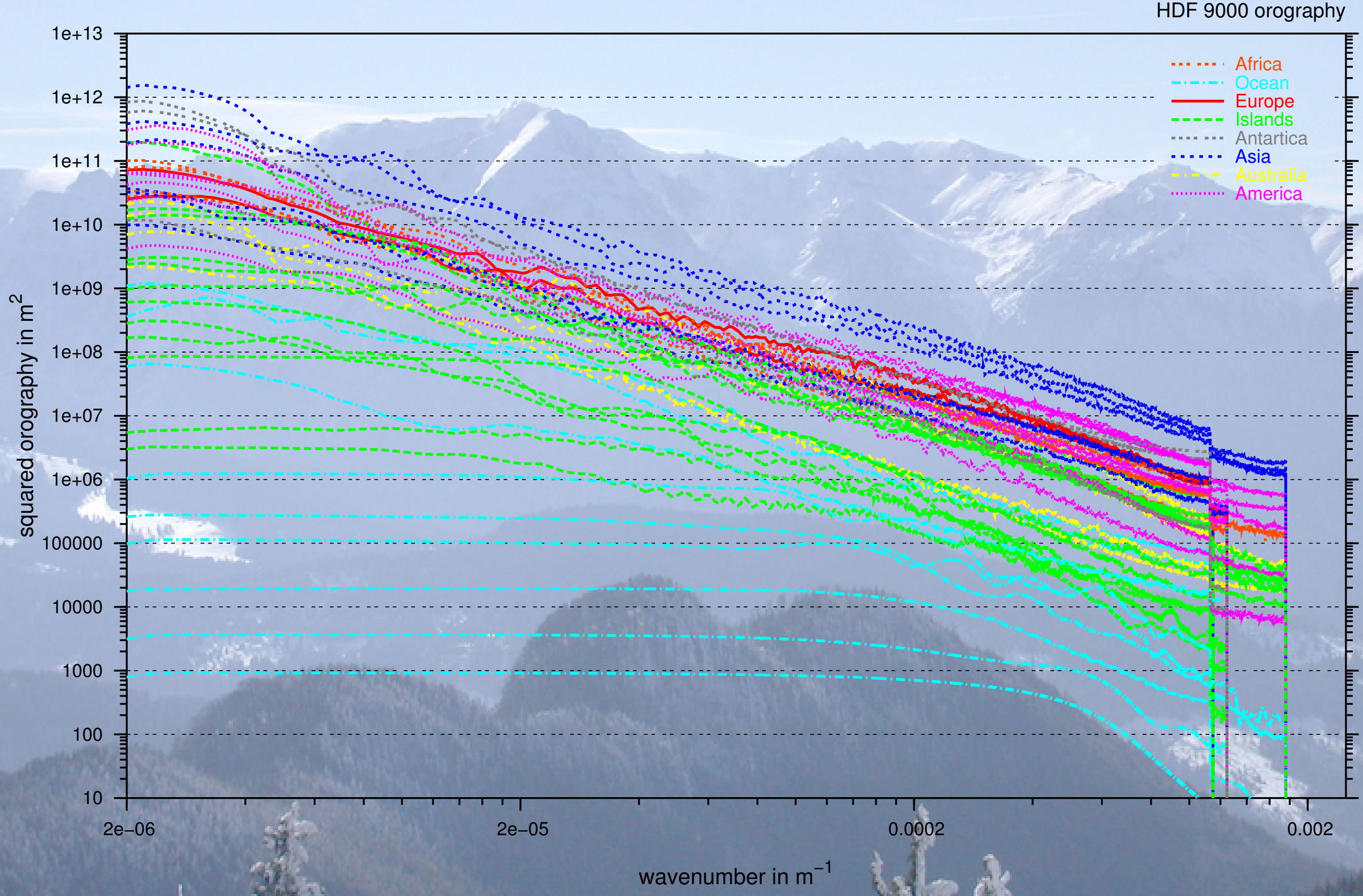
Parametrization of small-scale orography (SSO) effects

The drag due to the small-scale orographic features is parametrized as

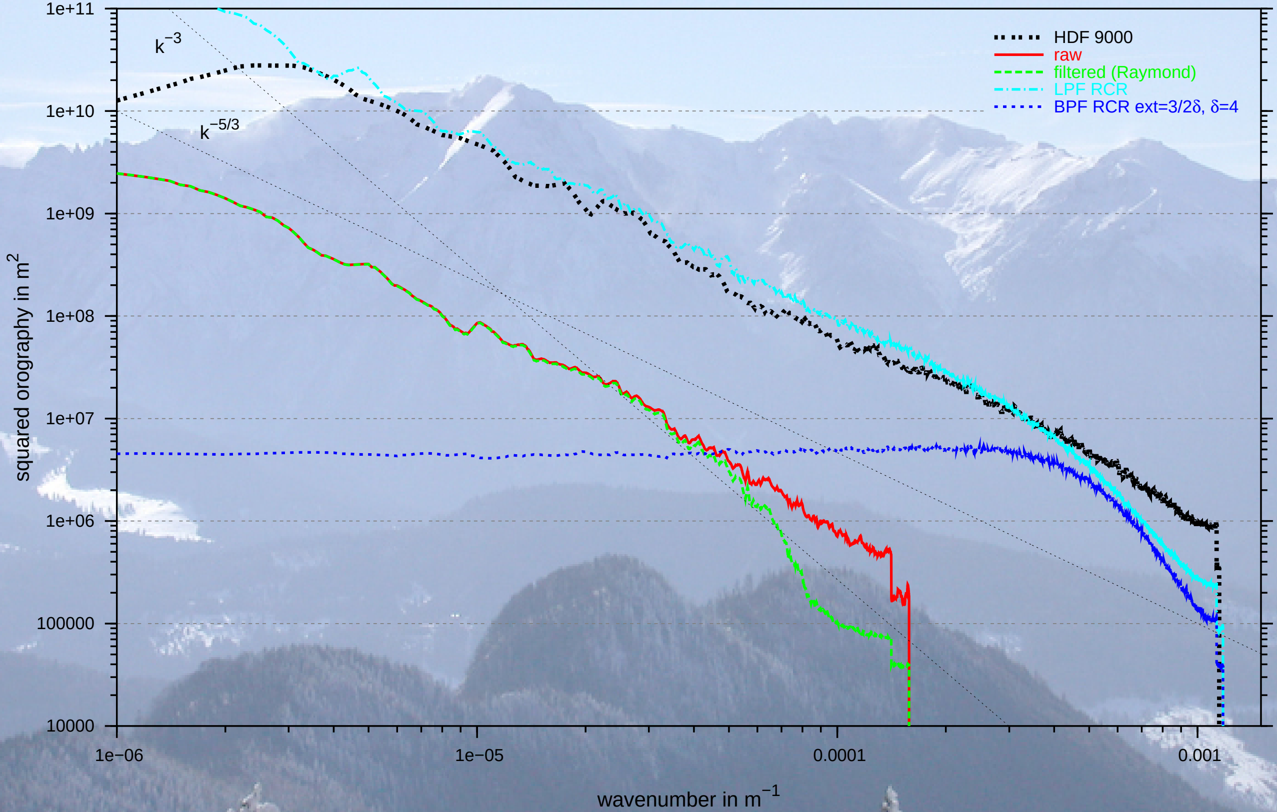
$$\vec{\tau}_{os}(z) = C_o \frac{\vec{\tau}_{ts}}{\rho_s} s_t^2 \tag{4}$$

where C_o is an orographic drag coefficient and s_t denotes the mean maximum small-scale slope (tangent) over the grid-square. According to Eq. (4), the surface orographic stress $\vec{\tau}_{os} = C_o \frac{\vec{\tau}_{ts}}{\rho_s} s_t^2$ is parallel to the turbulent stress $\vec{\tau}_{ts}$, which is determined by the wind and stability in model's surface layer. The vertical decay of the orographic stress is taken care by the turbulence parametrizations.

Two-dimensional spectra

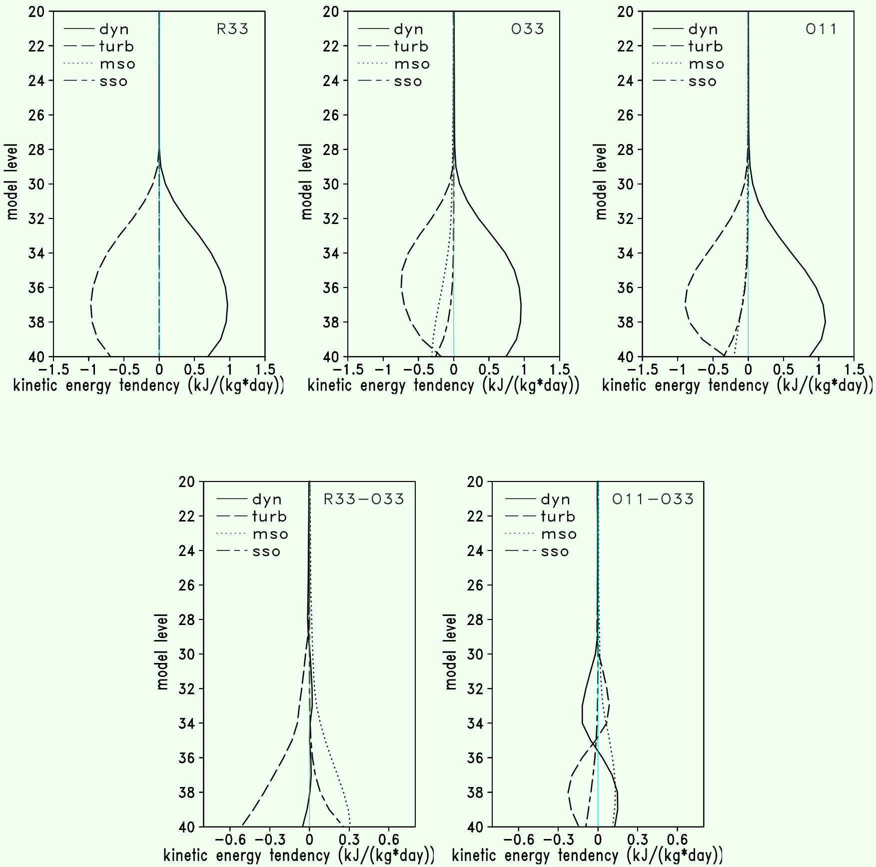


Filtered spectra



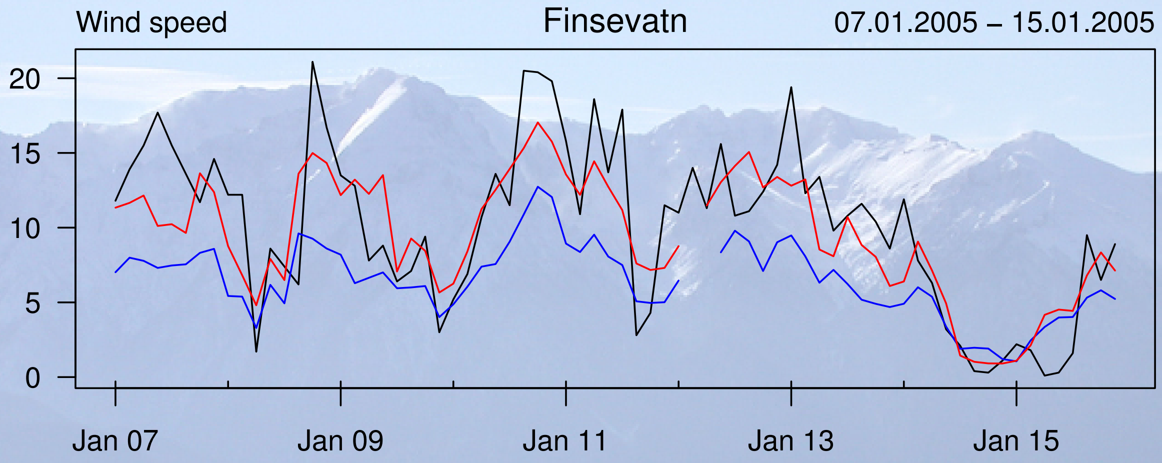
Home Page
Title Page
Page 22 of 30
Go Back
Full Screen
Close
Quit

About physics-dynamics interactions

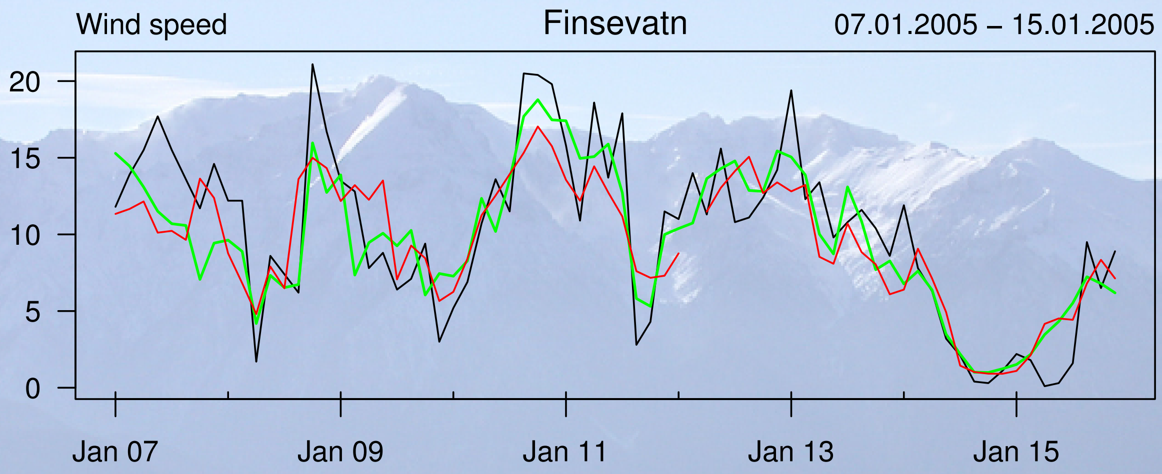


Home Page
Title Page
Page 23 of 30
Go Back
Full Screen
Close
Quit

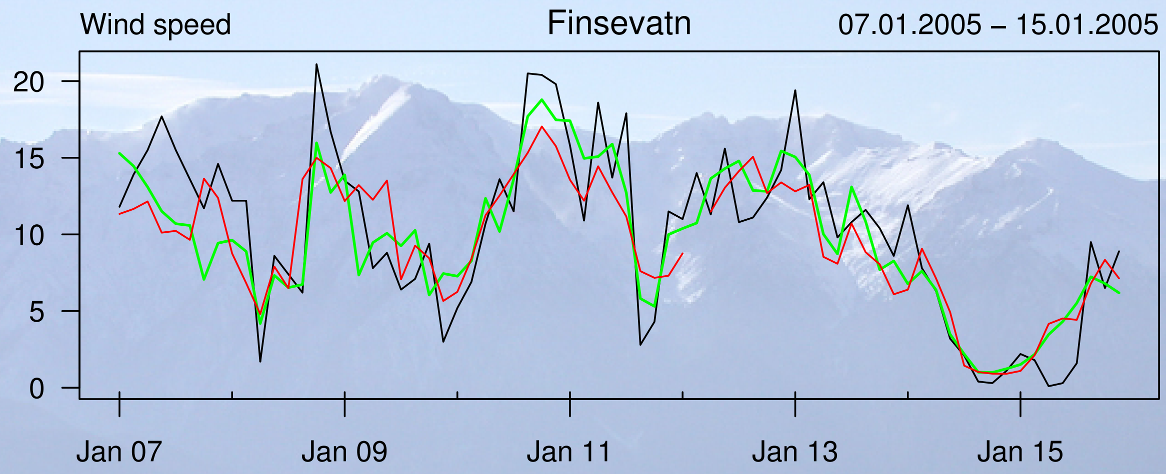
Norwegian validations of MSO-SSO



	Min	Mean	Max	Std	N
synop: 00,...,21 /3	0.1	10.2	21.1	5.5	72
NOR: 00+3,...,+24 /3	1.1	6.5	12.7	2.4	70
NOF: 00+3,...,+24 /3	0.9	9.5	17	4	71



	Min	Mean	Max	Std	N
synop: 00,...,21 /3	0.1	10.2	21.1	5.5	72
SM4: 00+3,...,+24 /3	1	9.7	18.8	4.5	72
NOF: 00+3,...,+24 /3	0.9	9.5	17	4	71



	Min	Mean	Max	Std	N
synop: 00,...,21 /3	0.1	10.2	21.1	5.5	72
SM4: 00+3,...,+24 /3	1	9.7	18.8	4.5	72
NOF: 00+3,...,+24 /3	0.9	9.5	17	4	71

Home Page
Title Page
Page 25 of 30
Go Back
Full Screen
Close
Quit

Concluding remarks

Orography defines the lower boundary condition and vertical coordinate of a NWP model. Mountains are a source of wave-like and turbulent disturbances and local circulations in the atmosphere.

Model dynamics is able to resolve orography forcing larger than (4-8)Δx. Effects due to smaller features need to be parametrized.

Different scales and physical processess create subgrid-scale momentum fluxes and (surface) drag. Scale-dependent parametrizations are needed.

Historically, the different parametrizations have been developed independently. During model simulations, they interact and effects may compensate each other and resolved-scale dynamical processess.

A challenge for parametrizations in fine-scale models: unified handling of turbulent breaking of buoyancy waves in free atmosphere and planetary boundary layer.

Some recent papers

Geleyn J.F., F. Bouyssel, B. Catry, I. Beau, R. Brozkova, D. Drvar and L. Gerard, 2006. The mountain drag/lift parameterisation scheme in ARPEGE/ALADIN. Submitted to Tellus.

Rontu L., 2006. A study on parametrization of orography-related momentum fluxes in a synoptic-scale NWP model. Tellus A, 58, 68-81.

Rontu L., K. Sattler and M. Homleid, 2006. Parametrization of mesoscale and small-scale orography effects in HIRLAM - final tests. HIRLAM Newsletter 50, mm-nn.

Thank you!

Home Page

Title Page

Page 29 of 30

Go Back

Full Screen

Close

Quit

Vorticity equation

Application of the gradient operator $\vec{k}\cdot\nabla_\zeta\times$ gives an equation for the vertical component of absolute vorticity η as a sum of relative ($\xi = \vec{k}\cdot\nabla_\zeta\times\vec{v}$) and planetary ($f$) vorticity, $\eta = \xi + f$

$$\underset{(a)}{\frac{\partial\xi}{\partial t}} = \underset{(b)}{-\vec{v}\cdot\nabla_\zeta\eta} \quad \underset{(c)}{-\dot{\zeta}\frac{\partial\eta}{\partial\zeta}}$$

$$\underset{(d)}{-\eta\nabla_\zeta\cdot\vec{v}} \quad \underset{(e)}{-\vec{k}\cdot\nabla_\zeta\dot{\zeta}\times\frac{\partial\vec{v}}{\partial\zeta}} \quad \underset{(f)}{-\frac{1}{p_s}\frac{\partial J(p,\Phi)}{\partial\zeta}}$$

$$\underset{(g)}{-\frac{g}{p_s}\frac{\partial(\vec{k}\cdot\nabla_\zeta\times\vec{\tau})}{\partial\zeta}} \tag{5}$$

Here, $\dot{\zeta} = \frac{\mathrm{d}\zeta}{\mathrm{d}t}$ is the vertical velocity in the ζ coordinate system and $J(a,b)$ denotes a Jacobian, defined as $J(a,b)=\frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial b}{\partial x}\frac{\partial a}{\partial y}$. The hydrostatic assumption $1/\rho = -\frac{\partial\Phi}{\partial p}$ was used in derivation of Eq. (5). The terms of the vorticity equation represent the local time change of vorticity (a), horizontal (b) and vertical (c) advection, stretching (d), tilting induced by the nonuniform vertical velocity (e), change of vorticity due to baroclinicity (f) and frictional forces (g).

Home Page

Title Page

Page 30 of 30

Go Back

Full Screen

Close

Quit

Vertically integrated vorticity equation

Eq. (5) can be integrated over an atmospheric column from the surface $p = p_s$ (ζ=1) to the top of atmosphere $p = 0$ (ζ=0). At the upper boundary $\vec{\tau}$=0, at the surface $\vec{\tau}$=$\vec{\tau}_s$. Denoting the integral by a hat, $\hat{\varphi}=\int_o^{p_s} \varphi \frac{dp}{g}$, we get

$$\begin{aligned}
&\widehat{\frac{\partial \xi}{\partial t}} \quad + \widehat{\vec{v} \cdot \nabla_\zeta \eta} \quad + \widehat{\dot{\zeta} \frac{\partial \eta}{\partial \zeta}} \\
&+ \widehat{\eta \nabla_\zeta \cdot \vec{v}} \quad + \widehat{\vec{k} \cdot \nabla_\zeta \dot{\zeta} \times \frac{\partial \vec{v}}{\partial \zeta}} \\
&= -J(p_s, h) \quad - \vec{k} \cdot \nabla_\zeta \times \vec{\tau}_s,
\end{aligned} \tag{6}$$

where h is the surface elevation. The term $J(h, p_s)$ results from vertical integration of the baroclinic term (term (f) in Eq. (5)) and represents the joint effect of baroclinicity and orography. Over a level surface this term disappears. It represents the grid-scale surface torque explicitly resolved by the model. In Eq. (6) both (terrain-related) source terms have been written on the right-hand side of the equation, to balance the local change and redistribution terms on the left.