# INCLUDING CORIOLIS EFFECTS IN THE PRANDTL MODEL FOR KATABATIC FLOW

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# OBJECTIVES

- INTRODUCTION
- ROTATING PRANDTL MODEL
- ASYMPTOTIC TIME-DEPENDENT MODEL AND NUMERICAL RESULTS
- CONCLUSION

# KATABATIC FLOWS - IMPORTANCE

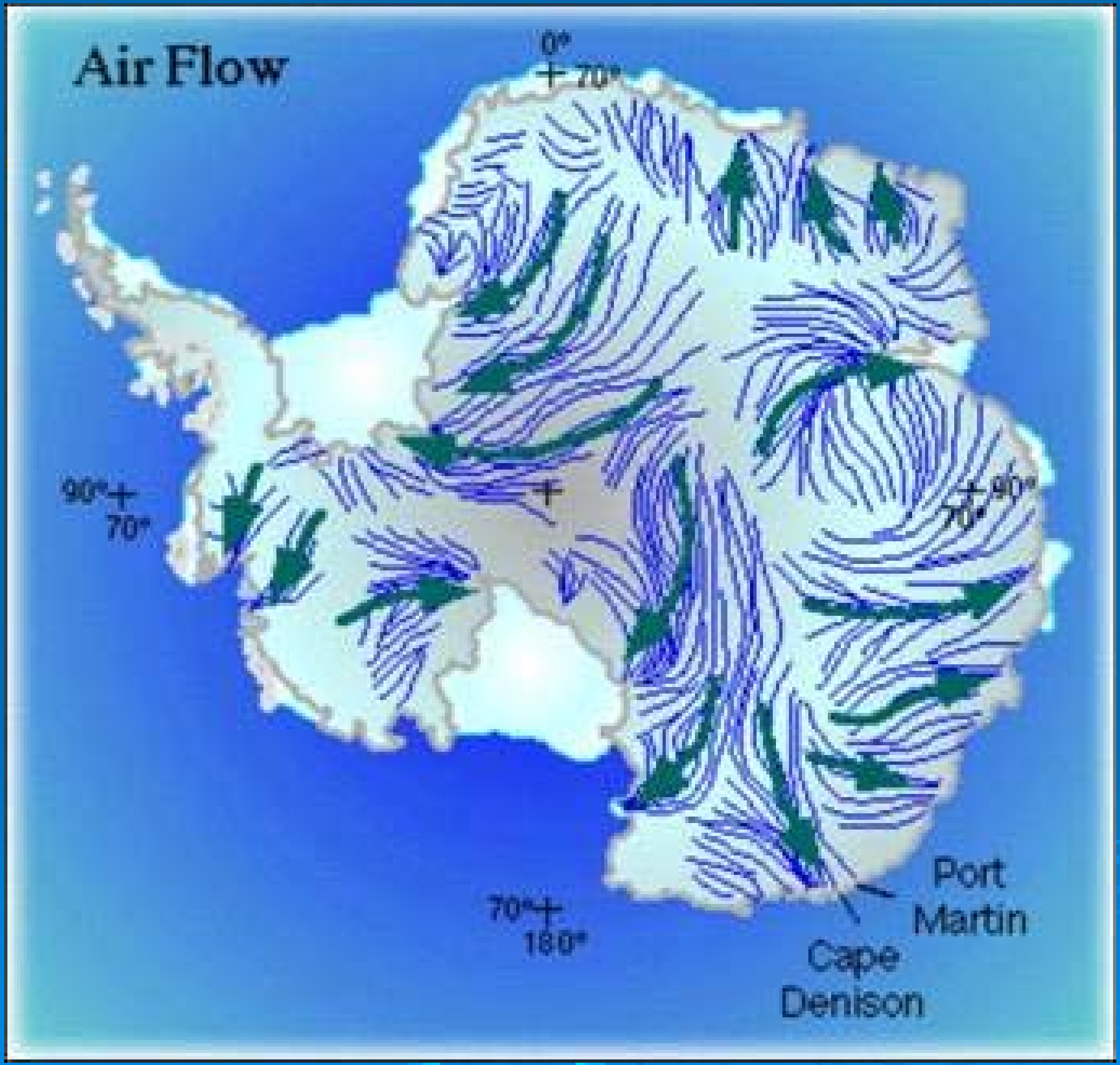
- GLACIERS – climate barometers
- LOCAL CIRCULATIONS – urban effects, traffic, convection…
- CUMMULATIVE EFFECTS IN CLIMATE PROCESSES – to be properly parameterized in climate models

KATABATICS + f
long-lasting ABL

0

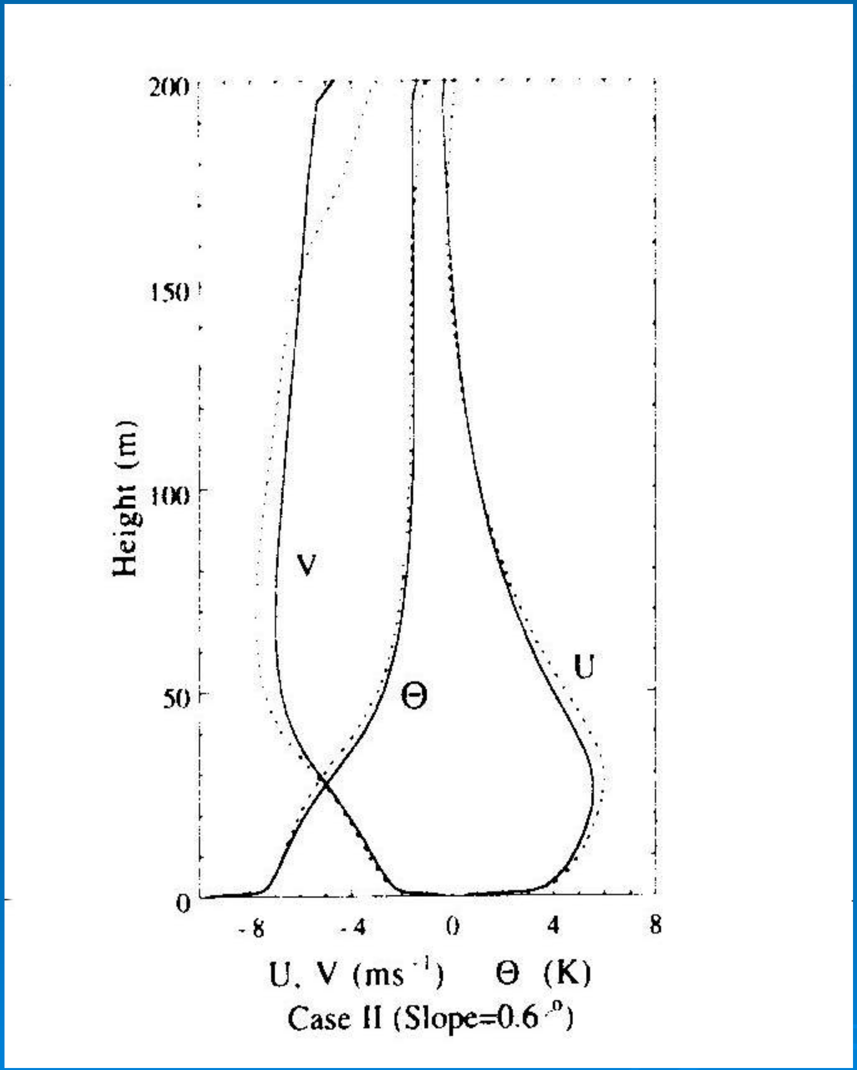
500 km

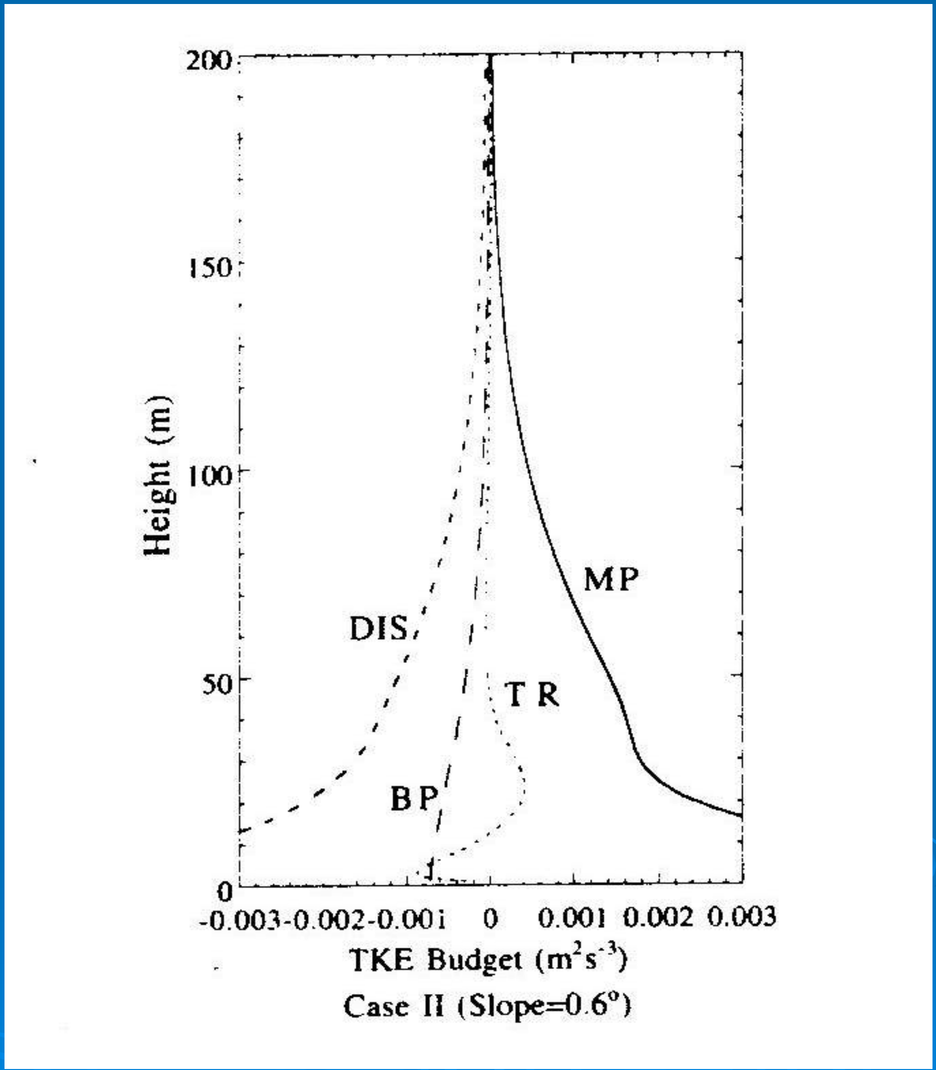
1200 km




4560 km

# Katabatic wind with rotation

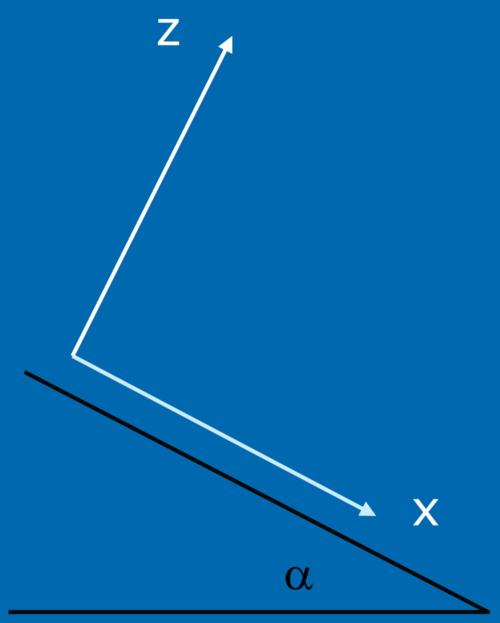



B. Denby, BLM 1999, H-O-C simulations

# PRANDTL MODEL+ CORIOLIS EFFECT

- hydrostatic balance
- linear (no advection)
- clockwise rotation ($\alpha < 0$)
- constant K*




* Parmhed et al., QJ04 - Prandtl model with K(z) may be treated via WKB method and applied to a real glacier (e.g. no problem with any large Ri and Pr but missing pressure gradient and variable advection)

X:

$$0=\frac{g}{\theta_0}\sin\alpha\,\theta+f\cos\alpha\,V+KPr\frac{d^2U}{dz^2}$$

Y:

$$0=-fU\cos\alpha+KPr\frac{d^2V}{dz^2}$$

θ:

$$0=-U\gamma\sin\alpha+K\frac{d^2\theta}{dz^2}$$

B. C.

$$\theta(z=0)=\tilde{C},\quad U(z=0)=V(z=0)=0$$
$$\theta(z\to\infty)=U(z\to\infty)=V(z\to\infty)=0$$

$$F = (\theta, U, V)$$

$$\frac{d^2}{dz^2}\left(\frac{d^4F}{dz^4} + \sigma^4 F\right) = 0 \quad \Rightarrow \quad \frac{d^4F}{dz^4} + \sigma^4 F = az + b$$

- B. C. give a = b = 0

$$\tilde{C} = \frac{C}{1+\Delta} \qquad \Delta = \frac{f^2 \cot^2\alpha}{N^2 \Pr}$$

$$\sigma^4 = \frac{N^2 \Pr \sin^2\alpha}{K^2 \Pr^2}(1+\Delta)$$

# STEADY-STATE SOLUTIONS FOR U AND $\theta$

- Analogy to classic Prandtl model

$$\theta_s = \tilde{C} e^{\frac{-\sigma z}{\sqrt{2}}} \cos\left(\frac{\sigma z}{\sqrt{2}} + \Delta\right),$$

$$U_s = \frac{\tilde{C} K \sigma^2}{\gamma \sin\alpha} e^{\frac{-\sigma z}{\sqrt{2}}} \sin\left(\frac{\sigma z}{\sqrt{2}}\right)$$

$$V_s = \frac{\tilde{C} f \cot\alpha}{\Pr \gamma} \left(1 - e^{\frac{-\sigma z}{\sqrt{2}}} \cos\left(\frac{\sigma z}{\sqrt{2}}\right)\right)$$

# NUMERICAL UNSTEADY SOLUTIONS

- **Main time scale: T=2π/[Nsinα]**

X: $$\frac{\partial U}{\partial t} = \frac{g}{\theta_0}\sin\alpha\,\theta + f\cos\alpha\,V + KPr\frac{\partial^2 U}{\partial z^2}$$

Y: $$\frac{\partial V}{\partial t} = -fU\cos\alpha + KPr\frac{\partial^2 V}{\partial z^2}$$

θ: $$\frac{\partial \theta}{\partial t} = -U\gamma\sin\alpha + K\frac{\partial^2 \theta}{\partial z^2}$$

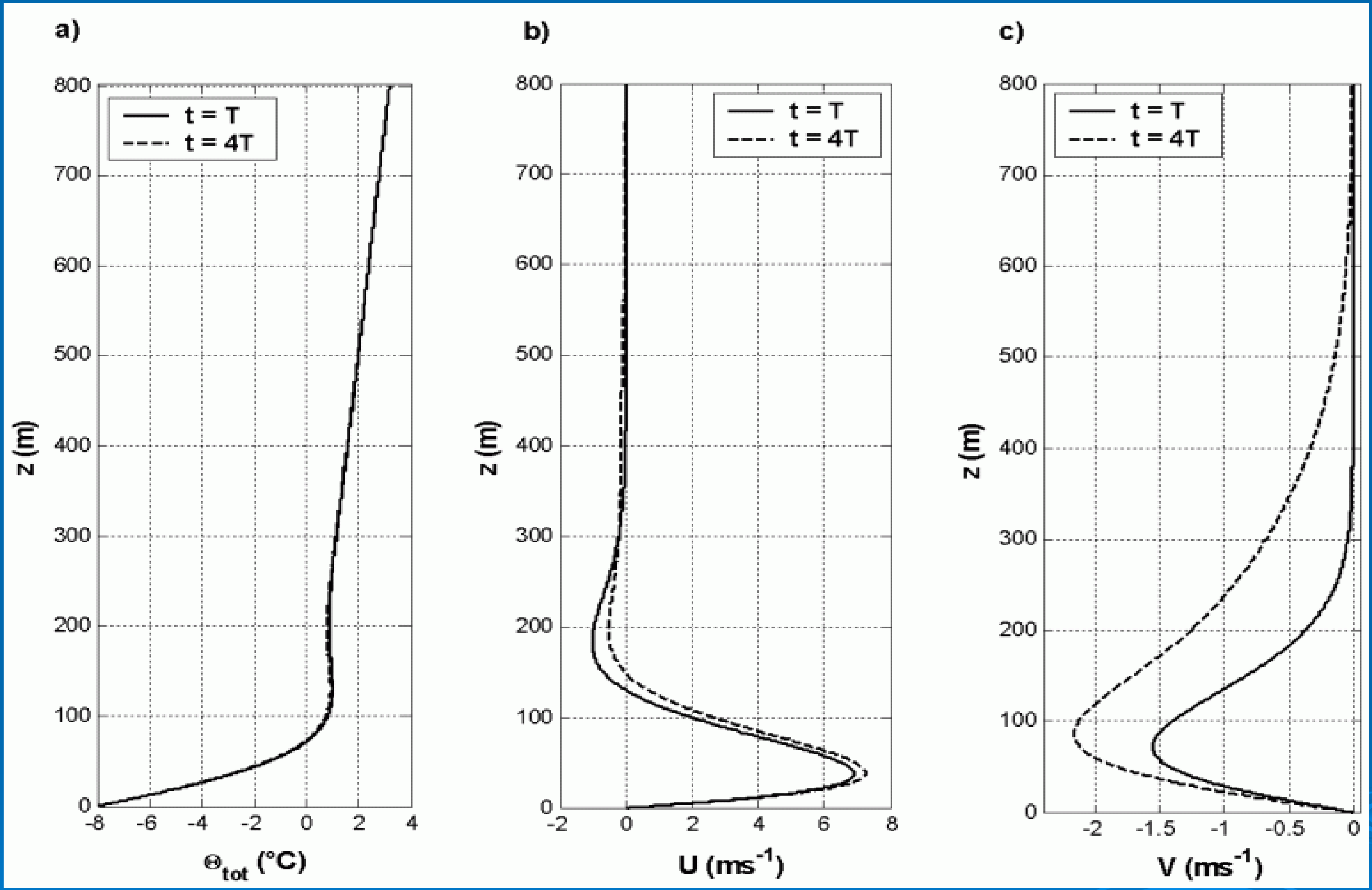



**Figure 1. Numerical solution for time-varying Prandtl model with f, T=2.1h, $\alpha$, $\gamma$, $K$, $Pr$, C, f = -4°, 4K/km, 1m$^2$s$^{-1}$, 1.1, -8K, 1.1·10$^{-4}$s$^{-1}$**

# QUASI-UNSTEADY SOLUTION

- Numerical results and scale analysis: U and θ become nearly steady after T≈1/Nsinα, but not V!
- Thus, we solve a simplified time-varying ‘Prandtl + f’ problem analytically and compare it to the fuller, numerical solution shown

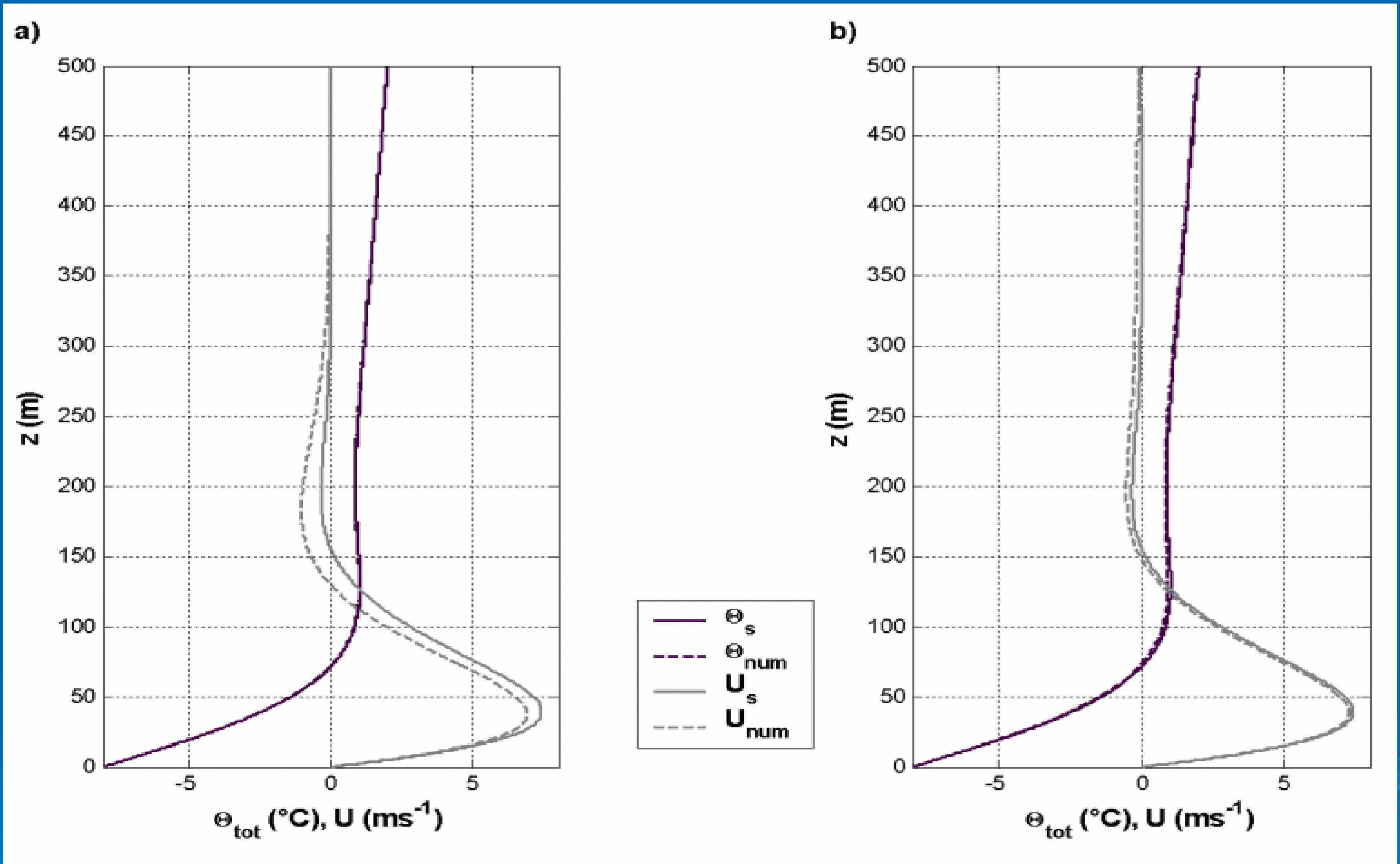



**Figure 2. Numerical (dashed) and approximate (solid) steady solutions for the Prandtl model, at (a) $t = T$ and (b) $t = 4T$. The rest as in Fig. 1.**

# V: diffusion equation forced by Prandtl model

$$\frac{\partial V}{\partial t} - K \Pr \frac{\partial^2 V}{\partial z^2} = -f \cos\alpha U_s, \; t > T$$

$$V = V_0 \left[ 1\text{-}erf\left( \frac{z}{2\sqrt{\tau K \Pr}} \right) - e^{\frac{-\sigma z}{\sqrt{2}}} \cot\left( \frac{\sigma z}{\sqrt{2}} \right) \right], \quad \tau = t\text{-}T$$

$$V_0 = \frac{Cf \cot\alpha}{\Pr\gamma}$$

- U(z,t) and θ(z,t) kept as before from the classic Prandtl model
- V-effect in the x-momentum, since weakest, is dropped:

  $f\,\theta_0/\alpha g\theta \sim O(10^{-2})$

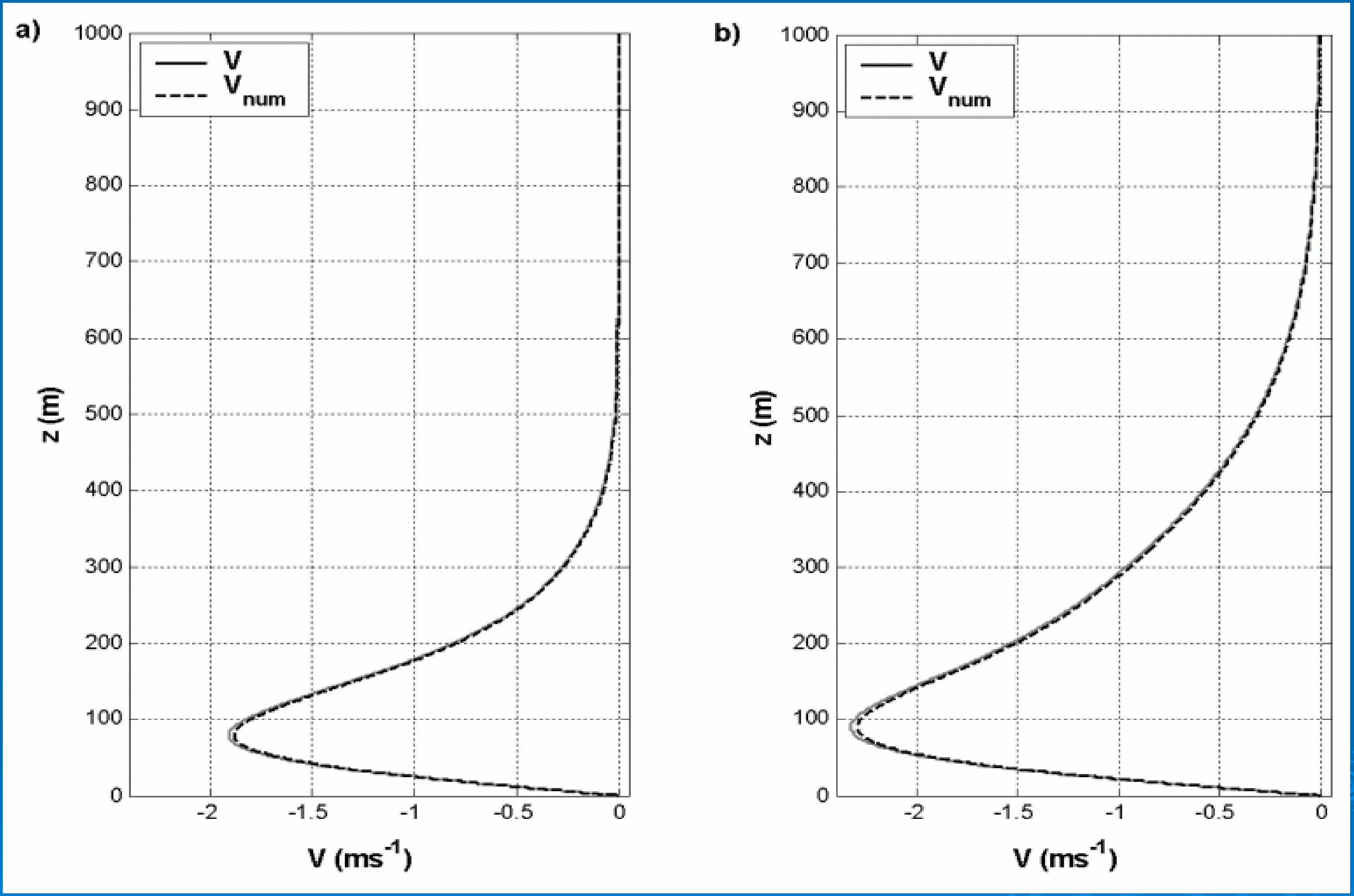



**Figure 3. Numerical (dashed) and time-dependent *V* (solid) solutions obtained for (a) *t* = 2*T* and (b) *t* = 6*T*. The rest as in Fig. 1.**

# CONCLUSION

- Steady Prandtl model + f is not equivalent to its time dependent counterpart
- U and $\theta$ reach steady state profiles after T
- V diffuses upwards in time without well defined time scale
- Approximate quasi-unsteady system $(U,V,\theta)$ is in agreement with numerical solution after $t > T$
- We got a closed form for new simple “kata-parameterization”