Large Eddy Simulations of stably stratified flows: technique, achievements and challenges

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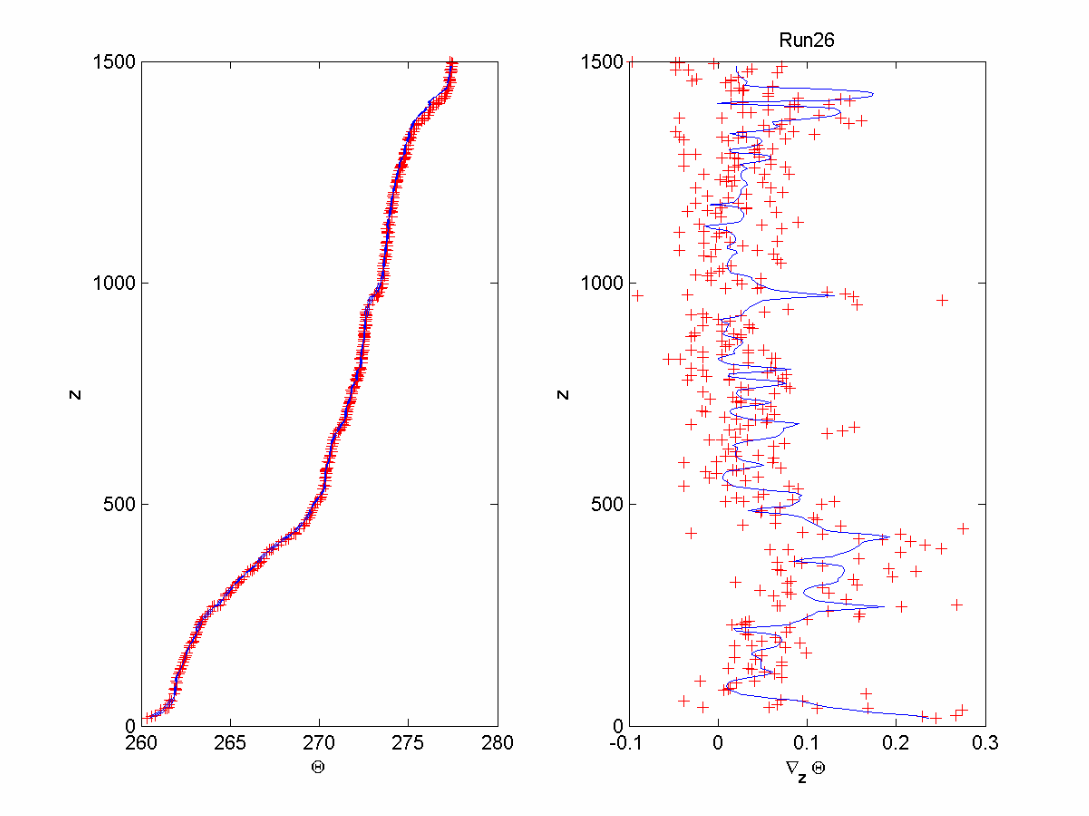
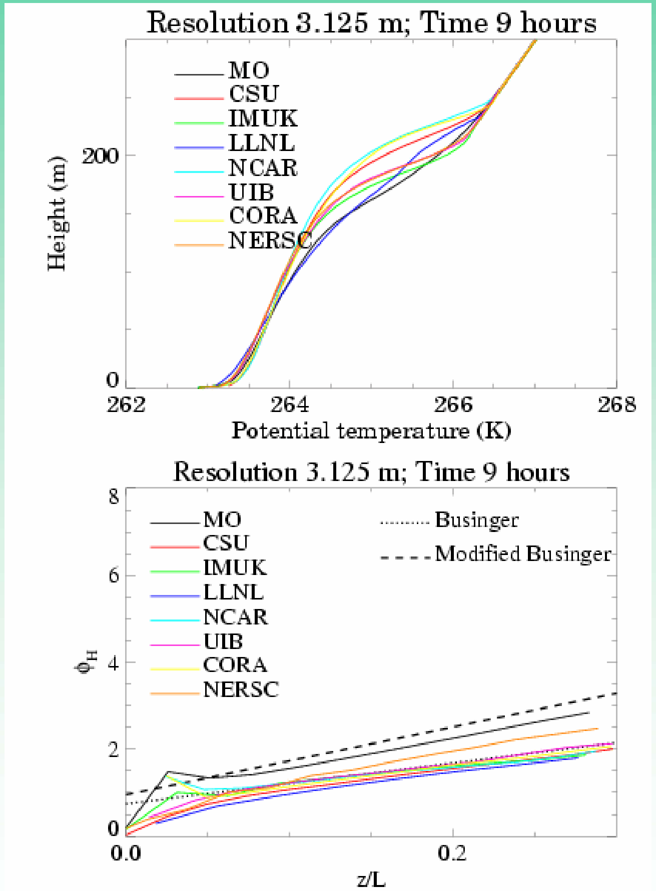
Large Eddy Simulations: What's it?

- Large Eddy Simulation is a numerical technique explicitly resolving large-scale, energetic motions in fluid
- Large Eddy Simulation is a feasible technique because energy, spatial and temporal scales of eddies are directly proportional in fluids
- Large Eddy Simulation is a subjective technique because applications determine "how large is large enough"

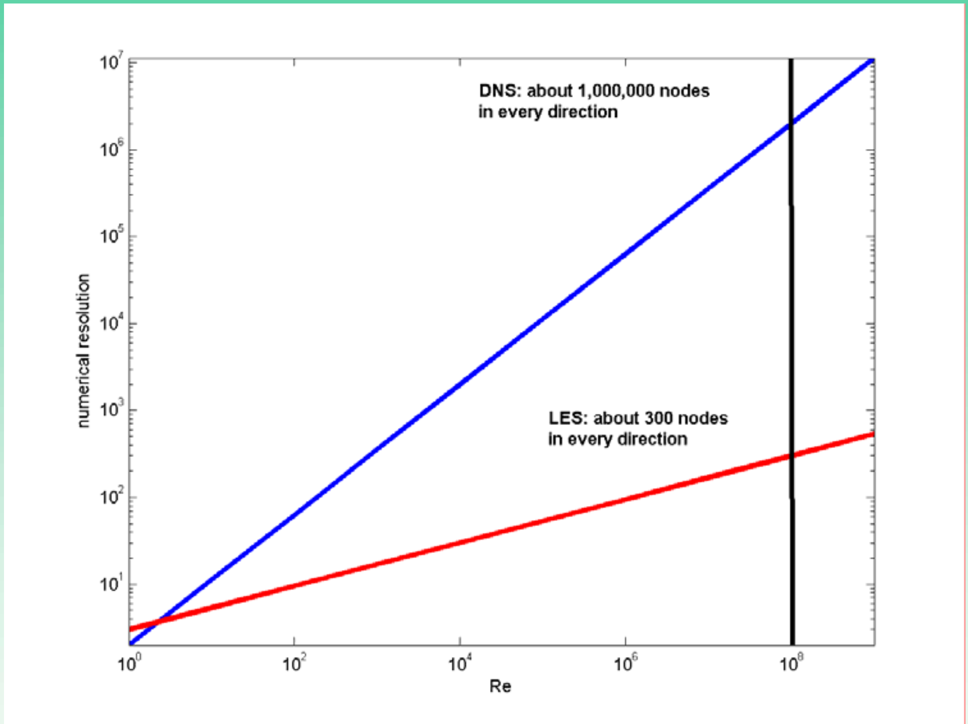
Why do we need it?





Compare quality of data (SHEBA) and LES (GABLS)!

Why do we need it?



After Mayers & Baelmans, 2004

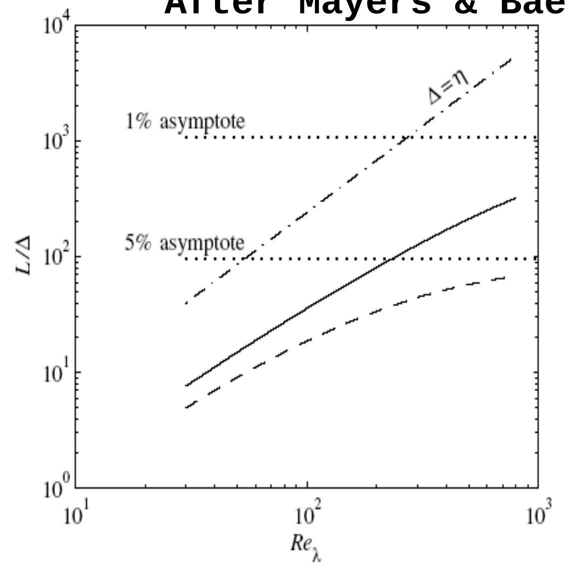


Figure 6. Required ratio of L/Δ as function of Re_λ (with $a = 2$) for different levels of subfilter energy, i.e. $E_{sgs}/E = 1\%$ (—) and $E_{sgs}/E = 5\%$ (- -), respectively. (-·) $\Delta = \eta$; (···) respective asymptotes according to (30).

LES strongly reduce the computational cost

but

Only slightly reduce accuracy

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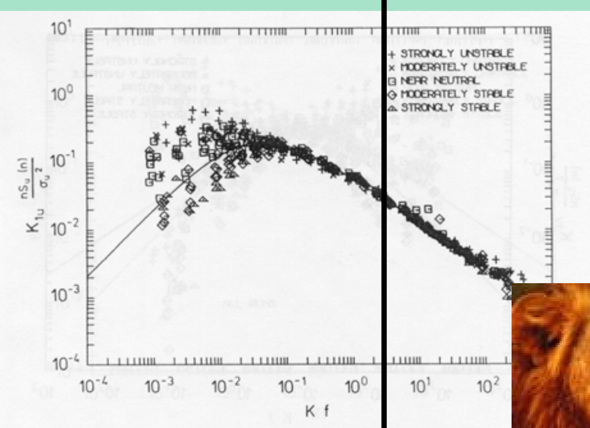
LES idea

?



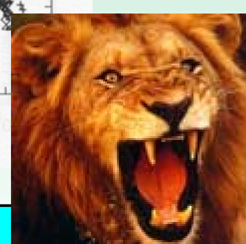
Applications

Applications

Applications

LES equations

Scale cut



Equations of Motions
Transport Equations

What Data Do LES Provide?

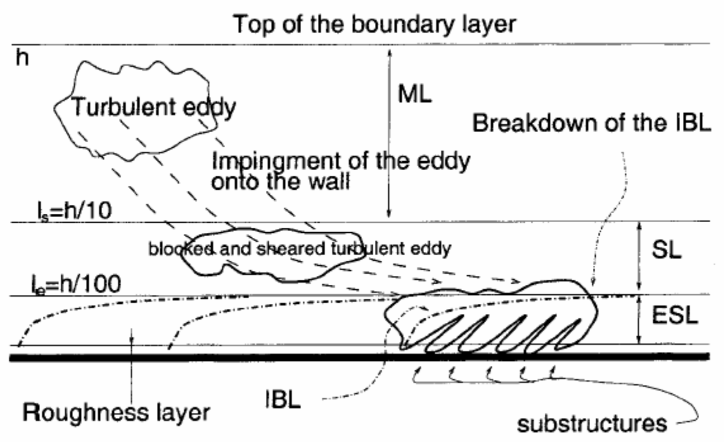


Figure 1. Sketch of a typical high Reynolds number boundary layer; $h \approx$ 1–2 km, $\ell_s \approx$ 100–200 m, $\ell_e \approx$ 10–20 m, the roughness length z_0 is less than 0.1 m over a field, less than 1 m over a typical city.

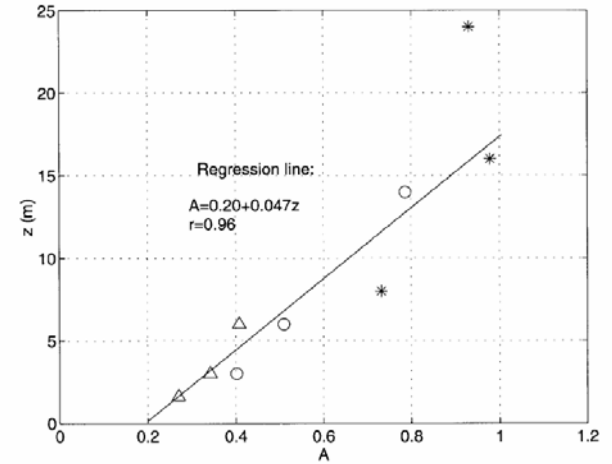


Figure 6. The scale parameter A of the expression $\Lambda_{max} = Au_*/f_c$ plotted against height. Data from: 'Laban's mills' (Högström, 1992), Δ; Lövsta (Högström, 1990), ○; Östergarnasholm (cf. Smedman et al., 1999), *. Note that each symbol represents a mean over many measurements from each site and measuring height.

- Hunt et al. (2001) suggested and Hoegstroem et al. (2002) found some observational support for a top-down turbulence generation mechanism for high Re boundary layers: small-scale, surface layer turbulence is just imprint of large-scale eddies impinging from the PBL core
- Top-down mechanism suggests that LES should be quite accurate tool to study turbulent properties of the PBL

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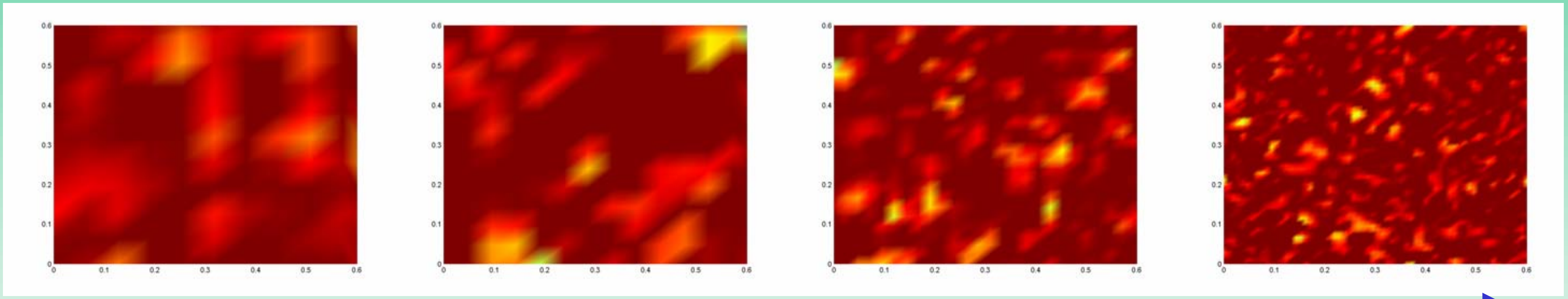
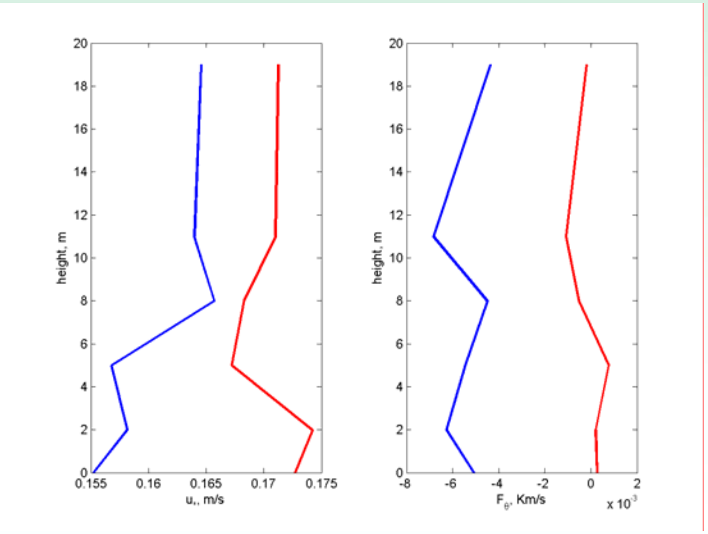
Top-Down Turbulence Generation Mechanism in SBL in very important!



Turbulence production in increasingly thinner SBL (LESNIC data from Database64)



Averaged wintertime (blue) and summertime (red) fluxes at SHEBA

LES: Milestones

- 1969 – Ladyzenskaja: existence and uniqueness theorem for regularized equations of motions
- 1972 – Deardorff: simulations of self-organized large eddies in convective boundary layers
- 1974 – Leonard: spectral fluxes in regularized non-linear equations
- 1980 – Bardina: demonstration of direct information cascade toward small scales in 3D turbulence
- 1986 – Germano: exact analytical closure or deconvolution for equations of motions
- 1993 – Zang, Street, Koseff: the first working approximate deconvolution, large-eddy model
- 2001 – HATS: Horizontal Array Turbulence Study to compare measured and modelled fluxes and variances
- 2004 – Guermond et al.: relation between spectral properties of dissipation and flow Re number

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LES: First Experience

J. Fluid Mech. (1970), *vol.* 41, *part* 2, *pp.* 453–480
Boeing Symposium on Turbulence

A numerical study of three-dimensional turbulent channel flow at large Reynolds numbers

By JAMES W. DEARDORFF
National Center for Atmospheric Research, Boulder, Colorado 80302

(Received 9 May 1969)

The three-dimensional, primitive equations of motion have been integrated numerically in time for the case of turbulent, plane Poiseuille flow at very large Reynolds numbers. A total of 6720 uniform grid intervals were used, with sub-grid scale effects simulated with eddy coefficients proportional to the local velocity deformation. The agreement of calculated statistics against those measured by Laufer ranges from good to marginal. The eddy shapes are examined, and only the u-component, longitudinal eddies are found to be elongated in the downstream direction. However, the lateral v eddies have distinct downstream tilts. The turbulence energy balance is examined, including the separate effects of vertical diffusion of pressure and local kinetic energy.

It is concluded that the numerical approach to the problem of turbulence at large Reynolds numbers is already profitable, with increased accuracy to be expected with modest increase of numerical resolution.

- How to deal sub-grid stress/diffusivity terms?
- How to deal with boundary conditions for large eddies?
- How to deal with transition to turbulence in flows?

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LES: First Success?

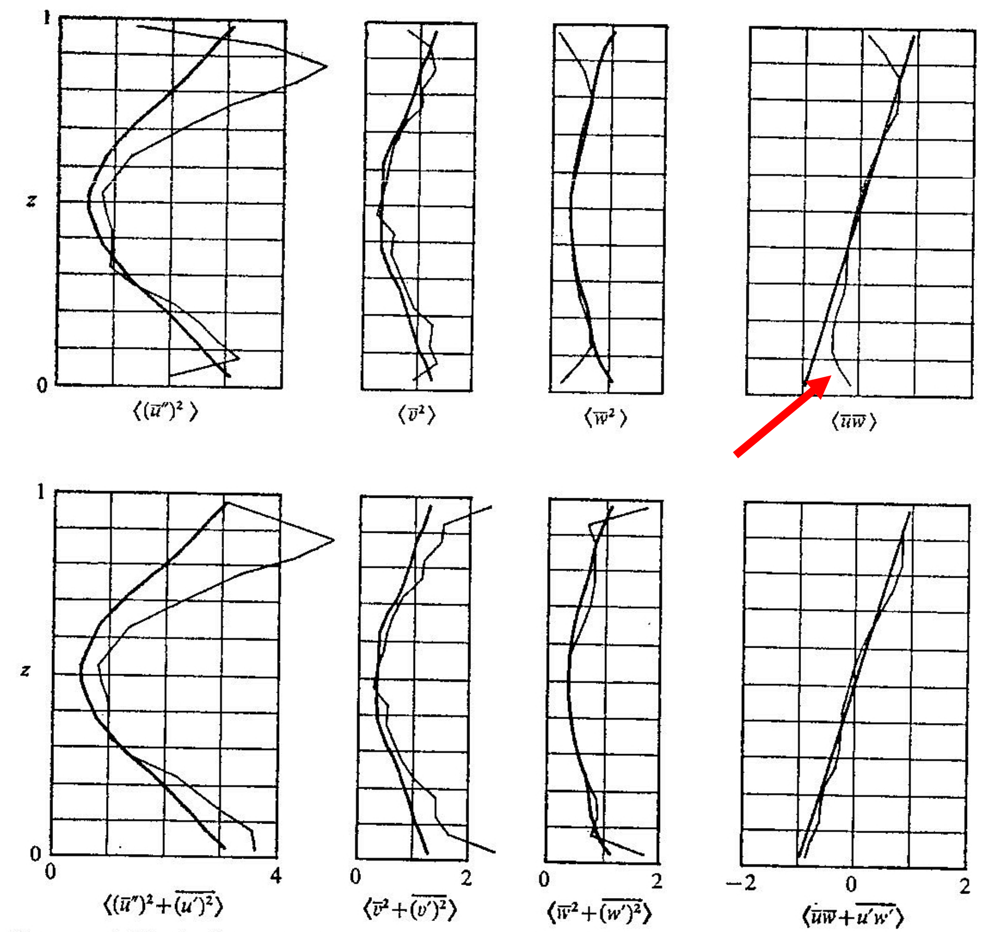
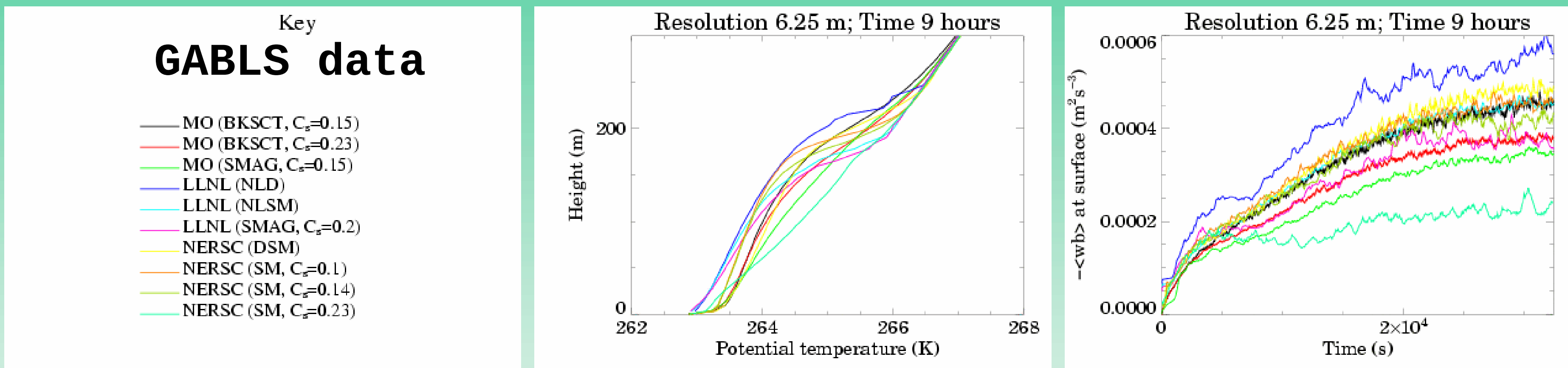


FIGURE 5. Vertical profiles of dimensionless, horizontally averaged turbulence intensities and Reynolds stress (thin curves). Upper portion shows the resolvable turbulence intensities and $\langle \bar{u}\bar{w} \rangle$; lower portion the total intensities after adding in the subgrid scale estimates. Heavy curves are from the measurement of Laufer (1950).

Is LES equal to direct numerical simulations with specific boundary conditions and a finite effective, eddy viscosity?
P. Mason (QJRMS, 1994)

Implications of effectively viscous fluid for stably stratified flows:
No Turbulence in Flows

Experience with Stably Stratified PBL



- Mason P. and Derbyshire S., QJRMS, 1990
- Brawn A.R., Mason P. and Derbyshire S., QJRMS, 1994
- Kosovic B. and Curry J., JAS, 2000
- Saiki E., Moeng C.-H. and Sullivan P., BLM, 2000

GEWEX Atmospheric Boundary Layer Study (GABLS)

- **First Detailed Intercomparisons of Atmospheric LES codes for a selected SBL case**
- **Advantages: Large number of participants and large amount of data for intercomparisons**
- **Disadvantages: No observational data to justify LES output and the only SBL case to compare**

Acronym	Institution	Investigators	Email
MO	Met Office, UK	Malcolm MacVean Bob Beare Anne McCabe	bob.beare@metoffice.gov.uk
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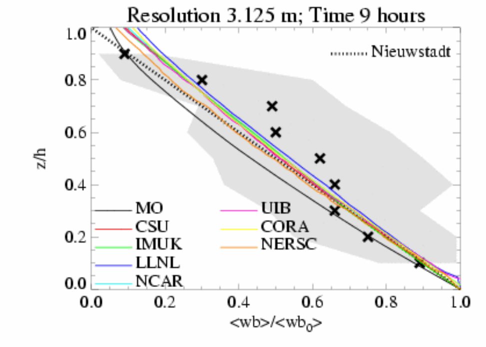
WU	Wageningen University, The Netherlands	Arnold Moene	arnold.moene@wur.nl
NRL	Naval Research Laboratory, Monterey, CA	Chris Golaz	golaz@nrlmry.navy.mil

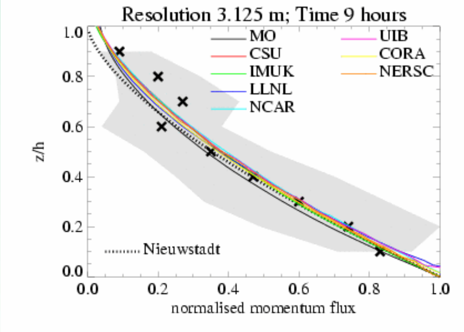
GABLS Results

- **All models produce successful simulations at 3 m and finer mesh**
- **Sub-grid models remain important in simulations at coarser meshes**

Table IV. Boundary layer heights for last hour of simulation and each of the models at different resolutions.

Model/Resolution	1 m	2 m	3.125 m	6.25 m	12.5 m
MO	164m	162m	171m	204m	263m
CSU	–	–	197m	211m	237m
IMUK	149m	162m	168m	158m	–
LLNL	–	–	169m	194m	257m
NERSC	–	–	179m	188m	204m
WVU	–	–	–	201m	197m
NCAR	–	197m	204m	–	–
UIB	–	–	173m	174m	191m
CORA	–	187m	195m	211m	–
WU	–	–	–	178m	158m (L)
COAMPS^{TM}	–	–	–	161m	–
ENSEMBLE MEAN	157m	177m	182m	188m	215m





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Resolution Issues

- **SBL is difficult to simulate because of wide range of time and spatial scale responsible for its formation**
- **Large scales: Inertial Oscillation, Internal Gravity Waves**
- **Small scales: Developed turbulence**
- **Computational time: $N^3=(4L/l)^3$**

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Where is Turbulence?

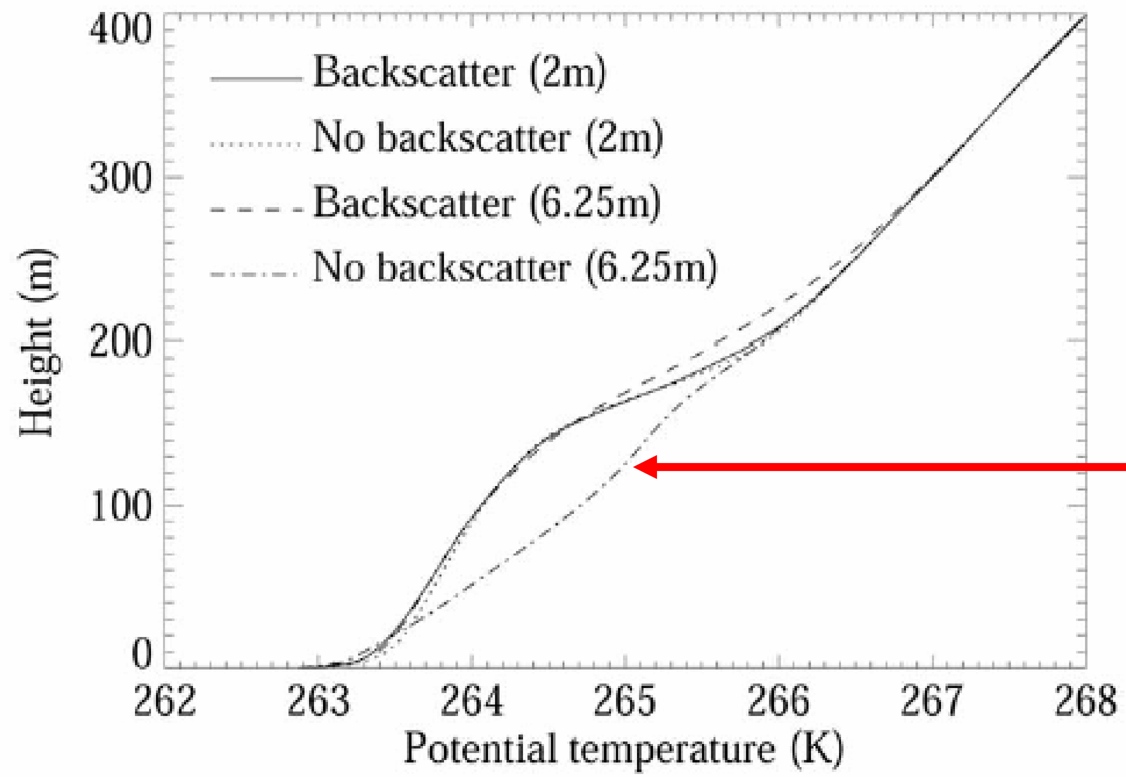

Figure 6. The sensitivity to including and excluding stochastic backscatter at 6.25-m and 2-m resolutions for the prescribed surface cooling runs.

Gradient based eddy-viscosity suppresses development of turbulence in SBL.

After Beare & McVean, 2004

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Turbulence Closure Problem

- Turbulence in stably stratified flows is small-scale
- How to deal with it:
  - Either refine numerical resolution
  - Or improve turbulence closures
  - Or both?

How does energy exchange between scales in the real stratified atmosphere?

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Horizontal Arrays Turbulence Study (HATS)

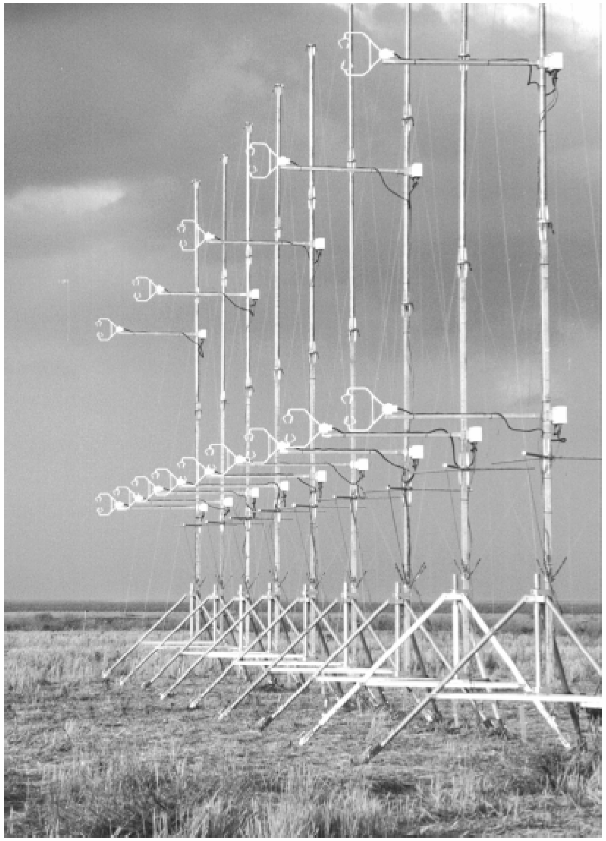

FIG. 1. Photograph of the setup of array 1 during the HATS experiment near Kettlamen City, California. Photo courtesy of Tom Horst, NCAR.

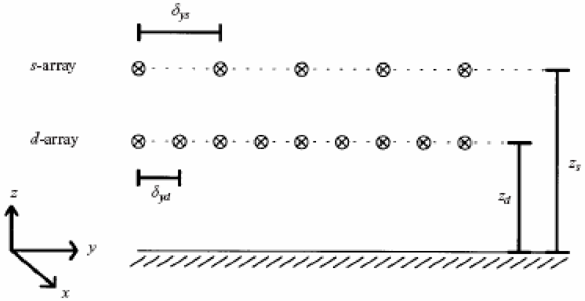

FIGURE 1. Sketch of the sonic deployment and the (x, y, z) coordinate system used for analysis. The sonic anemometers ⊗ in the double and single arrays are located at (z_d, z_s) above the surface, the lateral separation between individual sonic anemometers is $(\delta_{yd}, \delta_{ys})$.

Configuration	z_d	δy_d	z_s	δy_s	Curve
1	3.45	3.35	6.90	6.70	●
2	4.33	2.17	8.66	4.33	⊗
3a	8.66	2.17	4.33	1.08	✳
3b	8.66	2.17	4.33	1.08	○
4	4.15	0.50	5.15	0.62	◇

TABLE 1. Vertical location and lateral spacing of the sonic anemometers (in m).

HATS Aim:

- **to understand spectral energy fluxes in the stratified atmosphere**

LES: Rigorous Foundations

Original NSE
for incompressible Boussinesq fluid

$$\frac{\partial u}{\partial t} = -\nabla(uu + p) + Ro^{-1}\omega \times u - Ri_B \Theta$$

Convolution with low-pass filter operator (filtration)

$$\overline{u} = F * u = \int u(x',t)F(x-x',t)d^3x'$$

Regularized NSE for large scales

$$\frac{\partial \overline{u}}{\partial t} = -\nabla(\overline{u}\,\overline{u} + \overline{p}) + Ro^{-1}\omega \times \overline{u} - Ri_B \overline{\Theta} - \boxed{\nabla\left(\overline{uu} - \overline{u}\,\overline{u}\right)}$$

Existence and Uniqueness Theorem (Ladyzenskaja)

4.2. The Ladyženskaja model

Recalling that the Navier–Stokes equations are based on Newton's linear hypothesis, Ladyženskaja and Kaniel proposed to modify the incompressible Navier–Stokes equations to take into account possible large velocity gradients, [43, 42, 37].

Ladyženskaja introduced a nonlinear viscous tensor $\mathbf{T}_{ij}(\nabla \mathbf{u})$, $1 \leq i, j \leq 3$ satisfying the following conditions:

L1. $\mathbf{T}$ is continuous and there exists $\mu \geq \frac{1}{4}$ such that

$$\forall\ \xi \in \mathbb{R}^{3\times 3}, \quad |\mathbf{T}(\xi)| \leq c(1 + |\xi|^{2\mu})|\xi|. \tag{4.3}$$

L2. $\mathbf{T}$ satisfies the coercivity property:

$$\forall\ \xi \in \mathbb{R}^{3\times 3}, \quad \mathbf{T}(\xi) : \xi \geq c|\xi|^2(1 + c'|\xi|^{2\mu}). \tag{4.4}$$

L3. $\mathbf{T}$ possesses the following monotonicity property: there exists a constant $c > 0$ such that for all solenoidal fields ξ, η in $\mathbf{W}^{1,2+2\mu}(\Omega)$ either coinciding on the boundary Γ or being periodic,

$$\int_\Omega (\mathbf{T}(\nabla\xi) - T(\nabla\eta)) : (\nabla\xi - \nabla\eta) \geq c \int_\Omega |\nabla\xi - \nabla\eta|^2. \tag{4.5}$$

These conditions are actually satisfied in the case where

$$\mathbf{T}(\xi) = \beta(|\xi|^2)\xi \tag{4.6}$$

provided the viscosity function $\beta(\tau)$ is a positive monotonically-increasing function of $\tau \geq 0$ and for large values of τ the following inequality holds

$$c\tau^\mu \leq \beta(\tau) \leq c'\tau^\mu,$$

with $\mu \geq \frac{1}{4}$ and c, c' are some strictly positive constants. Smagorinsky's model obviously falls into the admissible category with $\beta(\tau) = \tau^{1/2}$.

Introducing now a (possibly small) positive constant $\varepsilon > 0$, the modified Navier–Stokes equations take the form

$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p - \nabla\cdot(\nu\nabla\mathbf{u} + \varepsilon\mathbf{T}(\nabla\mathbf{u})) = \mathbf{f}, \\ \nabla\cdot\mathbf{u} = 0 \\ \mathbf{u}|_\Gamma = 0, \quad \text{or } \mathbf{u} \text{ is periodic,} \\ \mathbf{u}|_{t=0} = \mathbf{u}_0. \end{cases} \tag{4.7}$$

The striking result from [43, 42] (see [37] for a similar result where monotonicity is also assumed) is the following theorem

Theorem 4.1. *Provided conditions* L1, L2, *and* L3 *are satisfied,* $\mathbf{u}_0 \in \mathbf{H}$ *and* $\mathbf{f} \in L^2(]0, +\infty[; \mathbf{L}^2(\Omega))$, *then* (4.7) *has a unique weak solution in*

$$L^{2+2\mu}(]0, t[; \mathbf{W}^{1,2+2\mu}(\Omega) \cap \mathbf{V}) \cap C^0([0, t]; \mathbf{H}) \quad \textit{for all } t > 0.$$

- **The solution exists and is unique if the regularizing term satisfy (Ladyzenskaja, 1969), e.g.**

$$\tau_{ij} = \varepsilon\beta\left|S_{ij}\right|^2 S_{ij} = \nu_t S_{ij};$$

$$\varepsilon, \beta > 0,\ \varepsilon\beta = 2(C_s\Delta)^2 f(\tau_{ij});$$

$$S_{ij} = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$

This is exactly the Boussinesq gradient approximation with the Smagorinsky-Lilly eddy-viscosity closure!

- **Solution weekly converge to a weak NSE solution**

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Ladyzenskaja Theorem States That:

- LES do provide the deterministic solution for realistic turbulent flows: Not only statistics but also individual structure development in LES is meaningful!
- Infinite Re, turbulent fluids possess an effective, natural viscosity: Flow Re is scale dependent and lower for larger eddies

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In Theory:
No Closure Needed !

Exact form of the sub-filter stress term (Germano, 1986):

$$u = \overline{u} - \varepsilon^2 \nabla^2 \overline{u}$$

This differential filter with $\varepsilon = a/\sqrt{24}$

is consistent with the Gaussian filter:

$$G(x - x'; a) = \left(\frac{6}{\pi a^2}\right)^{3/2} \exp\left(-\frac{6(x - x')^2}{a^2}\right)$$

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In Practice:
A Closure Needed !

Taylor series give another exact form of the stress:

$$\left(\overline{u_i u_j} - \overline{u_i}\,\overline{u_j}\right) = \sum_{k=1}^{3} \Delta^2 \frac{\partial \overline{u_i}}{\partial x_k} \frac{\partial \overline{u_j}}{\partial x_k} + \frac{\Delta^4}{2!} \sum_{m=1}^{3} \frac{\partial^2 \overline{u_i}}{\partial x_k \partial x_m} \frac{\partial^2 \overline{u_j}}{\partial x_k \partial x_m} + \dots$$

- **This is an ill-posed mathematical problem**
- **Convergence is very slow**
- **Result is sensitive to the numerical errors at the smallest grid scales.**

1 term (Clark's closure)	2 terms	3 terms
0.61	0.80	0.89

Correlations between exact and truncated stresses

Scale Distribution [0, π] of Numerical Errors in Advection Term

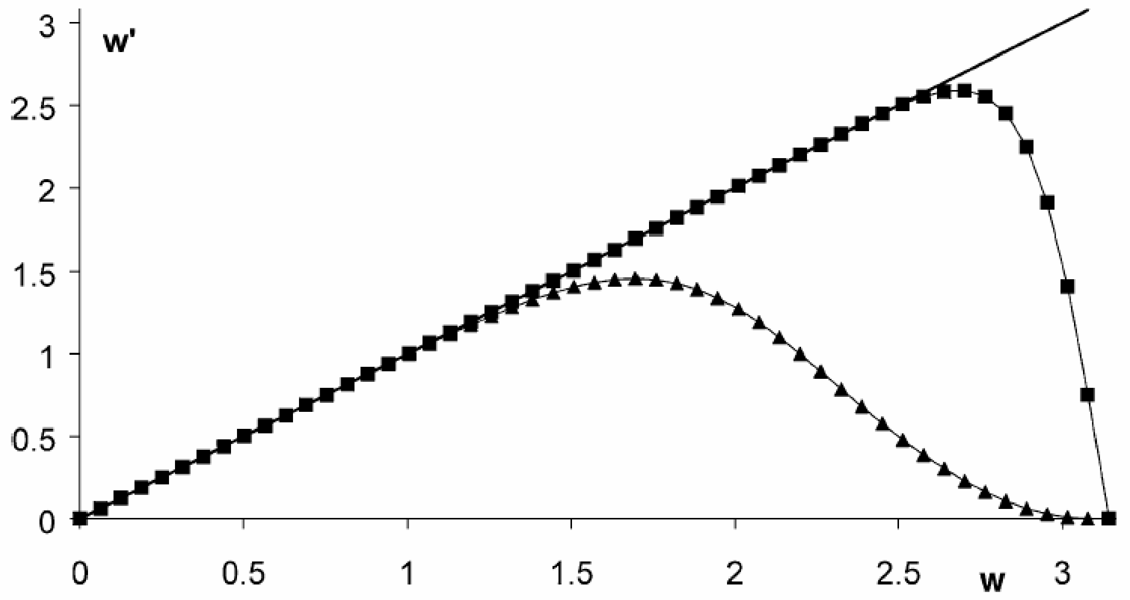

FIG. 4. MWN for derivative/filtering. —, exact; -■-, derivative only (7); -▲-, derivative + filtering (7)*(36).

After, Fedioun et al, 2001

Consistent SGS Model: Derivation

Decompose on resolved and sub-filter variables: $u_i = \overline{u}_i + u'_i$

Substitute:

$$\left(\overline{u_i u_j} - \overline{u}_i \overline{u}_j\right) = \underbrace{\left(\overline{\overline{u}_i \overline{u}_j} - \overline{u}_i \overline{u}_j\right)}_{\substack{\textbf{Known} \\ \text{Up to 50\% of total stress}}} + \underbrace{\left(\overline{\overline{u}_i u'_j} + \overline{\overline{u}_j u'_i}\right) + \overline{u'_i u'_j}}_{\substack{\textbf{Unknown} \\ \text{About 50\% of total stress}}} = L_{ij} + C_{ij} + R_{ij}$$

HATS Observed Weights of SGS Model Terms

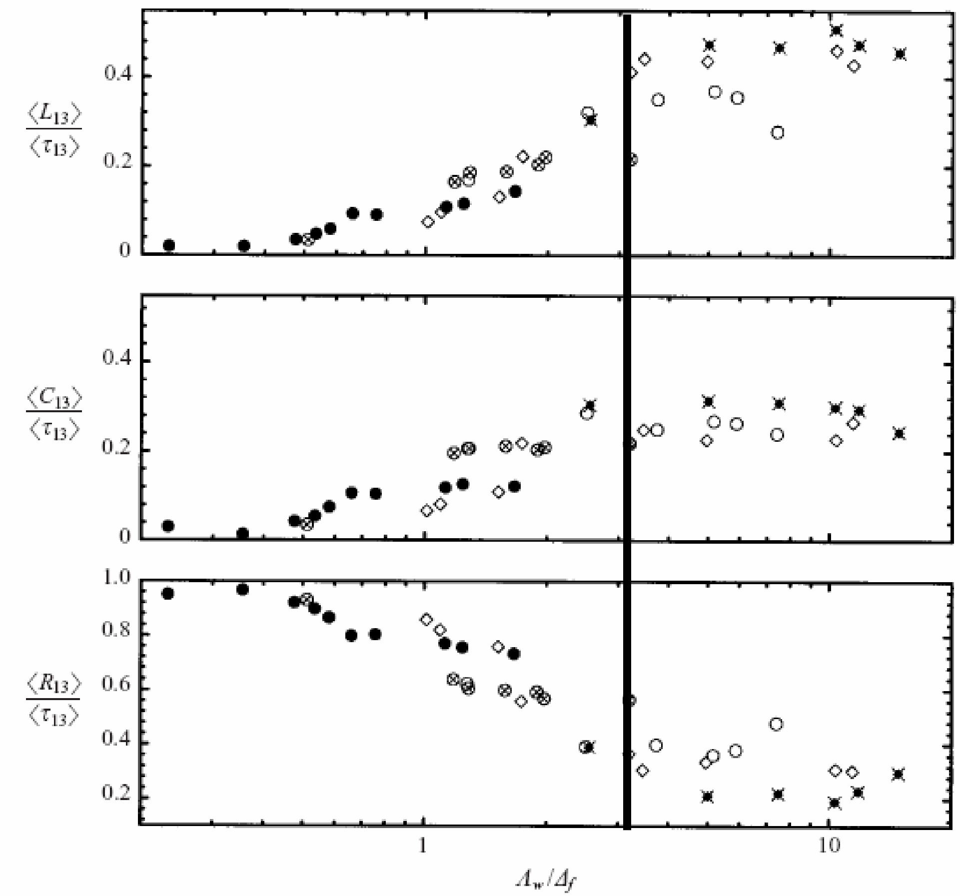

FIGURE 12. Decomposition of τ_{13} flux into modified-Leonard, cross-, and Reynolds terms.

After Sullivan et al, JFM, 2003

At certain resolution almost exact closure is possible.

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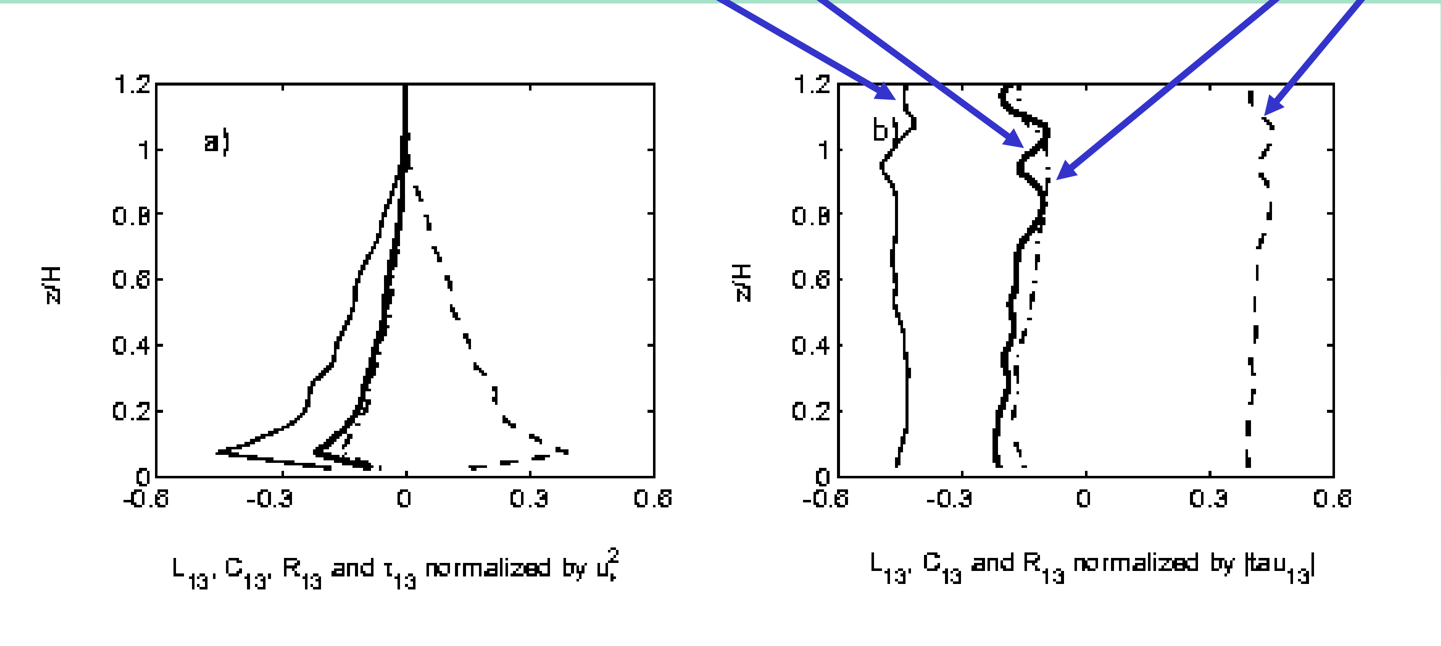
Terms of SGS Model in Actual Simulations

R_{13} - Almost cancel each other - C_{13}

t_{13} - Almost equal - L_{13}



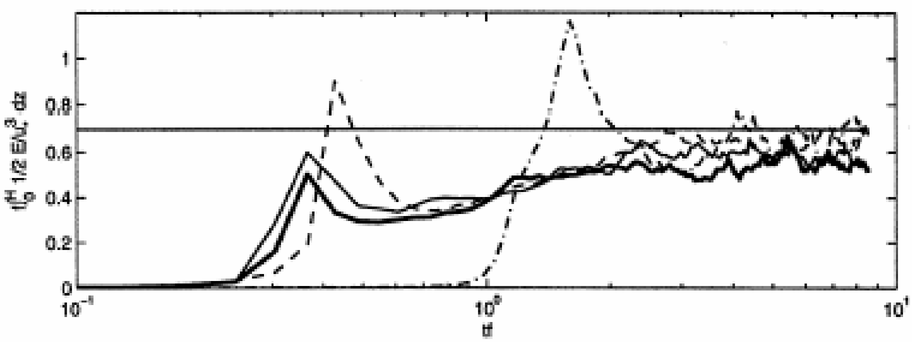
Efficient SGS Model

- **Definition:**

A SGS model is more efficient if it allows simulating higher Re flows at a given mesh

- **Controversy:**

To conduct steady-state LES, at any given mesh, N, should be assured energy dissipation rate equal to energy generation rate, 1/Cs, so that total LES Re=N/Cs=const



In all runs N=64 but Cs are different. Gradient models have problems with early perturbation development

Figure 14. Time evolution of the integral non-dimensional TKE: — run PBL64-DMM; — run PBL64-DSM; – – – run PBL64-TSM0; –·– run PBL64-TSM1. The straight line shows the mean steady state level of the integral non-dimensional TKE in [17].

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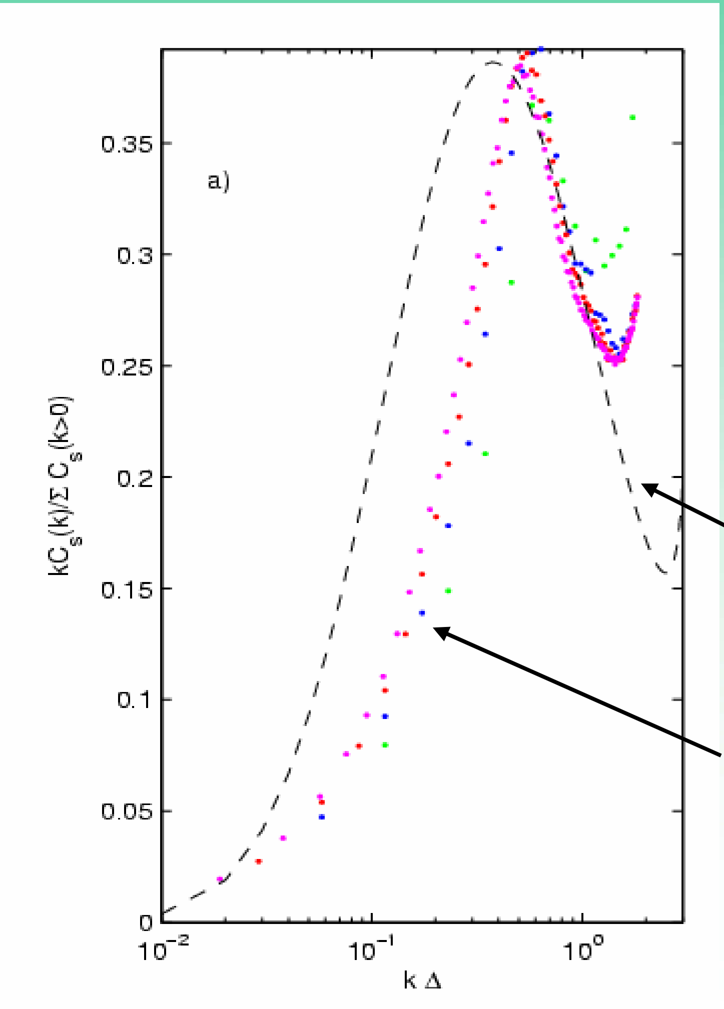
Efficient SGS Model: Spectra



- **Answer:**
Viscosity should be concentrated on the smallest resolved scales!

Statistically "ideal" SGS model

Dynamic-mixed SGS model used in the LESNIC code

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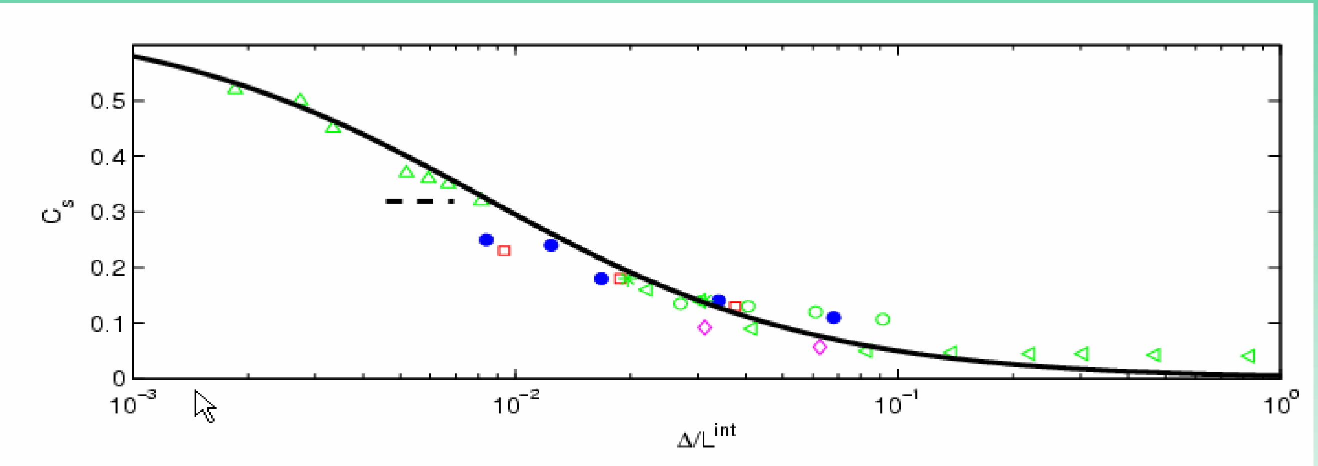
Dynamic-Mixed SGS Model: Integral Assessment



- DMM is robust
- DMM is mathematically consistent:
  - resolved Leonard term is explicitly calculated
  - unresolved Cross and Reynolds terms are modelled with the regularizing, local Smagorinsky model
- DMM is efficient:
  - effective eddy-viscosity is concentrated at the smallest resolved scales

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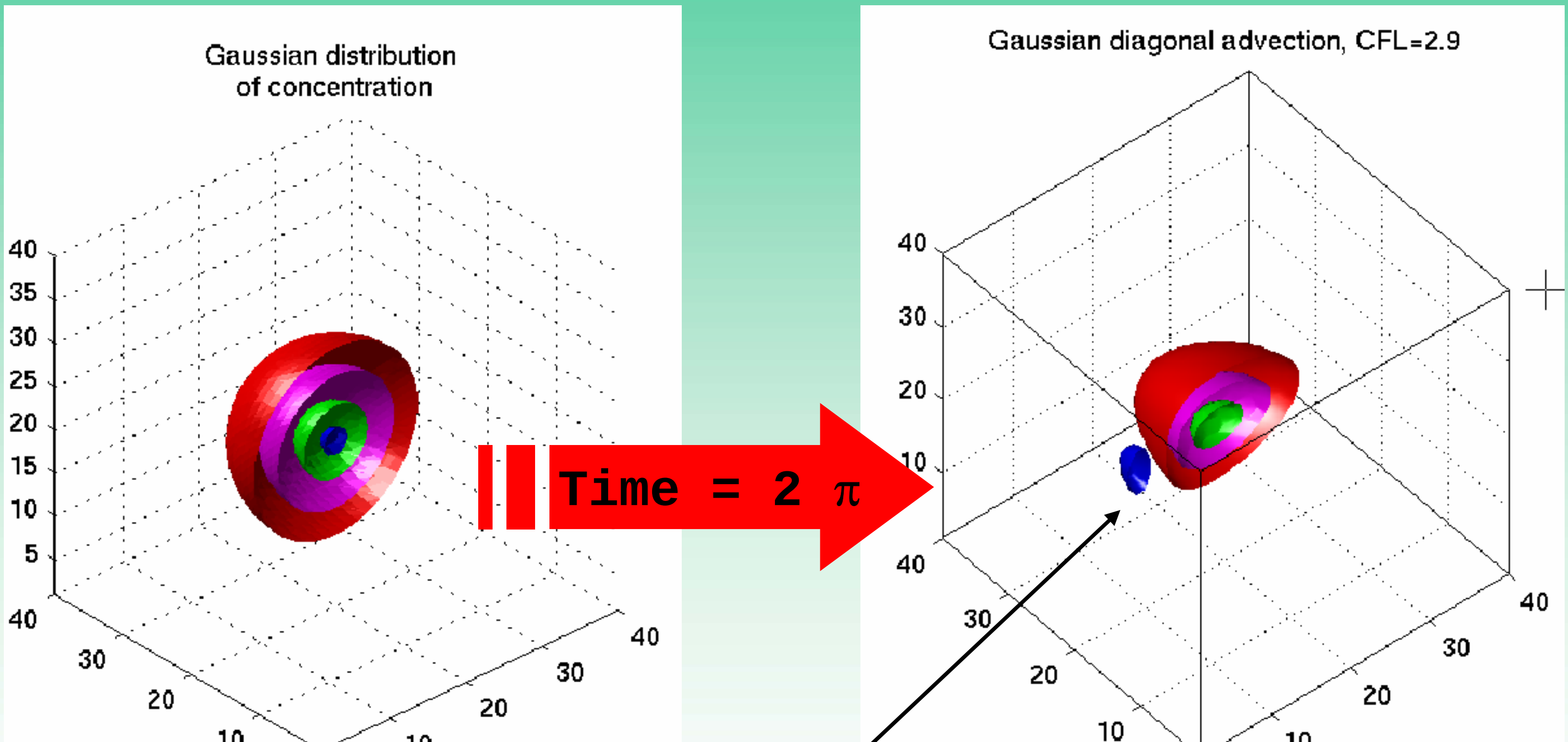
LESNIC v2.13
Large Eddy Simulation NERSC Improved Code

- Esau, I. N., J. Env. Fluid Mech., 4(3), 2004, 273-303
- Fedorovich, E., Esau, I., et al., Proceedings 16th AMS Conf. on PBL, 2004
- Beare, R. J., Esau, I., et al., Boundary Layer Meteorol. 2005, in press
- Esau & Lyons, Agricul. Forest Meteorol., 2002, 114(1-2), 3-13
- Esau, Proceedings 1st CliC meeting, Beijing, 2005

LESNIC: Numerical Schemes

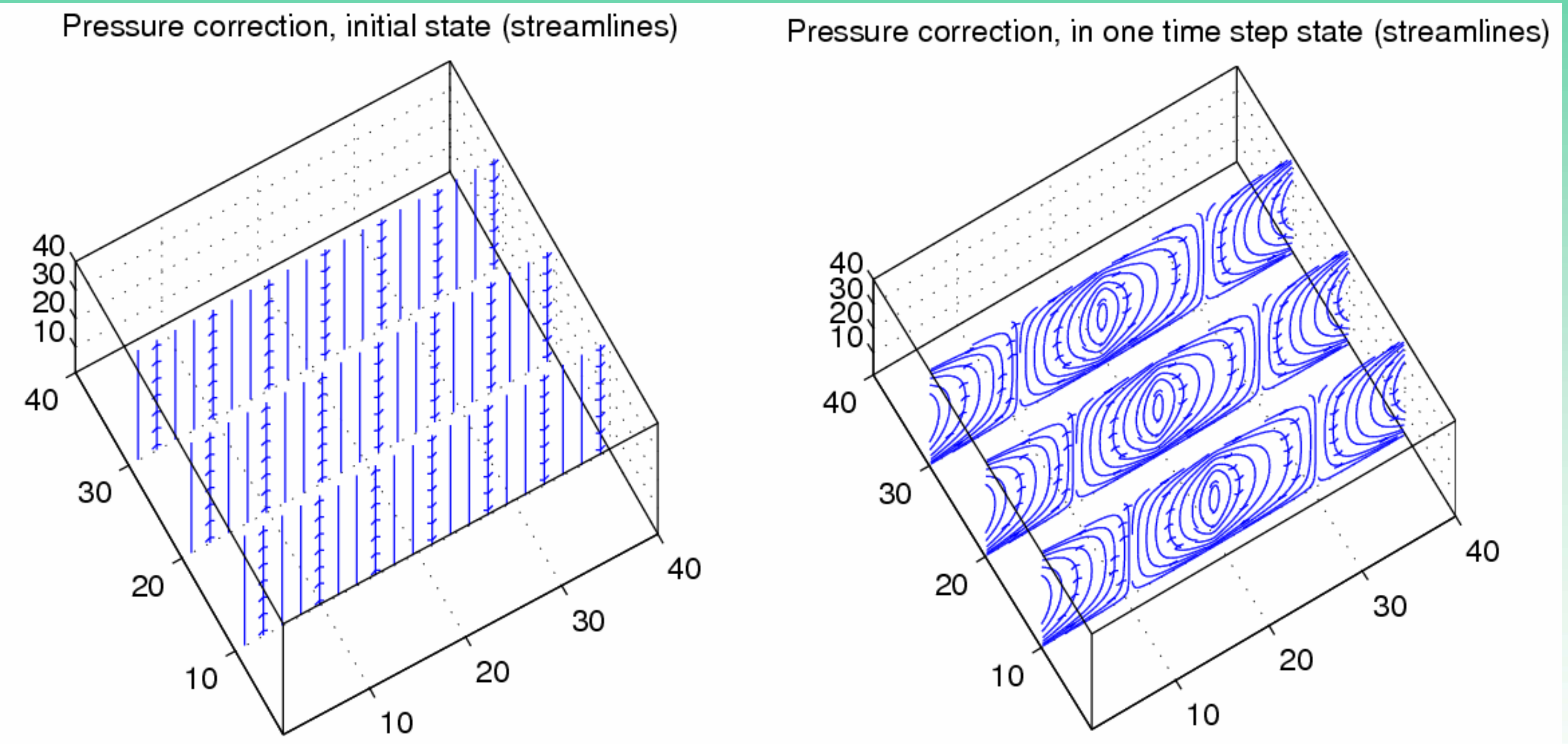
- 2nd order fully conservative central difference scheme for the skew-symmetric advection term
- 4th order Runge-Kutta scheme for time stepping
- Direct (Fourier-Tridiagonal solver) fractional-step pressure correction scheme for continuity
- Staggered C-type computational mesh, which demands only fluxes as boundary conditions

LESNIC: Advection



Central difference schemes are conservative (preserve total energy) but non-monotonic (do not preserve shape of fluctuations) – effect introduce artificial buoyancy flux in LES

LESNIC: Pressure Correction



Buoyancy forces work trough continuity equation –
Numerical errors in velocity divergence introduce artificial buoyancy flux in LES

LESNIC: Errors Summary

- **Artificial fluxes concentrating on the smallest resolved scales are harmful for turbulence closures**
- **Artificial fluxes can be comparable with physical fluxes in the case of strongly stratified, intermittent boundary layers**

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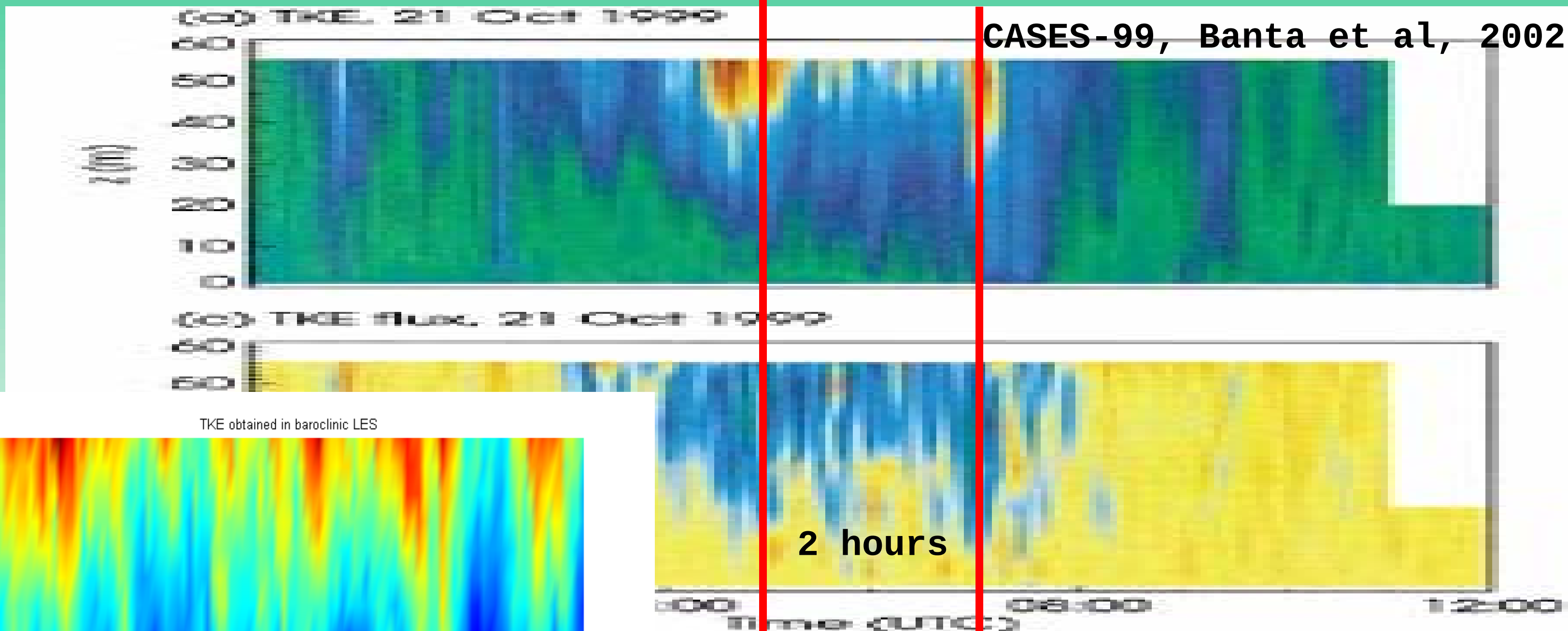
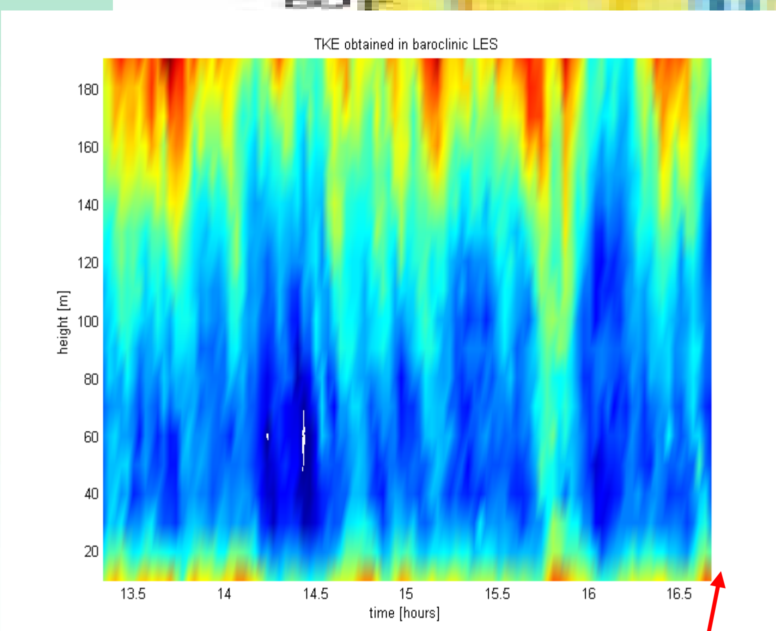
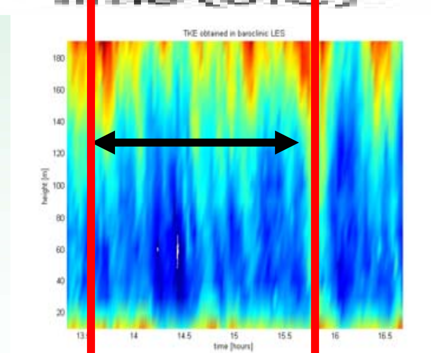
LESNIC: Illustration

CASES-99, Banta et al, 2002



2 hours





Artificial TKE generation

LESNIC: Turbulence Closure

- **Dynamic Mixed Model (DMM) by Vreman et al., 1994, 1997, which excludes needs for manual tuning of sub-grid parameters**

$$\tau = (\overline{uu} - \overline{u}\,\overline{u}) = (\widehat{\overline{u}\,\overline{u}} - \hat{\overline{u}}\,\hat{\overline{u}}) - 2l^2\left|\overline{S}\right|\overline{S},$$

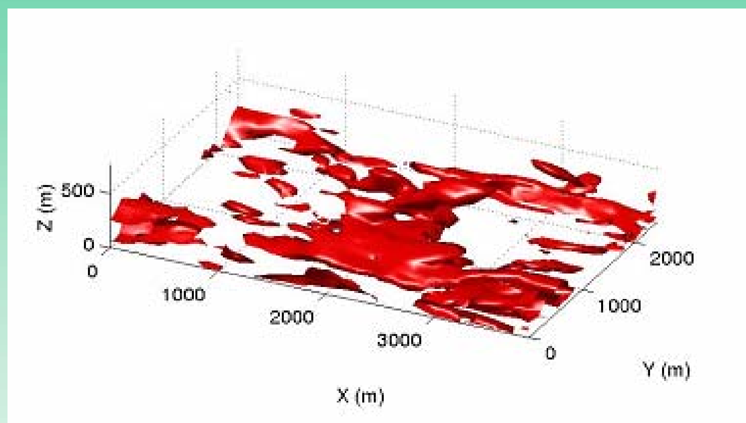
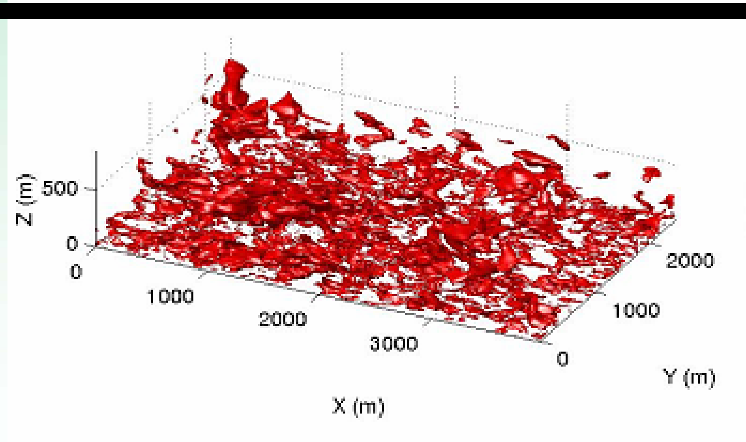
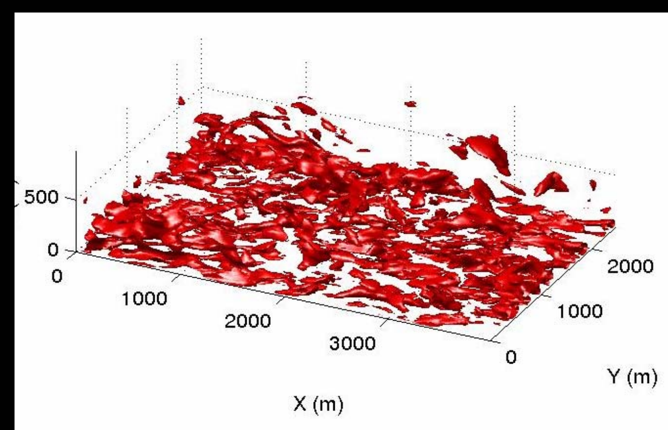
$$l = C_S\Delta$$

$$M_{ij} = \widehat{\left(\left|\overline{S}_{ij}\right|\overline{S}_{ij}\right)} - \alpha\left|\hat{\overline{S}}_{ij}\right|\hat{\overline{S}}_{ij},$$

$$l^2 = \frac{1}{2}\frac{\left|(L_{ij} - H_{ij})M_{ij}\right|_2}{\left|M_{ij}\right|_2^2}$$

$$H_{ij} = \widehat{\overline{\hat{\overline{u}}_i\,\hat{\overline{u}}_j}} - \hat{\overline{\hat{\overline{u}}}}_i\,\hat{\overline{\hat{\overline{u}}}}_j - \left(\hat{\overline{\overline{u}}}_i\,\hat{\overline{\overline{u}}}_j - \left(\widehat{\overline{\overline{u}}_i\,\overline{\overline{u}}_j}\right)\right)$$

DMM versus Smagorinsky

	DMM	Smagorinsky
N=32		No Turbulence
N=128		

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Shear-driven PBL Parameter Phase Space

- Regularized Equations + Boundary Conditions = Unique Solution
- Turbulence measures are universal, albeit non-linear and unknown, functions of prescribed external governing parameters
- Primary LES application is to find those universal functions

$$\frac{\partial U}{\partial z} = \frac{u_*}{z} f\left(\frac{z}{L}\right),$$

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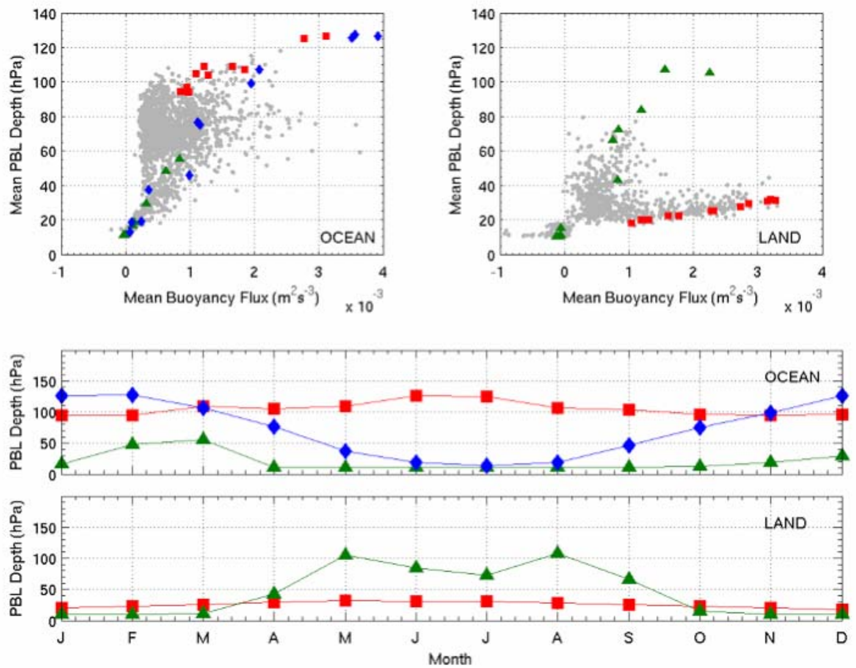
Wrong Way to Do Analysis



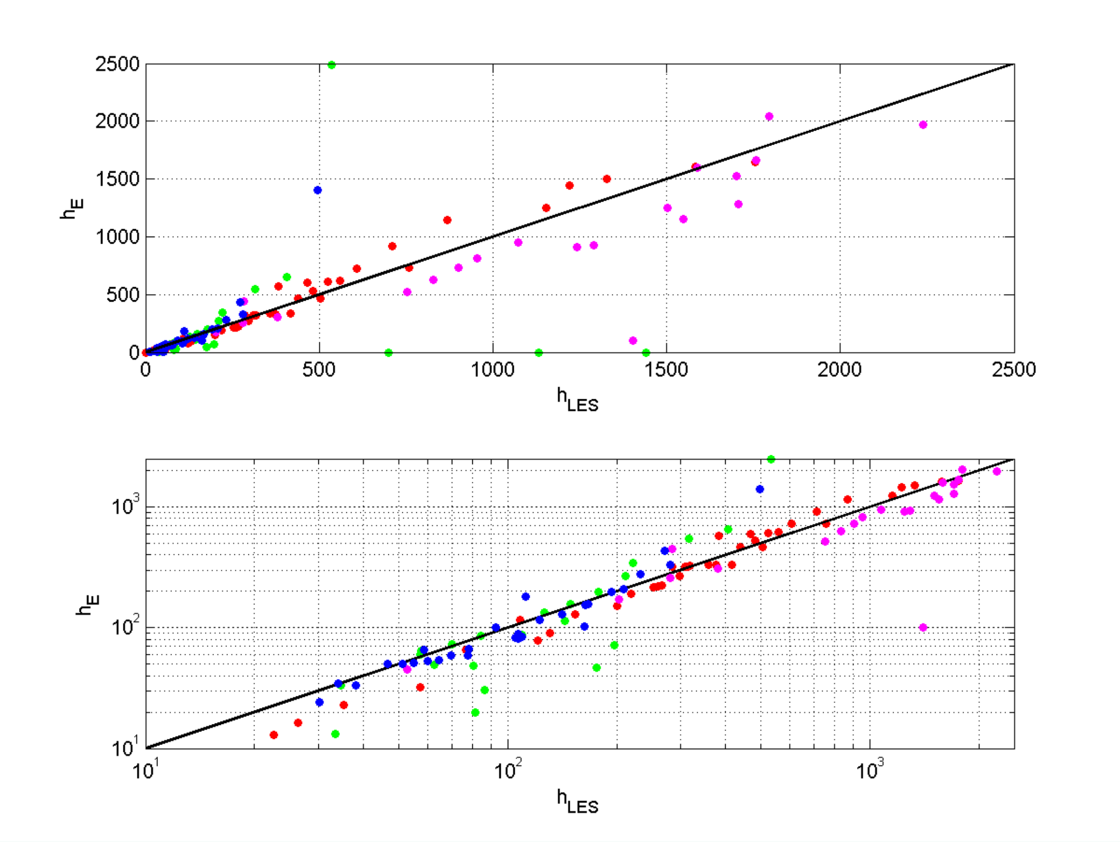
$$\hat{h}_o = 60 - 6005\,\mathcal{B}_H + 1.4 \times 10^5\,\mathcal{B}_E + 32130\,\mathcal{B}_R - 1.83\,\sigma - 12.6\,M_B$$

$$\hat{h}_l = 13 + 4434\,\mathcal{B}_H + 0.4 \times 10^5\,\mathcal{B}_E + 56003\,\mathcal{B}_R + 0.04\,\sigma - 2.2\,M_B$$

- data collection without due regards to their physical nature
- matching statistical regression without due regards to the method limitations

UCLA, Department of Atmospheric & Oceanic Sciences

Right Way to Do Analysis



- start from governing parameters
- consider physical asymptotes
- find proper class of universal functions
- match coefficients with data

$$h_E = C_h \frac{u_*}{|f|}$$

$$C_h = C_R\left(1 + C_0\mu_\Gamma\right)\left(1 + \frac{C_R^2 C_{uN}}{C_S^2}\mu_N + \frac{C_R^2}{C_S^2}\mu\right)^{-1/2}$$

Governing Parameters

What a LES run needs to start:

- $\Delta P = - f\, Ug$ [m s^{-2}] – horizontal pressure gradient

- Fs [K m s^{-1}] – surface temperature flux
- $\Delta\Theta$ [K m^{-1}] – vertical temperature gradient
- f [s^{-1}] – Coriolis parameter

- z_0 [m] – surface roughness
- β [m s^{-2} K^{-1}] – effective gravity

π-theorem:
6 (parameters) – 3 (dimensional units) =
3 (non-dimensional groups)

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Non-Dimensional Numbers

Truly neutral PBL: $Ro = -\Delta P / (f^{2} z_0) = Ug / (f\ z_0)$

Conventionally neutral PBL: $Ri = -\Delta P / (\beta\ \Delta\Theta\ z_0) = f\ Ug / (N^2 z_0)$

Nocturnal PBL: $Rs = -\Delta P / ((\beta\ Fs)^{2/3} z_0^{1/3}) =$

$= f\ Ug / ((\beta\ Fs)^{2/3} z_0^{1/3})$

"*Universal description of the large-scale turbulence is still missing partially because of considerable statistical scatter in measurements in nature.*"
(Monin and Yaglom, 1974)

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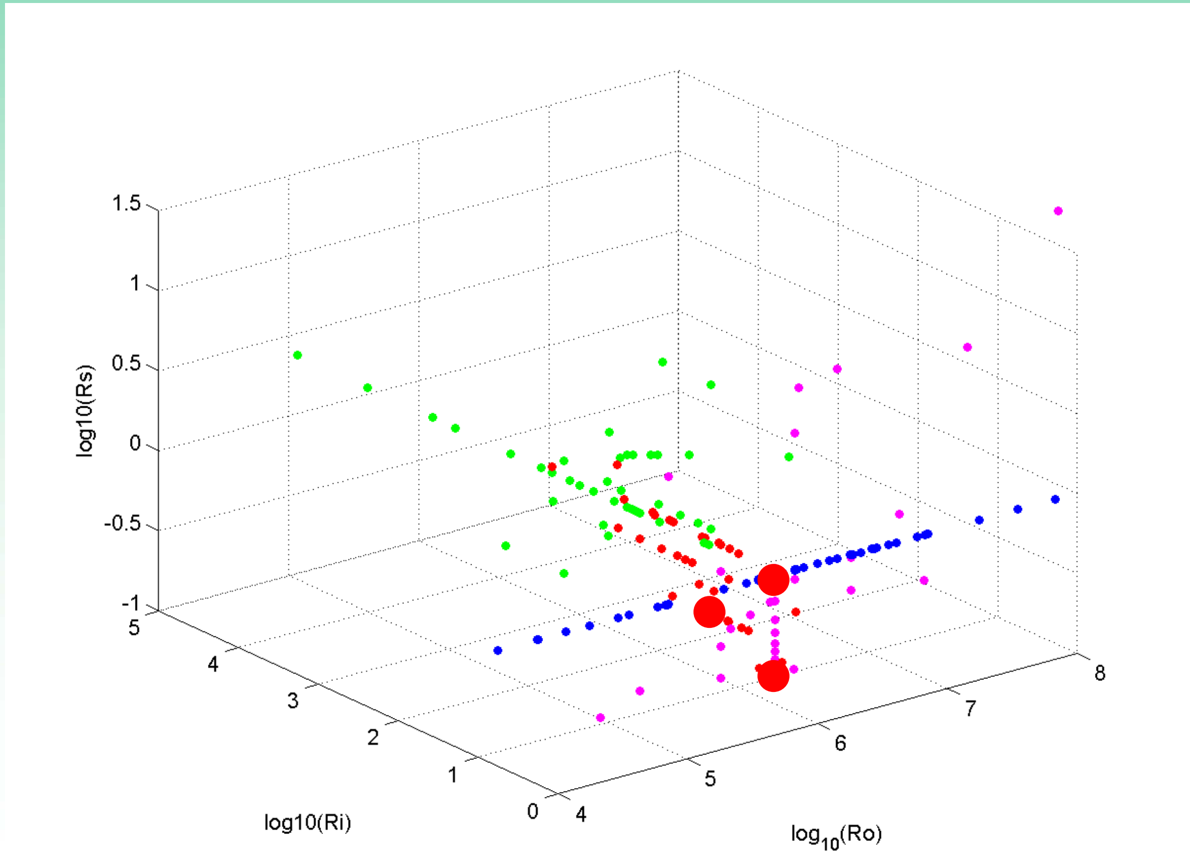
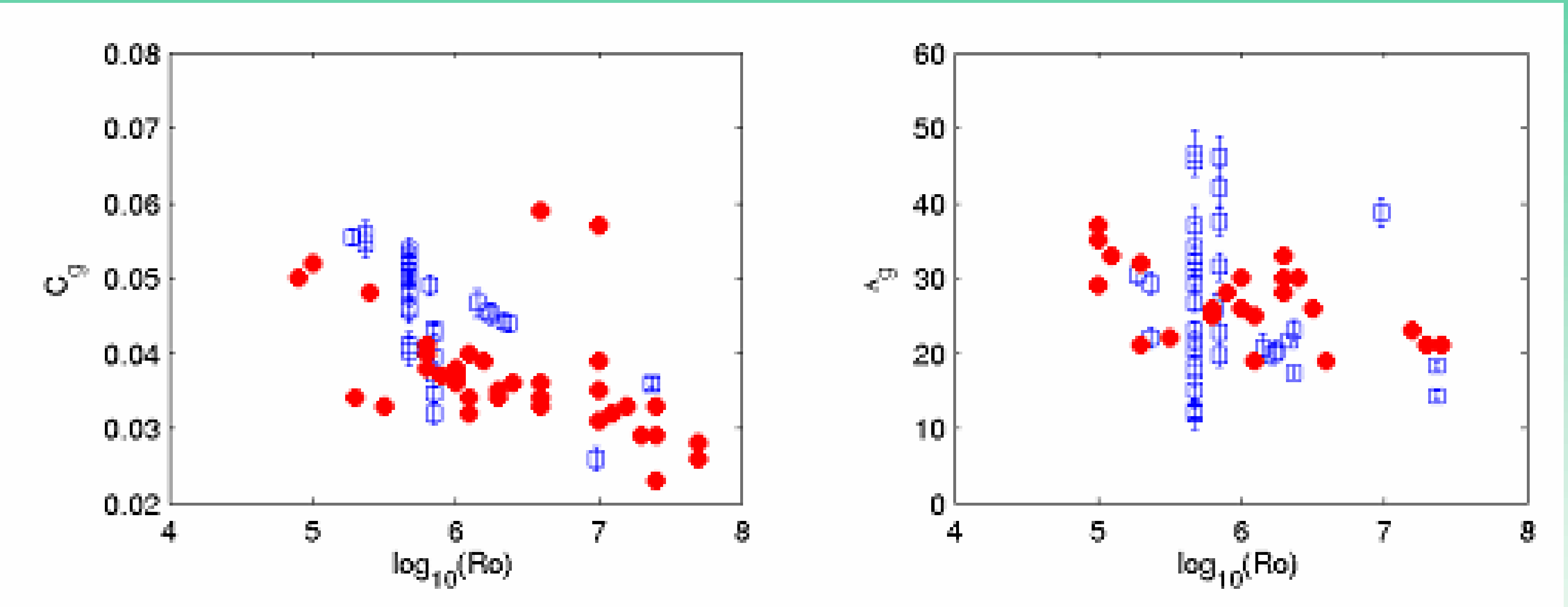
LESNIC Database64

- **Numerous LES had been published.**
- **They covered only few sports ● in the phase space.**

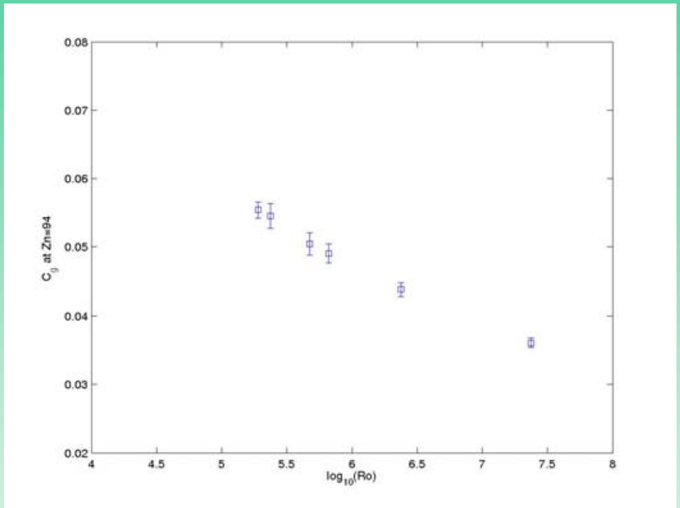


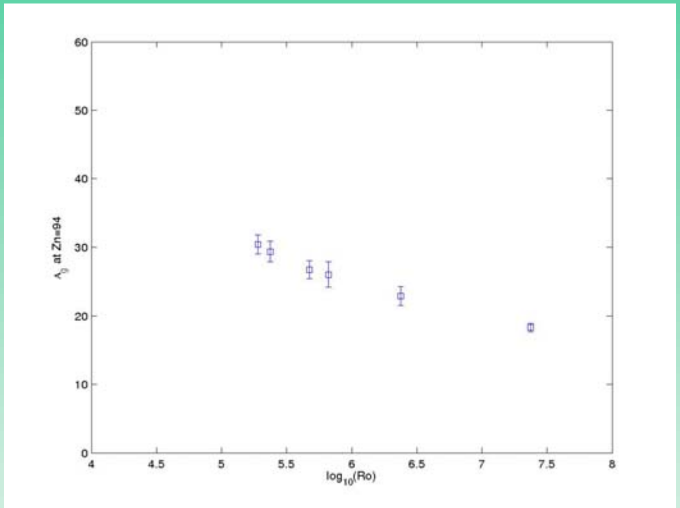
Database64: Applications

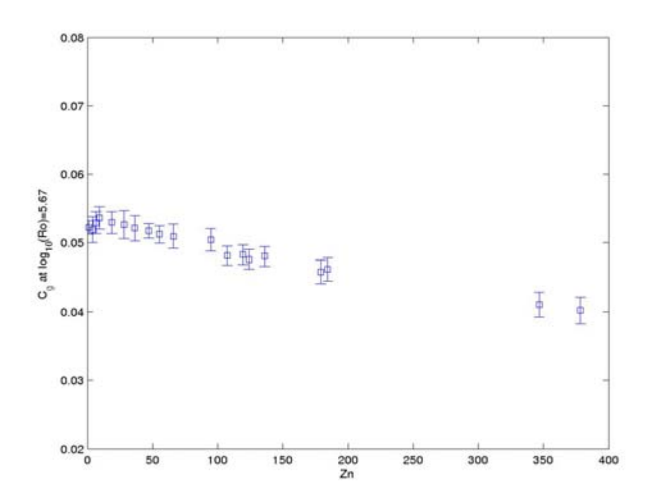


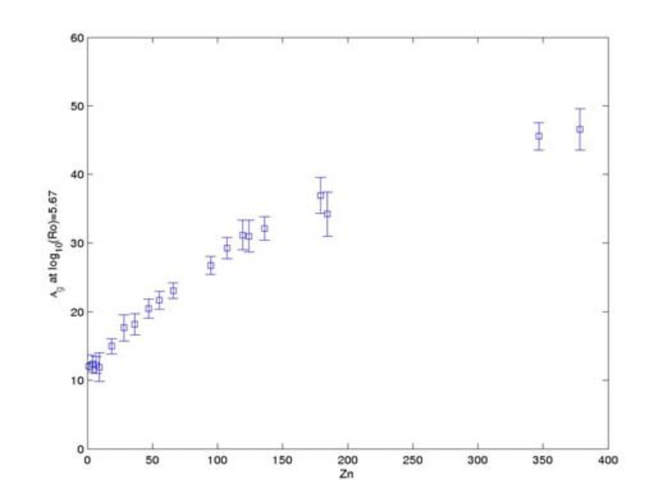
Geostrophic drag and cross-isobaric angle on traditional charts versus Rossby number. Symbols:
red – atmospheric near-neutral data (Hess & Garratt, BLM, 2002);
blue – conventionally neutral data LESNIC

Database64: Applications









SBL: New Views

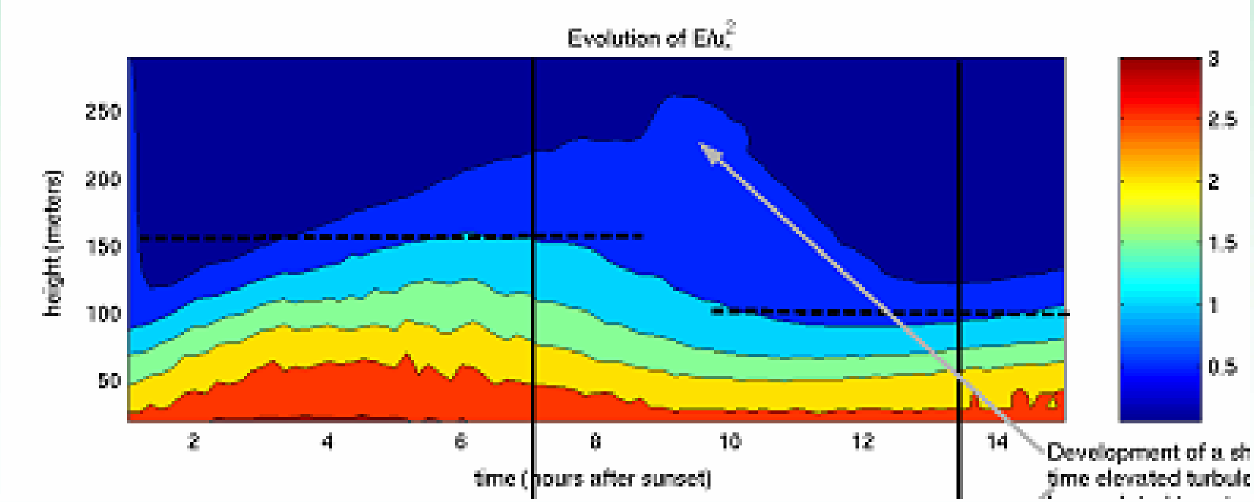
- **Two (possibly three) kinds of SBL:**
  - nocturnal SBL developing against neutrally stratified atmosphere

  - long-lived SBL developing against stably stratified atmosphere

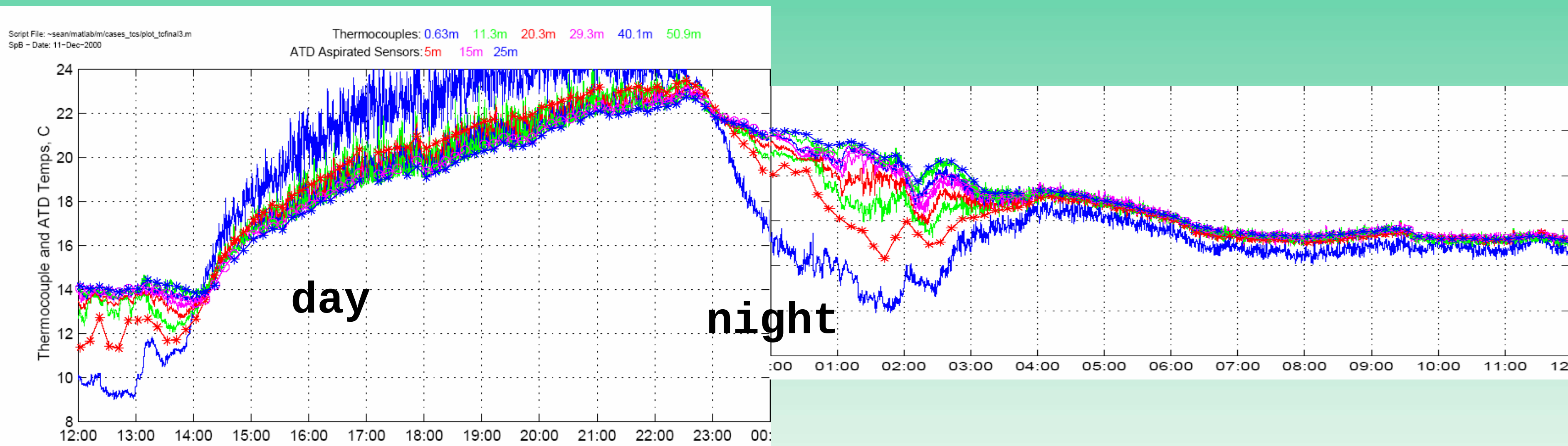
  - intermittent or buoyancy-dominated SBL sporadically developing against very stable stratification? (see Challenges)

SBL: New Views

- Turbulence is not just decay in the SBL
- Turbulence eventually rebounds due to linear growth of optimal structures

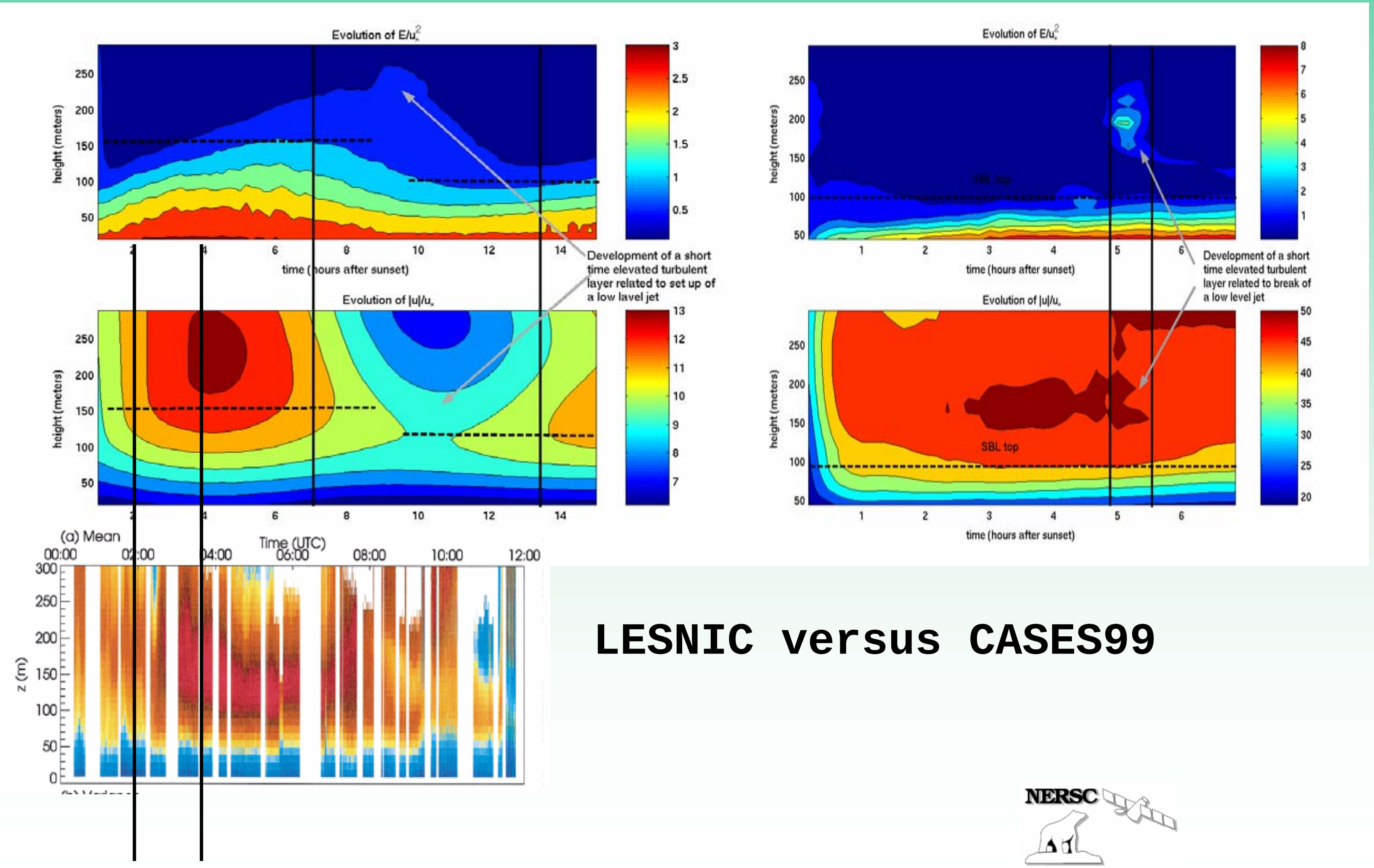


CASES99: Turbulence Rebound



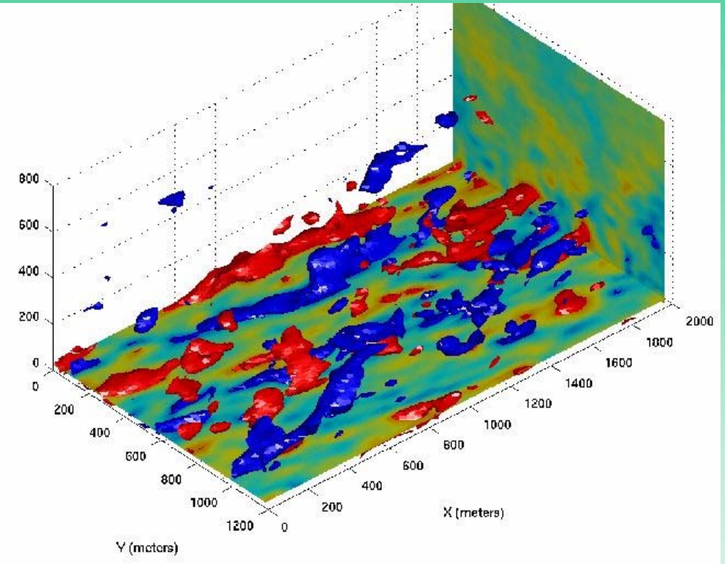
28/10 1999, Kansas

SBL: Low Level Jet



LESNIC versus CASES99

SBL: 3D Turbulence Structure



- Eddies do not look like PANCAKES flatten in the vertical direction
- Eddies look like fat WORMS snaking along with the mean flow

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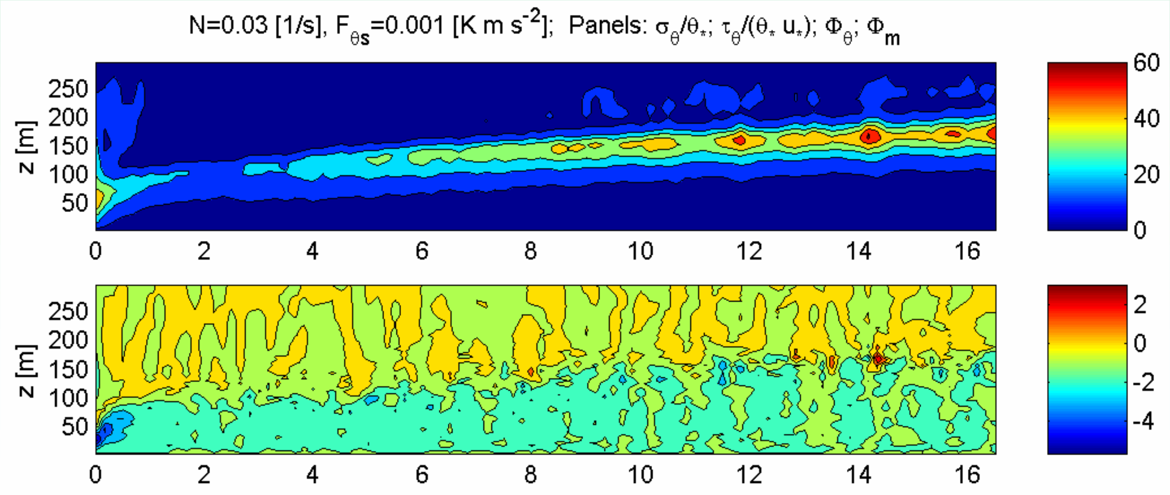
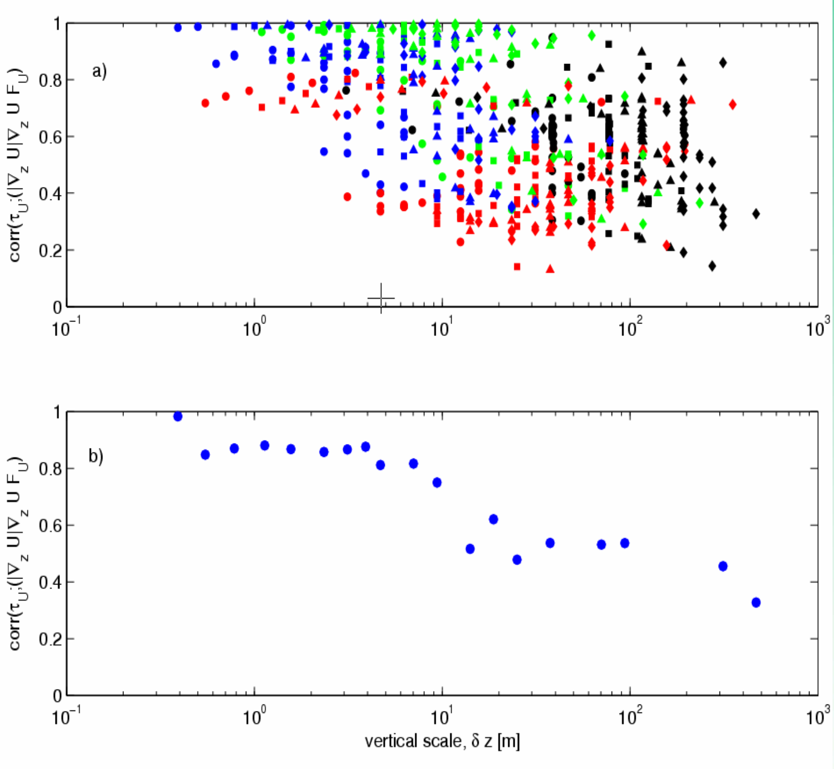
SBL: New Views

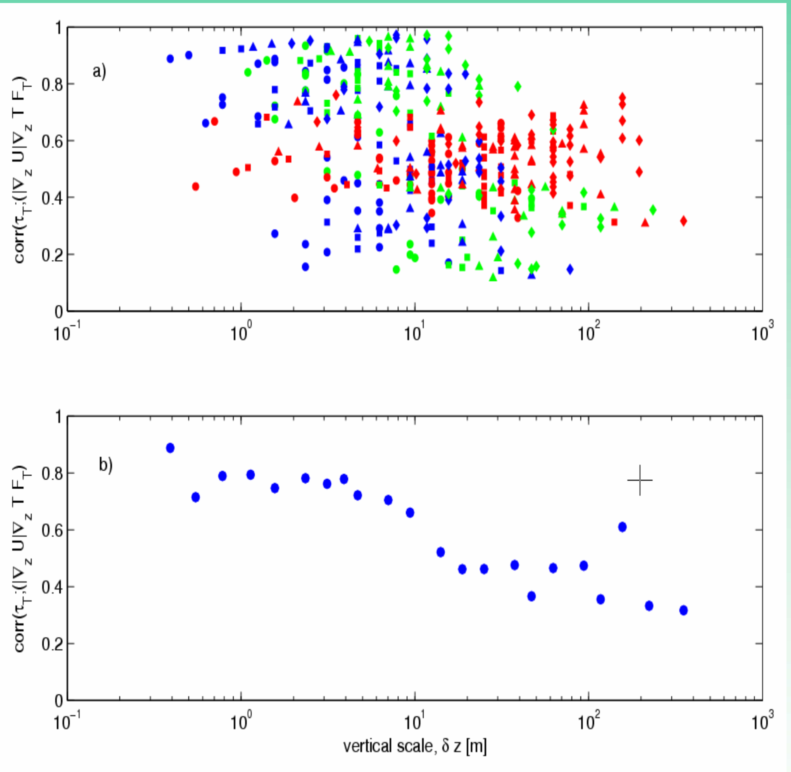
- Long-lived SBLs develop strong capping inversion at the top
- Due to large gradients across the inversion temperature/scalar fluxes at the top can exceed those at the bottom



SBL: Capping Inversion

Applications to Large-Scale Modelling





Flux-Gradient relationship is not applicable at scales, which LSMs could afford to run

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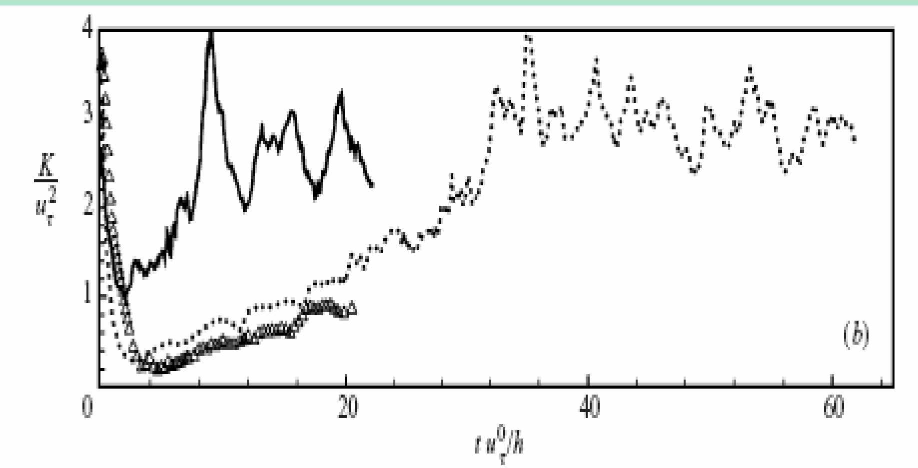
Challenges: Very Stable PBL

- **Intermittent or buoyancy-dominated SBL sporadically developing against very stable stratification?**



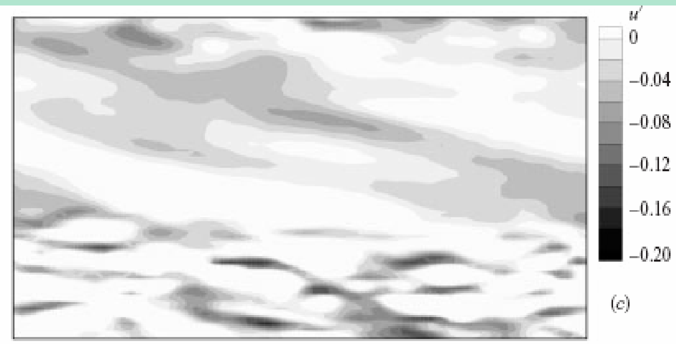


FIGURE 14. Low-speed streaks at $z^+ \simeq 12$: (a) neutral flow ($tu_\tau/h = 20$); (b) moderate stable stratification (C2) ($tu_\tau/h = 20$); (c) strong stable stratification (C5) ($tu_\tau/h = 20$). Here u' is the fluctuation with respect to the plane-averaged velocity and, thus, regions with velocity lower than the plane-averaged velocity are displayed in various shades of grey.

Evolution of averaged TKE and snapshot of the velocity fluctuations in buoyancy-dominated LES run (after Armenio and Sarkar, JFM, 2002)

Challenges: Mixing Efficiency

- **Sub-grid closure for density (temperature and heavy scalar) is far less developed than that for momentum**
- **All existing schemes are using variations of turbulent Prandtl number in closures**

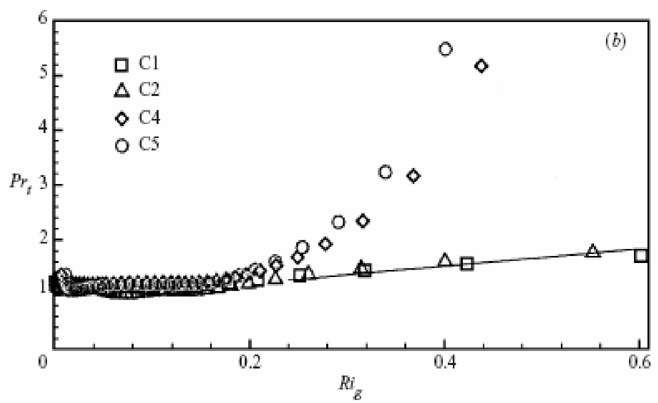
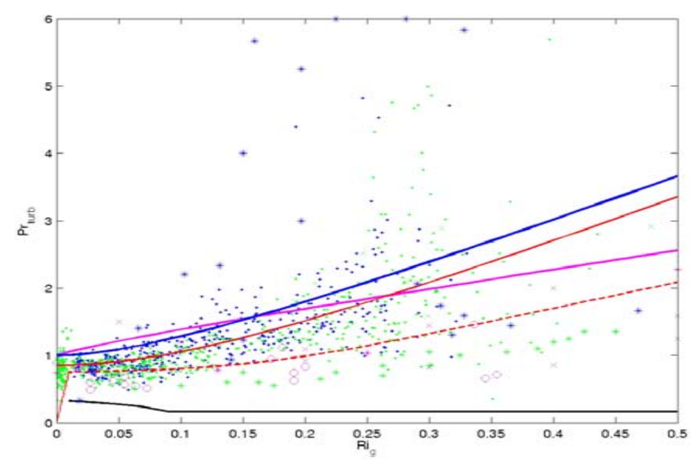


FIGURE 17. The influence of gradient Richardson number on (*a*) the ratio, B/P (a surrogate for mixing efficiency), and (*b*) the turbulent Prandtl number, Pr_t. Cases C1, C2, C4, C5 correspond to Ri_b = 0.032, 0.0685, 0.188, 0.297, respectively.



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Challenges: Add Complexity

- With a few exceptions, LES still run for idealized flows governing by prescribed forces
- Boundary conditions are still too simple to represent realistic surface properties
- Real initial conditions are not assimilated in the LES
- Microphysical processes are given through rudimentary description
- LES domains are usually far too small to study any transitional, advective and 2D turbulence effects

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Conclusions

- Modern LES is rigorous, internally consistent numerical technique to study turbulence in stratified high Re flows
- Considerable efforts are still needed to add model complexity required by environmental applications