The influence of self-correlation on the flux-gradient relationships in the Stable Boundary Layer

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Flux-gradient relationships (1)

- Applied in meteorological models to calculate vertical fluxes from mean profiles as a function of stability

- Originate from Monin-Obukhov Similarity (MOS) theory
  - homogeneous land surface
  - stationary situation (turbulence is continuous)
  - based on dimensional analysis (Buckingham-Pi theorem)

- Dimensionless height : $\frac{z}{\Lambda} = -\frac{zgk}{\theta}\frac{\overline{w'\theta'}}{u_*^3}$ note: Λ is *local* Obukhov length

- Dimensionless shear : $\phi_m = \frac{kz}{u_*}\frac{\partial U}{\partial z}$

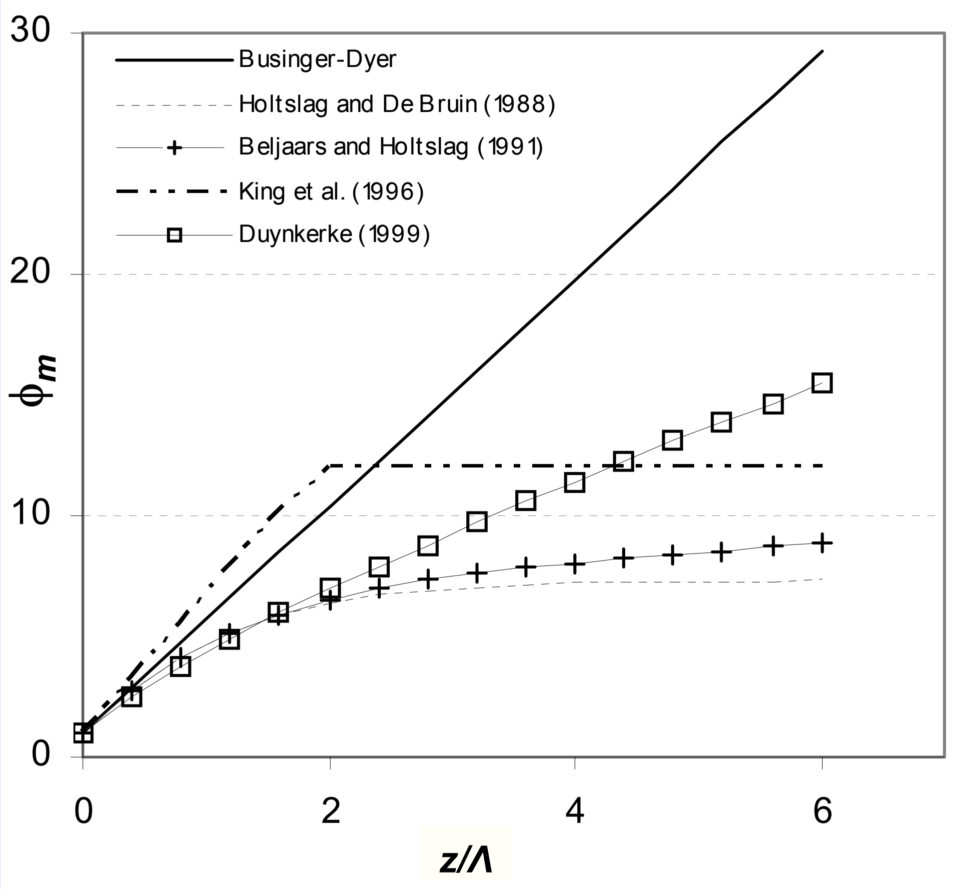
- Dimensionless lapse rate : $\phi_h = \frac{kz}{\theta_*}\frac{\partial \theta}{\partial z} = -\frac{kzu_*}{\overline{w'\theta'}}\frac{\partial \theta}{\partial z}$

Flux-gradient relationships (2)

Shape of the $\phi_x(z/\Lambda)$ -function must be found from observations.

Here uncertainty and scatter are introduced.

$$\overline{w'\varphi'} = -\frac{K_{neutral}}{\phi_\varphi}\frac{\partial \varphi}{\partial z}$$



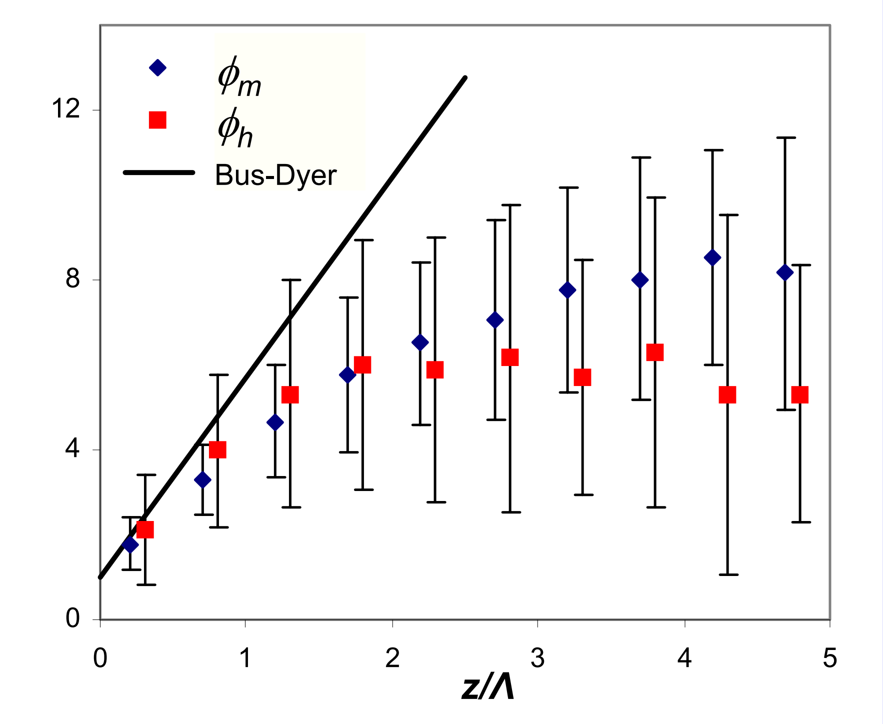
Dimensionless shear, ϕ_m, in the SBL

Dimensionless shear (ϕ_m) and lapse rate (ϕ_h)

Scatter in $\phi_h(z/\Lambda)$ is larger than in $\phi_m(z/\Lambda)$

Possible sources of scatter:

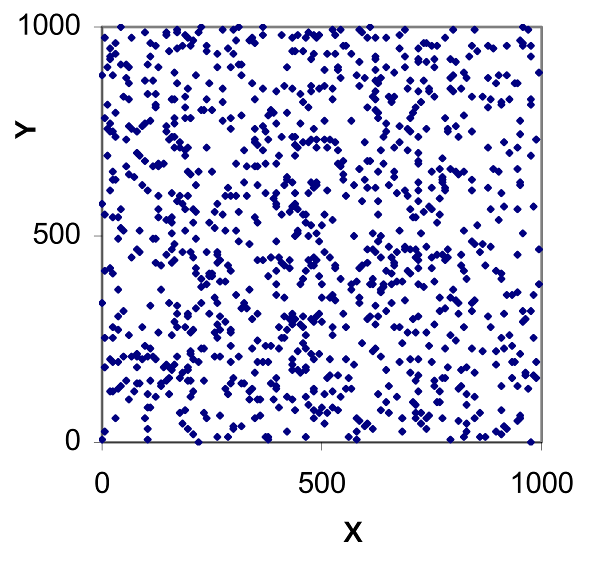
- Absence of no-slip BC for *T*
- Contaminating processes
  (gravity waves / LLJet)
- Radiative flux-divergence
- Violation assumptions MOS
- Estimation of the gradients
- Observational problems
- Self-correlation

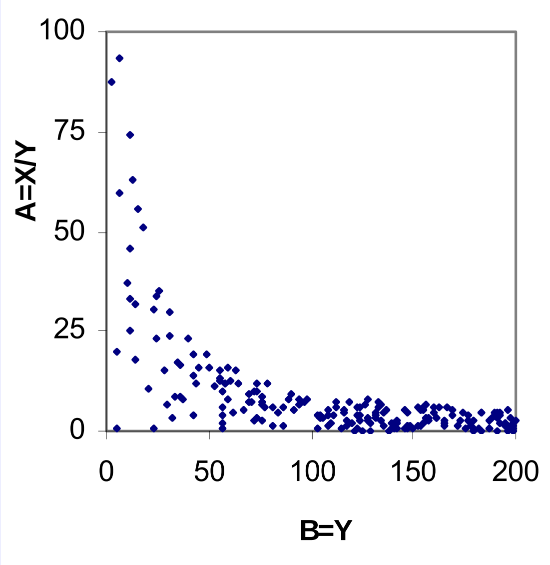


Self-correlation in theory

- Arises when one parameter A (ratio, product, …), is correlated with a second parameter B (ratio, product, …) and the 2 parameters have some common element(s).
- Plots where self-correlation is involved reflect the mathematical relation between the common variables, rather then something fundamental about atmospheric physics.
- Self-correlation often results from (inappropriate) use of dimensional analysis

- Example:



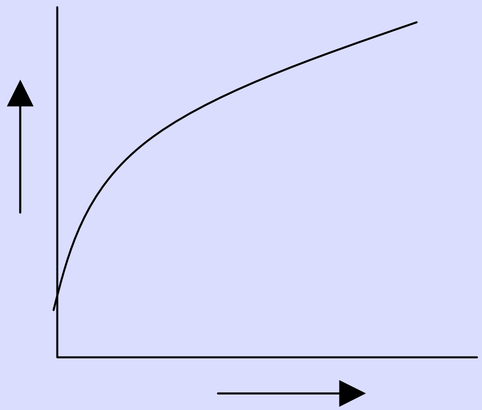


Kenney, 1982

Common elements in z/Λ, ϕ_m and ϕ_h

$$\phi_m = \frac{kz}{u_*}\frac{dU}{dz}$$

$$\phi_h = -\frac{kzu_*}{\overline{w'\theta'}}\frac{\partial\theta}{\partial z}$$



$$\frac{z}{\Lambda} = -\frac{zgk}{\theta}\frac{\overline{w'\theta'}}{u_*^3}$$

Note that $u_* = \sqrt{\frac{\tau}{\rho}}$ and $\overline{w'\theta'} = \frac{H}{\rho c_p}$

<u>Hypotheses</u>: The difference in scatter between ϕ_m and ϕ_h is caused by a different impact of self-correlation on the ϕ-functions, resulting from both τ *and H*

<u>Sensitivity analysis</u>:

- In which direction does a reference point shift, when we impose relative errors on τ and *H?*
- What is the influence of the *ratio* $\frac{\partial\tau/\tau}{\partial H/H}$ on the *type* of self-correlation?

Andreas and Hicks, 2002

Sensitivity analysis - technique

- We're interested in $\left(\frac{\partial \phi_{m,h}}{\partial \left(z/\Lambda \right)}\right)$ as a result of imposed errors on the common elements τ and *H*

- Method:

$$\partial \phi_m = \partial \left(\frac{kz}{(\tau/\rho)^{1/2}} \frac{dU}{dz} \right) = \alpha \cdot \partial \left(\frac{1}{\tau^{1/2}} \right)$$

$$\partial \phi_h = \partial \left(\frac{kz(\tau/\rho)^{1/2}}{H/(\rho c_p)} \frac{d\theta}{dz} \right) = \beta \cdot \partial \left(\frac{\tau^{1/2}}{H} \right)$$

$$\partial \left(z/\Lambda \right) = \partial \left(-\frac{zgk}{\theta} \frac{H/(\rho c_p)}{(\tau/\rho)^{3/2}} \right) = \gamma \cdot \partial \left(\frac{H}{\tau^{3/2}} \right)$$

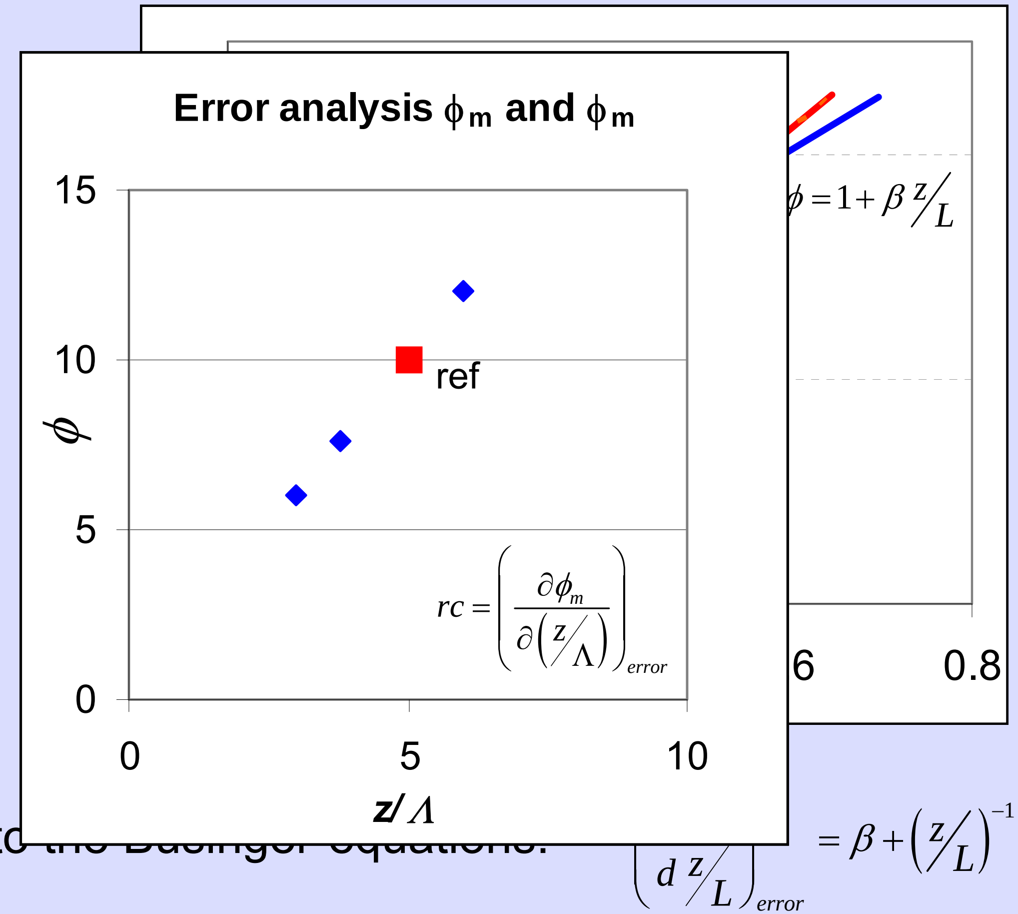
The ratio $\frac{\partial \tau/\tau}{\partial H/H}$ is substituted in the results

Does self-correlation make the difference?

Example Businger:

- Say $dH/H = d\tau/\tau$, then

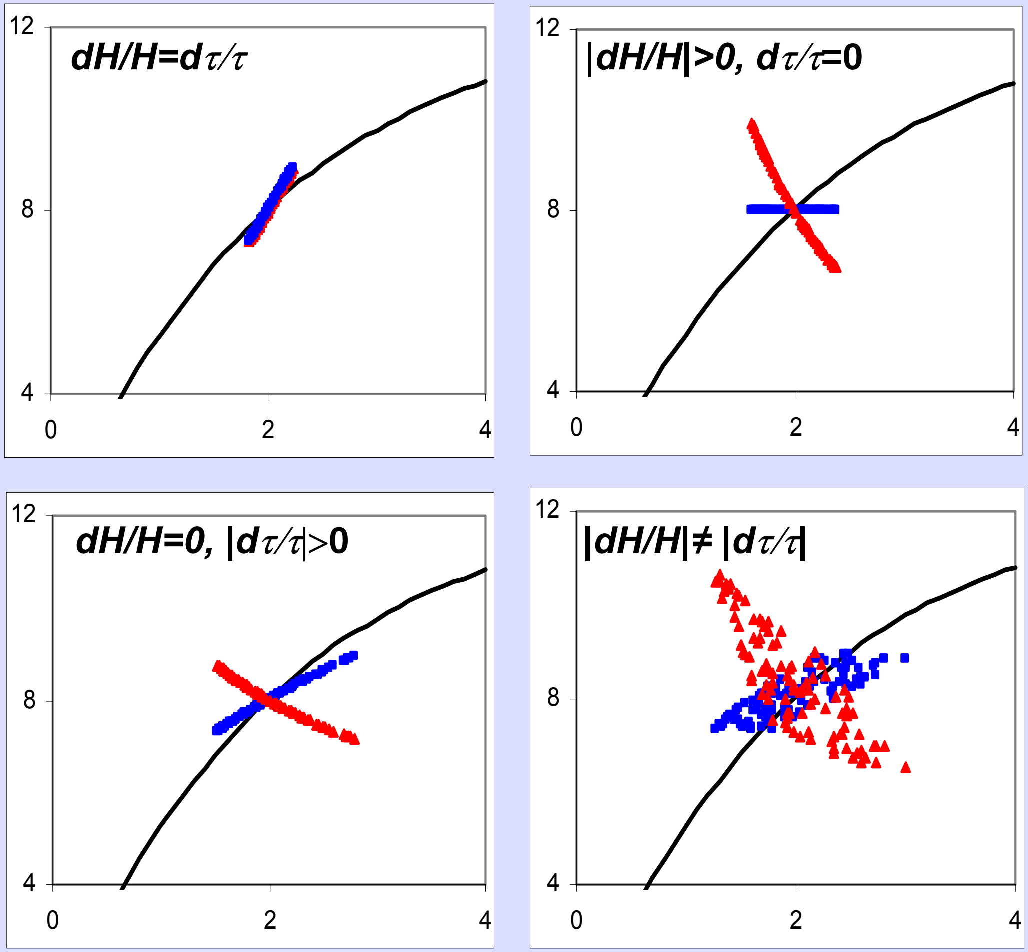
$$\left(\frac{\partial\phi_m}{\partial\left(z/\Lambda\right)}\right)_{error} = \left(\frac{\partial\phi_h}{\partial\left(z/\Lambda\right)}\right)_{error} = \frac{\phi_m}{z/\Lambda} = \frac{\phi_h}{z/\Lambda}$$



Applied to the Businger equations:

In general $d\tau/\tau \neq dH/H$!

- Simulation with random data relative to a reference point
- 4 Limit situations of $\frac{\partial \tau / \tau}{\partial H / H}$
- Max error is 20%



4 x $\phi_{m,h}$ vs (z/Λ) ϕ_m ϕ_h

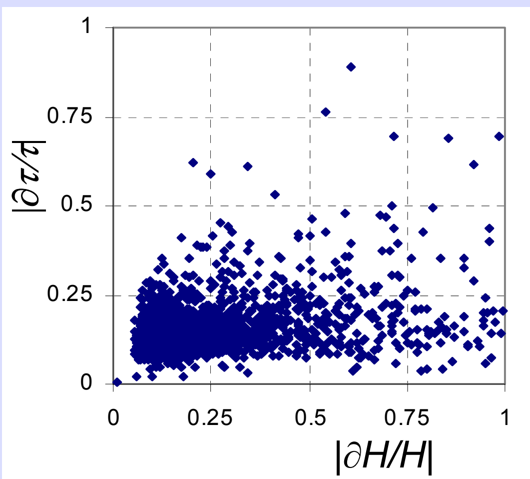
- ‘ Perpendicular’ vs ’parallel’ shift
- In the SBL $\phi_h(z/\Lambda)$ will show more scatter than $\phi_m(z/\Lambda)$ due to self-correlation
- Ratio of the errors in τ and *H* is important

Stability dependence of self-correlation

- observations -

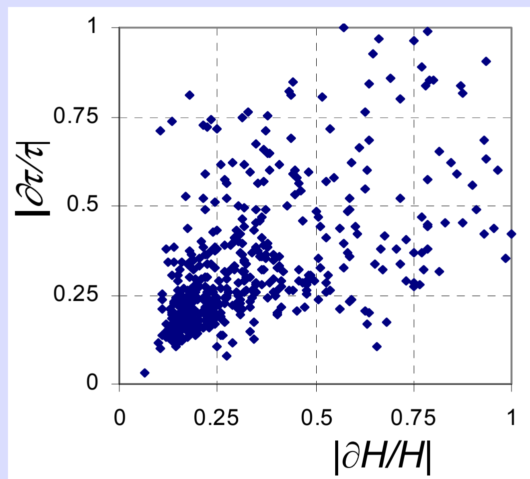
Is the ratio $\frac{\partial\tau/\tau}{\partial H/H}$ affected by stability?

Near Neutral



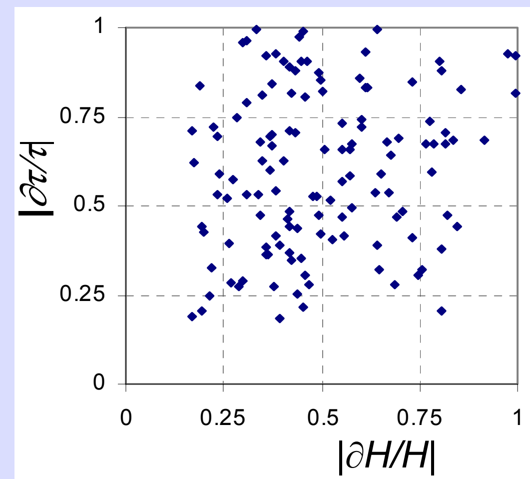
$0 < z/\Lambda < 0.1$

Weakly Stable



$0.5 < z/\Lambda < 1$

Very Stable



$z/\Lambda > 2$

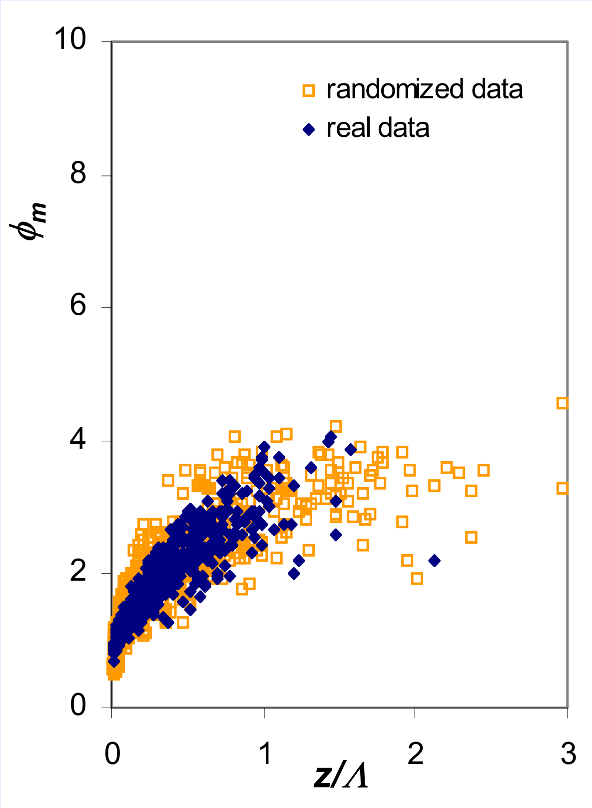
- A constant ratio of $\partial\tau/\tau$ and $\partial H/H$ cannot a priori be assumed
- So type of self-correlation depends on stability

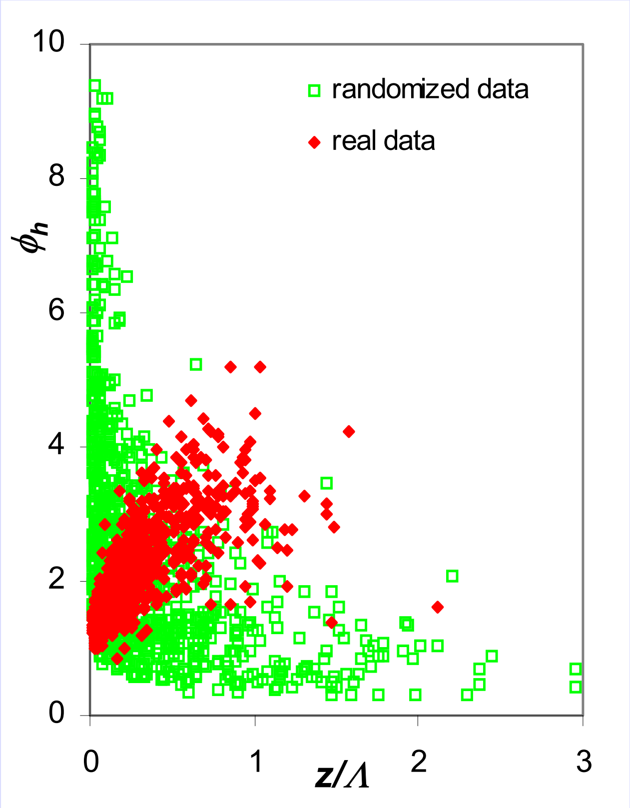
Real and randomized data

Construct a randomized dataset by using the original observations as a pool of values to draw from at random (Klipp & Mahrt, '04).

Randomize:

u_*, H, du/dz, dθ/dz





Conclusions

- Flux-gradient relationships are sensitive to self-correlation because τ and H occurs in both axes
- It is mathematically explained from sensitivity analyses that for the SBL scatter in $\phi_h(z/\Lambda)$ is larger than in $\phi_m(z/\Lambda)$.
  → Note: for the *<u>un</u>stable regime*, the effect is mostly *reversed*
- The effect of self-correlation depends highly on the ratio of $\partial\tau/\tau$ and $\partial H/H$
- The ratio of $\partial\tau/\tau$ and $\partial H/H$ varies with stability. As a consequence the type of self-correlation depends also on stability.