Impact of data assimilation on modelled snow-atmosphere interactions

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Nonlinear Dynamical Model

Model state vector $\mathbf{x}_k$ at time k

Model inputs $\mathbf{u}_k$ at time k

Model parameters $\mathbf{a}$

Update model state to time $k + 1$: $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{a})$

Model state estimates true state $\mathbf{x}_t$

Error covariance matrix: $\mathbf{B}_k = \left\langle (\mathbf{x}_k - \mathbf{x}_t)^T (\mathbf{x}_k - \mathbf{x}_t) \right\rangle$

Observations

Vector of observable quantities $\mathbf{y}$ related to models states by observation operator

$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$

Observed value $\mathbf{y}_o$ estimates true value $\mathbf{y}_t$

Error covariance matrix: $$\mathbf{R} = \left\langle (\mathbf{y}_o - \mathbf{y}_t)^T (\mathbf{y}_o - \mathbf{y}_t) \right\rangle$$

Observation operator may be nonlinear

Jacobian matrix $\mathbf{H} = \dfrac{\partial \mathbf{h}}{\partial \mathbf{x}}$ gives linearization $\mathbf{h}(\mathbf{x} + \boldsymbol{\delta}\mathbf{x}) \approx \mathbf{h}(\mathbf{x}) + \mathbf{H}\,\boldsymbol{\delta}\mathbf{x}$

Sequential Data Assimilation

Given observations $\mathbf{y}_o$ and background state $\mathbf{x}_b$, what is the "best" estimate $\mathbf{x}_a$ of the true state at the observation time?

Choose $\mathbf{x}_a$ to minimize the cost function

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}_k^{-1}(\mathbf{x}-\mathbf{x}_b) + \frac{1}{2}[\mathbf{h}(\mathbf{x})-\mathbf{y}_o]^T \mathbf{R}_k^{-1}[\mathbf{h}(\mathbf{x})-\mathbf{y}_o]$$

"Variational" methods minimize J iteratively

Direct methods solve $\left.\frac{\partial J}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_a} = 0$

$$\frac{\partial}{\partial \mathbf{x}}(\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) = 2\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)$$

$$\frac{\partial}{\partial \mathbf{x}}[\mathbf{y}_o-\mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1}[\mathbf{y}_o-\mathbf{h}(\mathbf{x})] = 2\mathbf{H}^T \mathbf{R}^{-1}[\mathbf{y}_o-\mathbf{h}(\mathbf{x})]$$

Sequential Data Assimilation

$$\left.\frac{\partial J}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_a} = \mathbf{B}^{-1}(\mathbf{x}_b - \mathbf{x}_a) + \mathbf{H}^T \mathbf{R}^{-1}[\mathbf{y}_o - \mathbf{h}(\mathbf{x}_a)] = 0$$

$$\mathbf{h}(\mathbf{x}_a) \approx \mathbf{h}(\mathbf{x}_b) + \mathbf{H}(\mathbf{x}_a - \mathbf{x}_b)$$

Analysis equation:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}[\mathbf{y}_o - \mathbf{h}(\mathbf{x}_b)]$$

Optimal interpolation: fixed state error covariance for all k

Extended Kalman Filter (EKF):

Prognostic equation $\mathbf{B}_{k+1} = \mathbf{F}\mathbf{B}_k\mathbf{F}^T + \mathbf{Q}_k$

with tangent linear model $\mathbf{F} = \dfrac{\partial \mathbf{f}}{\partial \mathbf{x}}$

Ensemble Kalman Filter (EnKF)

Ensemble of forecasts: $\mathbf{x}_{k+1}^{i} = \mathbf{f}(\mathbf{x}_{k}^{i}) + \mathbf{w}_{i} \qquad i = 1,\ldots,N$

Ensemble mean: $\bar{\mathbf{x}}_{k} = \frac{1}{N}\sum_{i}\mathbf{x}_{k}^{i}$

Approximate background covariance with ensemble covariance

$$\mathbf{B}_{k} = \left\langle (\mathbf{x}_{k} - \bar{\mathbf{x}})^{T} (\mathbf{x}_{k} - \bar{\mathbf{x}}) \right\rangle$$

Ensemble of perturbed observations:

$$\mathbf{y}_{o}^{i} = \mathbf{y}_{o} + \mathbf{v}_{k}^{i} \qquad \langle \mathbf{v} \rangle = 0 \qquad \left\langle \mathbf{v}^{T}\mathbf{v} \right\rangle = \mathbf{R}$$

Snow Surface Mass and Energy Balance Model

Mass balance $\frac{dS}{dt} = P_S - E_S - M$

Energy balance $(1-\alpha)SW_{\downarrow} + LW_{\downarrow} - \sigma T_S^4 = H + L_s E_S + L_f M$

Solve for T_S with M=0 if $T_S < T_{\mathrm{melt}}$, and solve for M if $T_S = T_{\mathrm{melt}}$

Albedo $\frac{d\alpha}{dt} = -\tau^{-1}\alpha$ if $T_S = T_{\mathrm{melt}}$

Two state variables: albedo α, snow water equivalent mass S

Three parameters:

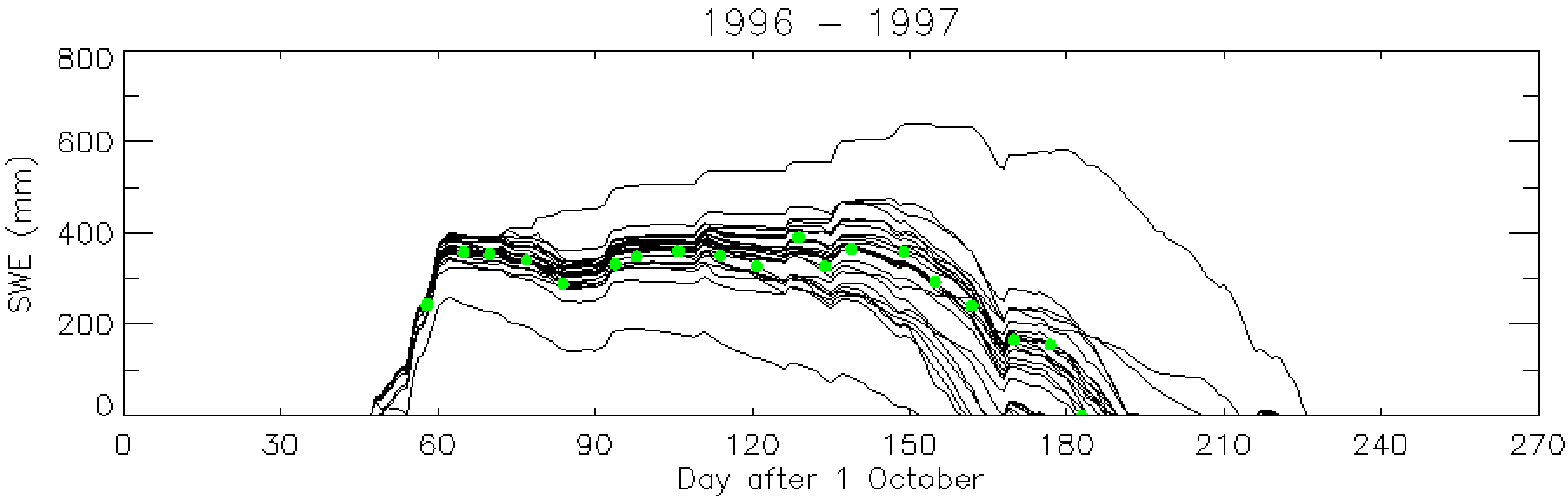
$\alpha_{fs} = 0.84$ fresh snow albedo

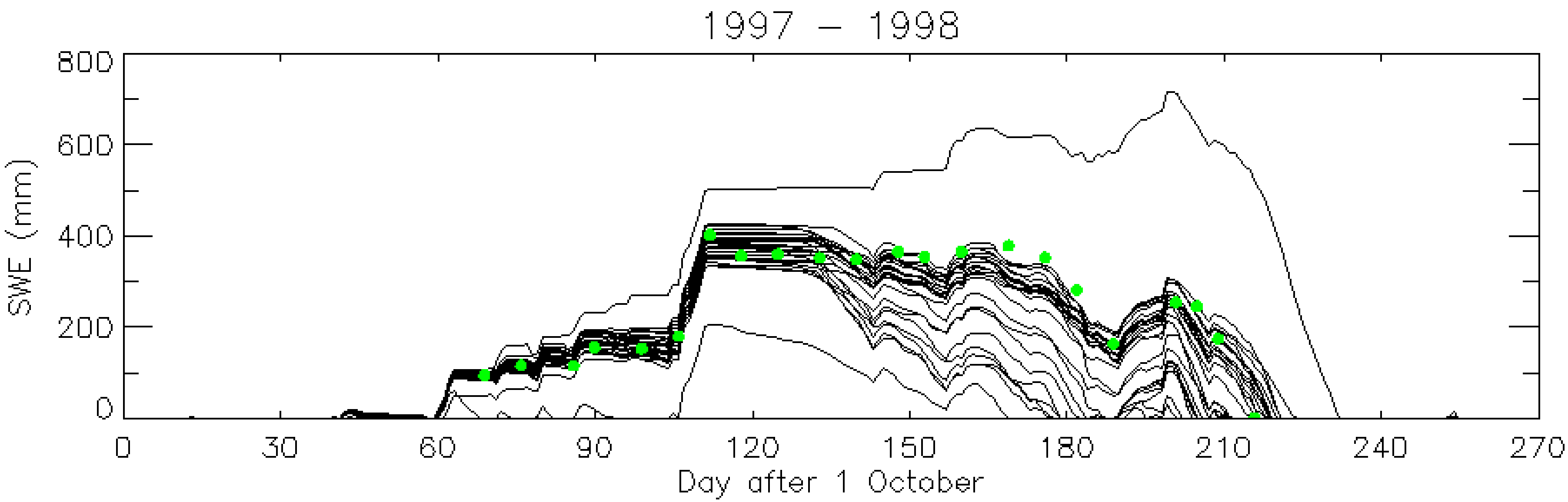
$\tau = 100$ h albedo decay time constant

$z_0 = 10^{-3}$ m surface roughness length

26 SnowMIP Models at Col de Porte

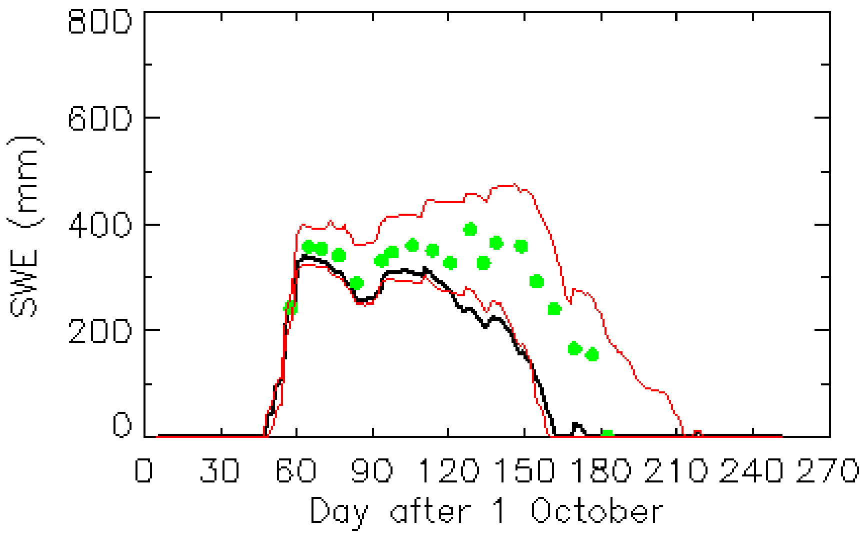
Data from Météo-France

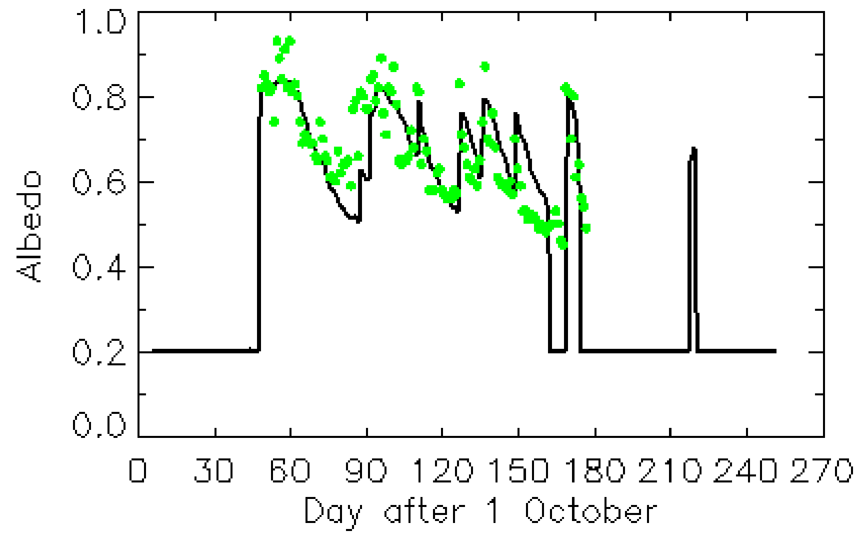




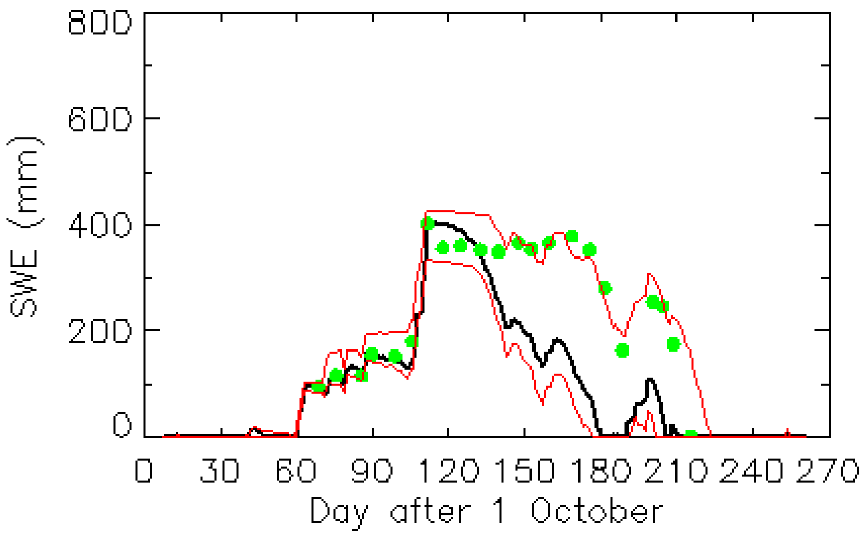
Snow Surface Mass and Energy Balance Model

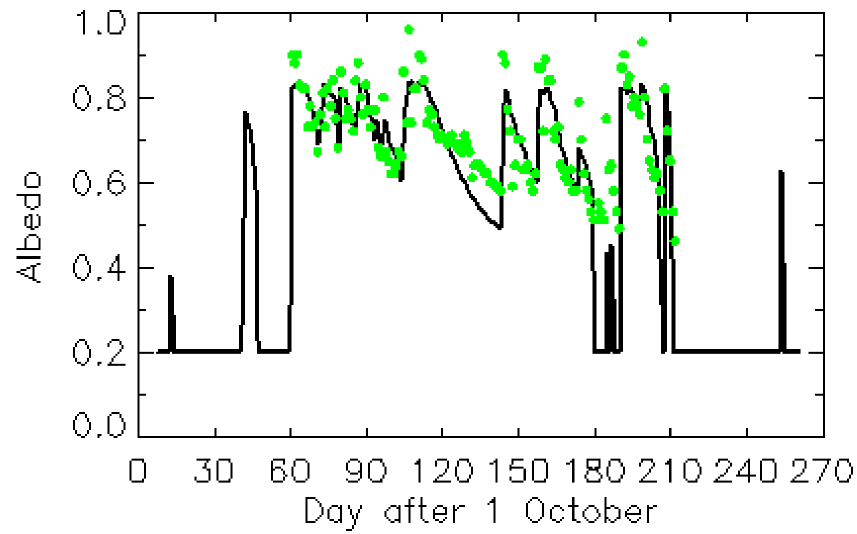
1996 – 1997





1997 – 1998





Assimilating SWE Observations

State vector $\mathbf{x} = \begin{pmatrix} \alpha \\ S \end{pmatrix}$ $\quad \mathbf{B} = \begin{pmatrix} b_{\alpha\alpha} & b_{S\alpha} \\ b_{S\alpha} & b_{SS} \end{pmatrix}$

SWE observation operator $\mathbf{H} = (0 \quad 1)$

with error variance r_S

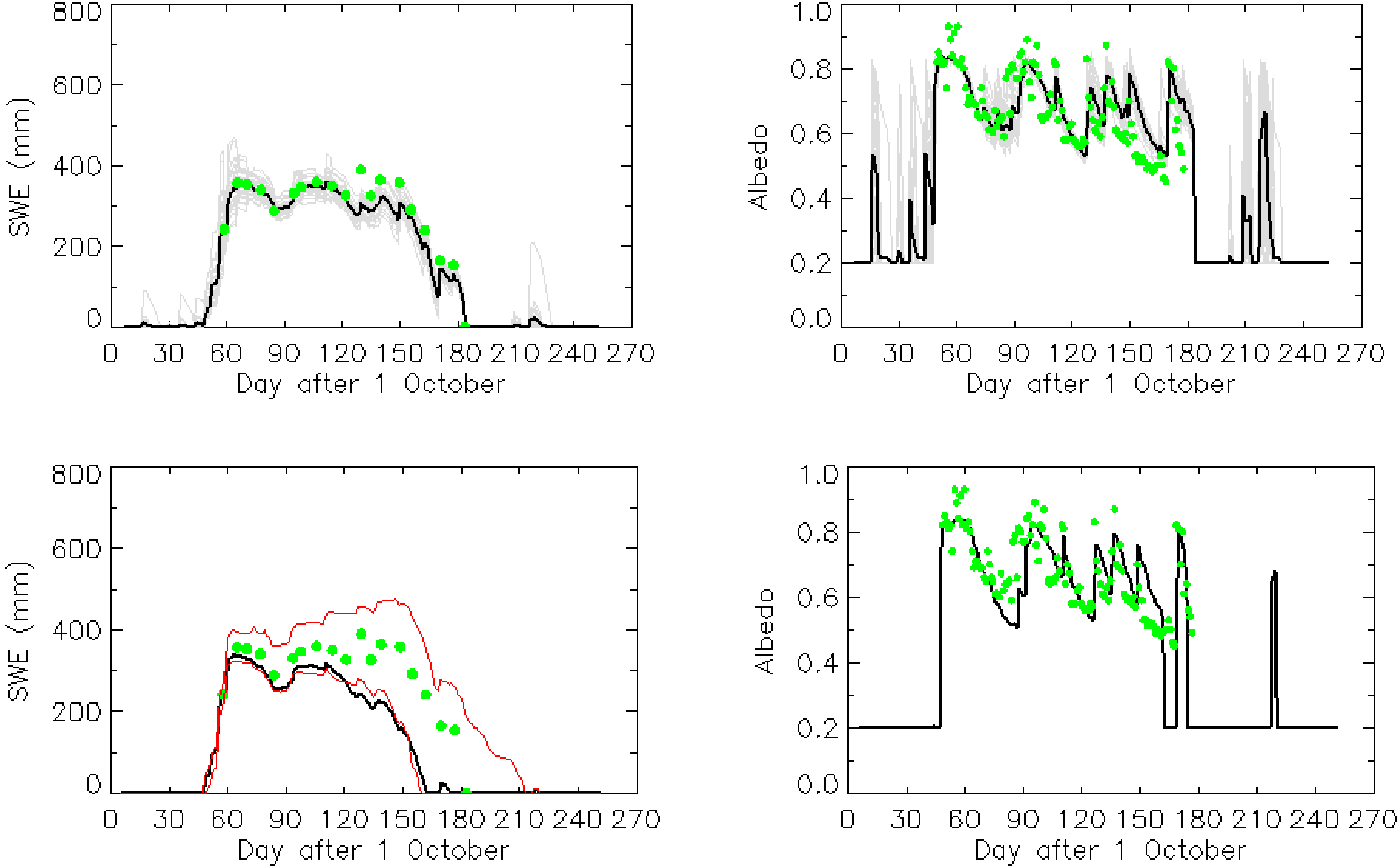
$$\mathbf{BH}^T = \begin{pmatrix} b_{S\alpha} \\ b_{SS} \end{pmatrix} \qquad \mathbf{HBH}^T + \mathbf{R} = b_{SS} + r_S$$

Analysis equations $S_a = S_b + \dfrac{b_{SS}}{b_{SS} + r_S}(S_o - S_b)$

$$\alpha_a = \alpha_b + \frac{b_{S\alpha}}{b_{SS} + r_S}(S_o - S_b)$$

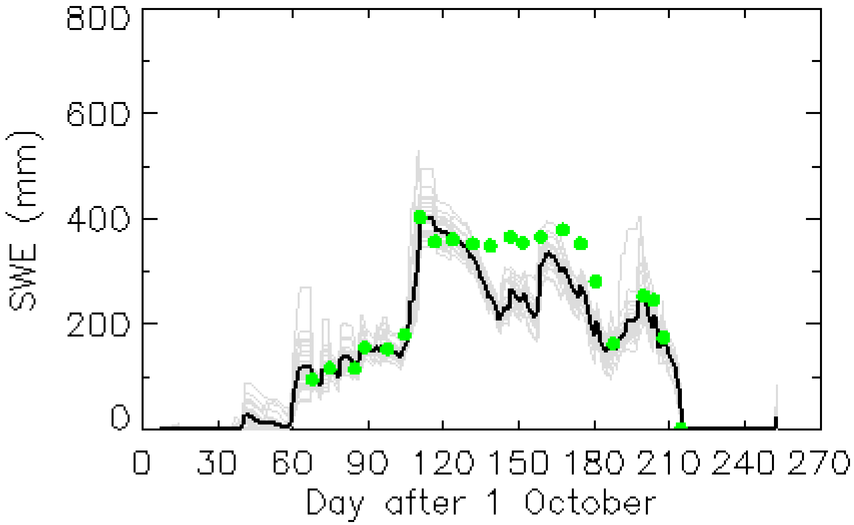
SWE Data Assimilation for Col de Porte

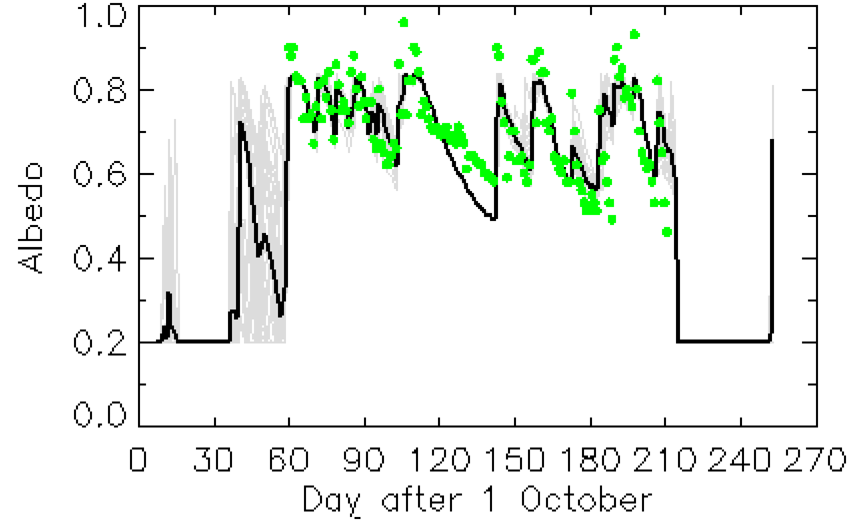
1996 – 1997

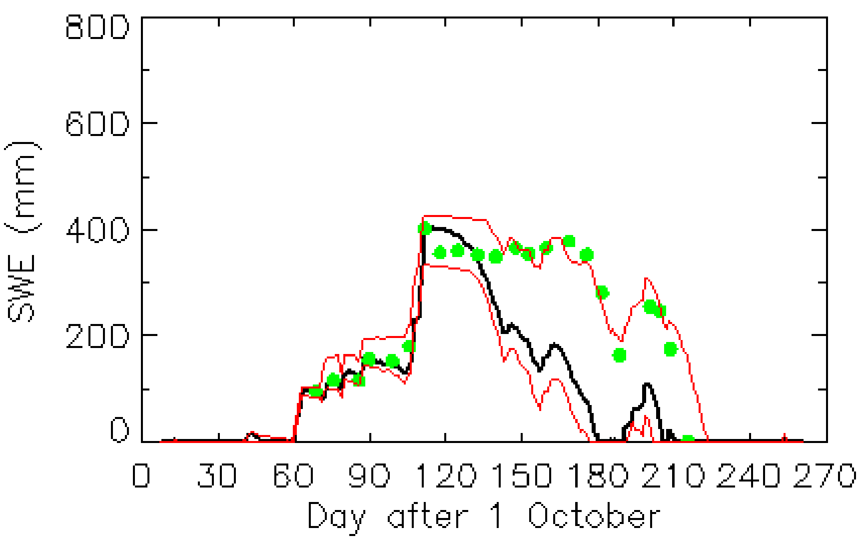


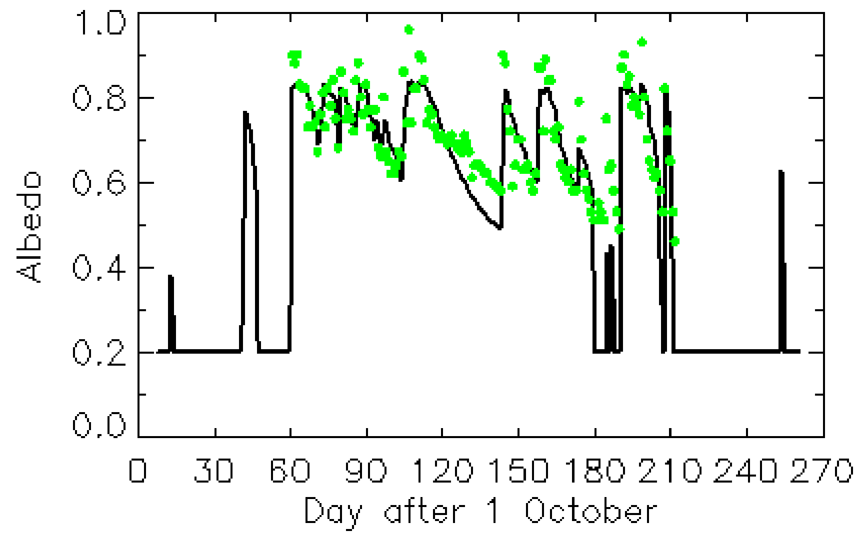
SWE Data Assimilation for Col de Porte

1997 – 1998









Assimilation of NoSREx Data

“Observations”

SnowScat SWE retrievals

NWP driving data

“Truth”

In situ SWE measurements

In situ meteorological data

Synthetic data experiments

“Truth” + “observation” error statistics