Atmospheric Planetary Boundary Layers (ABLs / PBLs) in stable, neural and unstable stratification: scaling, data, analytical models and surface-flux algorithms

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Part 1
Revised theory and improved parameterization of the Stably Stratified Atmospheric Boundary Layer (SBL) in climate, NWP, AQ, and wind energy models

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References

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Zilitinkevich S. S., and Esau I. N., 2005: Resistance and heat/mass transfer laws for neutral and stable planetary boundary layers: old theory advanced and re-evaluated. *Quart. J. Roy. Met. Soc*. **131**, 1863-1892.

Zilitinkevich, S., Esau, I. and Baklanov, A., 2007: Further comments on the equilibrium height of neutral and stable planetary boundary layers. *Quart. J. Roy. Met. Soc.* **133**, 265-271.

Zilitinkevich, S. S., and Esau, I. N., 2007: Similarity theory and calculation of turbulent fluxes at the surface for stably stratified atmospheric boundary layers. *Boundary-Layer Meteorol.* DOI : 10.1007/s10546-007-9187-4.

Motivation

NWP, climate and air pollution modeling require

- Surface fluxes (lower boundary conditions in all models)
  - surface layer
  - roughness layer
- SBL height
  - in advanced surface-flux scheme (especially for shallow SBLs)
  - in air-pollution modeling
- Turbulent fluxes in any stratification (to close Reynolds equations in all models)
  - critical Richardson number?
  - turbulent Prandtl number
  - where to go?
- Depth/strength of and fluxes within capping inversions (especially in Polar regions)

State of the art

Surface fluxes

Surface layer concept: τ, F_θ, F_q = constant

Local M-O (1954) scaling: $L = -u_*^3 / F_{bs}$

Roughness length $z_{0u} \sim h_0$: no stability effect

SBL height

Local (RM,1935)⇔Z(1974): $N|_{\text{free-flow}}$ neglected

Closure

Down-gradient, Kolmogorov (1941): $K_M, K_H, K_D \sim E_K^{1/2} l_T$

TKE and ,e.g., ε -budgets: TPE disregarded

Improvements: to avoid Ri_{cr} and correct Pr_{turb}

Capping inversions low interest / no parameterization

Data Mid latitudes → residual layers (N=0) → SBL = nocturnal BL

Basic types of the SBL

- Until recently ABLs were distinguished accounting only for $F_{bs}=F_*$:
  - neutral at $F_*=0$
  - stable at $F_*<0$

- Now more detailed classification:
  - truly neutral (TN) ABL: $F_*=0$, $N=0$
  - conventionally neutral (CN) ABL: $F_*=0$, $N>0$
  - nocturnal stable (NS) ABL: $F_*<0$, $N=0$
  - long-lived stable (LS) ABL: $F_*<0$, $N>0$

- Realistic surface flux calculation scheme should be based on a model applicable to all these types of the ABL

1.1 Mean profiles and surface fluxes

(Z and Esau, 2007)

Content

- Revision of the similarity theory for the stably stratified ABL
- Analytical approximations for the wind velocity and potential temperature profiles across the ABL
- Validation of new theory against LES and observational data
- Improved surface flux scheme for use in operational models

Turbulence in atmospheric models

- turbulence closure – to calculate vertical fluxes: $\vec{\tau}$ and F_θ through mean gradients: $d\vec{U}/dz$ and $d\Theta/dz$

- flux-profile relationships – to calculate the surface fluxes: $u_*^2 = \tau_* = \tau|_{z=0}$, $F_* = F_\theta|_{z=0}$ through wind speed $U_1 = U|_{z=z_1}$ and potential temperature $\Theta_1 = \Theta|_{z=z_1}$ at a given level z_1

- In NWP and climate models, the lowest computational level is z_1~30 m

Neutral stratification (no problem)

From logarithmic wall law:

$$\frac{dU}{dz} = \frac{\tau^{1/2}}{kz}, \quad \frac{d\Theta}{dz} = \frac{-F_\theta}{k_T \tau^{1/2} z}, \quad U = \frac{\tau^{1/2}}{k} \ln\frac{z}{z_{0u}}, \quad \Theta - \Theta_0 = \frac{-F_\theta}{k_T \tau^{1/2}} \ln\frac{z}{z_{0u}}$$

k, k_T von Karman constants; z_{0u} aerodynamic roughness length for momentum; Θ_0 aerodynamic surface potential temperature (at z_{0u}) [Θ_0-Θ_s through z_{0T}]

It follows: $\tau_1^{1/2} = kU_1(\ln z / z_{0u})^{-1}$, $F_{\theta 1} = -kk_T U_1(\Theta_1 - \Theta_0)(\ln z / z_{0u})^{-2}$

$\tau_1 = \tau_*$, $F_{\theta 1} = F_*$ when $z_1 \approx 30$ m << *h* → OK in neutral stratification

Stable stratification: current theory

(i) local scaling, (ii) log-linear Θ-profile → both questionable

- When z_1 is much above the surface layer → $\tau_1 \neq \tau_*$, $F_{\theta 1} \neq F_*$
- Monin-Obukhov (MO) theory → $L = \dfrac{\tau^{3/2}}{-\beta F_\theta}$ (neglects other scales)→

$$\frac{kz}{\tau^{1/2}}\frac{dU}{dz} = \Phi_M(\xi), \quad \frac{k_T \tau^{1/2} z}{F_\theta}\frac{d\Theta}{dz} = \Phi_H(\xi), \quad \text{where} \quad \xi = \frac{z}{L}$$

- $\Phi_M = 1 + C_{U1}\xi$, $\Phi_H = 1 + C_{\Theta 1}\xi$ from z-less stratification concept

$$U = \frac{u_*}{k}\left(\ln\frac{z}{z_{u0}} + C_{U1}\frac{z}{L_s}\right), \quad \Theta - \Theta_0 = \frac{-F_*}{k_T u_*}\left(\ln\frac{z}{z_{u0}} + C_{\Theta 1}\frac{z}{L_s}\right)$$

- Ri$\equiv \beta(d\Theta/dz)(dU/dz)^{-2}$ → Ri$_c$=$k^2 C_{\Theta 1} k_T^{-1} C_{U1}^{-2}$ (unacceptable)
- $C_{U1} \sim$2, $C_{\Theta 1}$ also ~ 2 (factually increases with $z \backslash L$)

Stable stratification: current parameterization

To avoid critical Ri modellers use **empirical, heuristic** correction functions to the neutral drag and heat/mass transfer coefficients

- Drag and heat transfer coefficients: $C_D = \tau/(U_1)^2$, $C_H = -F_{\theta s}/(U_1\Delta\Theta)$

- Neutral: C_{Dn}, C_{Hn} – from the logarithmic wall law

- To account for stratification, correction functions (dependent only of Ri):

$$f_D(\mathrm{Ri}_1) = C_D / C_{Dn} \quad \text{and} \quad f_H(\mathrm{Ri}_1) = C_H / C_{Hn}$$

$\mathrm{Ri}_1 = \beta(\Delta\Theta)z_1/(U_1)^2$ (surface-layer "Richardson number") is given parameter

Stable stratification: revised theory

Zilitinkevich and Esau (2005)→ two additional length scales besides *L*:

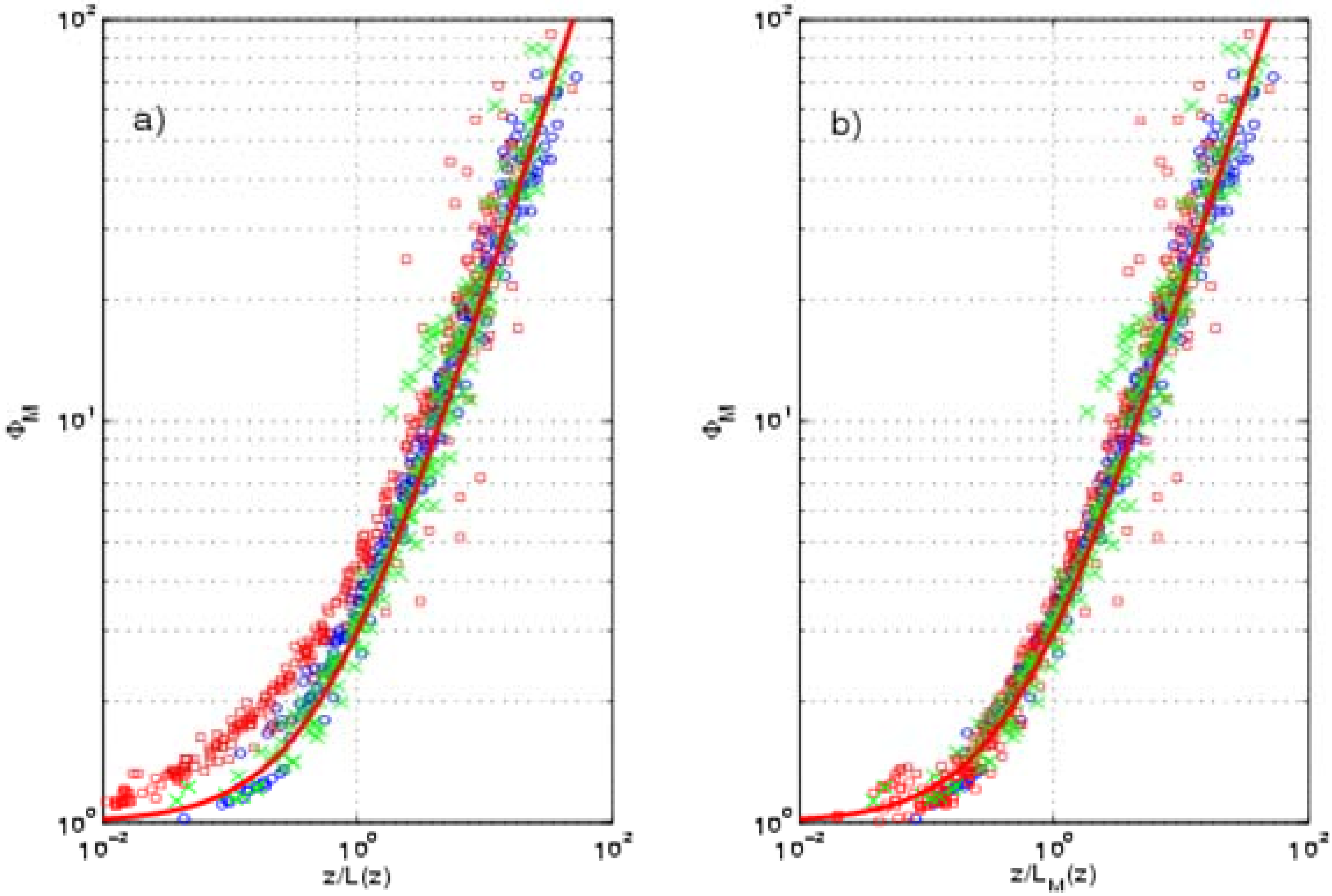
$$L_N = \frac{\tau^{1/2}}{N}$$ non-local effect of the free flow static stability

$$L_f = \frac{\tau^{1/2}}{|f|}$$ the effect of the Earth's rotation

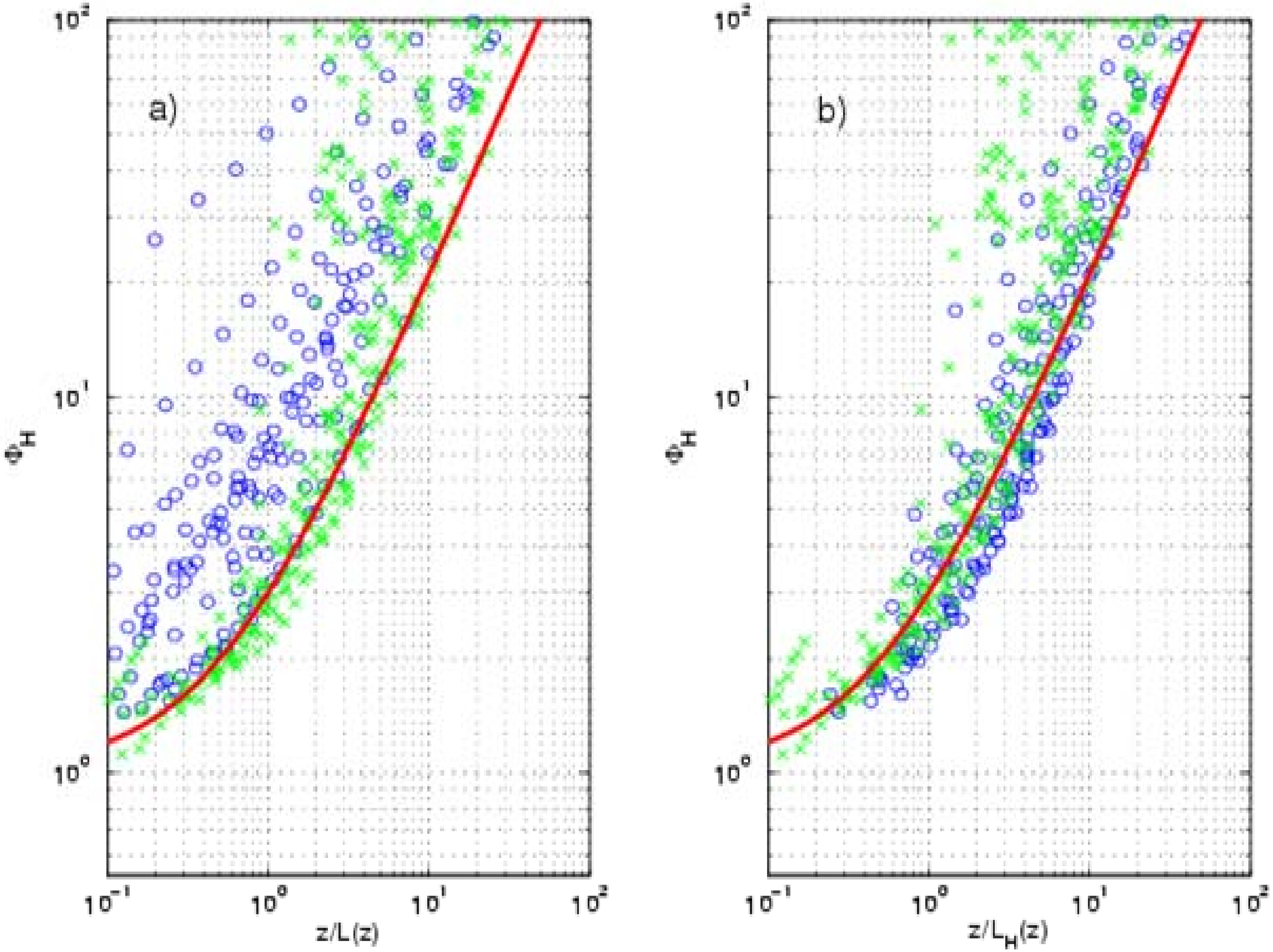
N is the Brunt-Väisälä frequency at $z>h$ (N ~10^{-2} s^{-1}), f is the Coriolis parameter

Interpolation: $$\frac{1}{L_*} = \left[\left(\frac{1}{L}\right)^2 + \left(\frac{C_N}{L_N}\right)^2 + \left(\frac{C_f}{L_f}\right)^2 \right]^{1/2}$$ where C_N=0.1 and C_f=1

$kz\tau^{-1/2}dU/dz$ vs. z/L (a), z/L_* (b) x *nocturnal*; o *long-lived*; □ *conventionally neutral*



$\Phi_H = (k_T \tau^{1/2} z/F_\theta) \mathrm{d}\Theta/dz$ vs. z/L (a), z/L_* (b) x nocturnal; o long-lived



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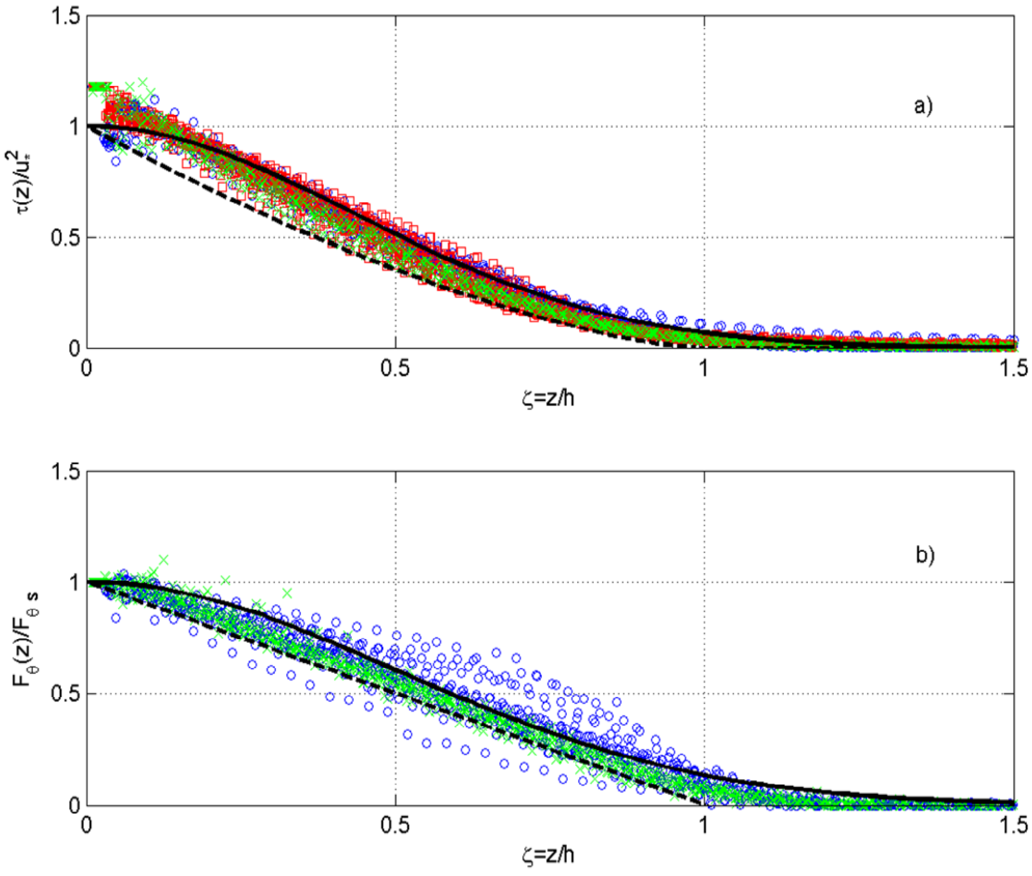
Vertical profiles of turbulent fluxes

LES turbulent fluxes: solid lines $\tau / u_*^2 = \exp(-\tfrac{8}{3} \varsigma^2)$, $F_\theta / F_{\theta s} = \exp(-2\varsigma^2)$

Approximation based on atmospheric data (e.g. Lenshow, 1988): dashed lines



New mean-gradient formulation (no critical Ri)

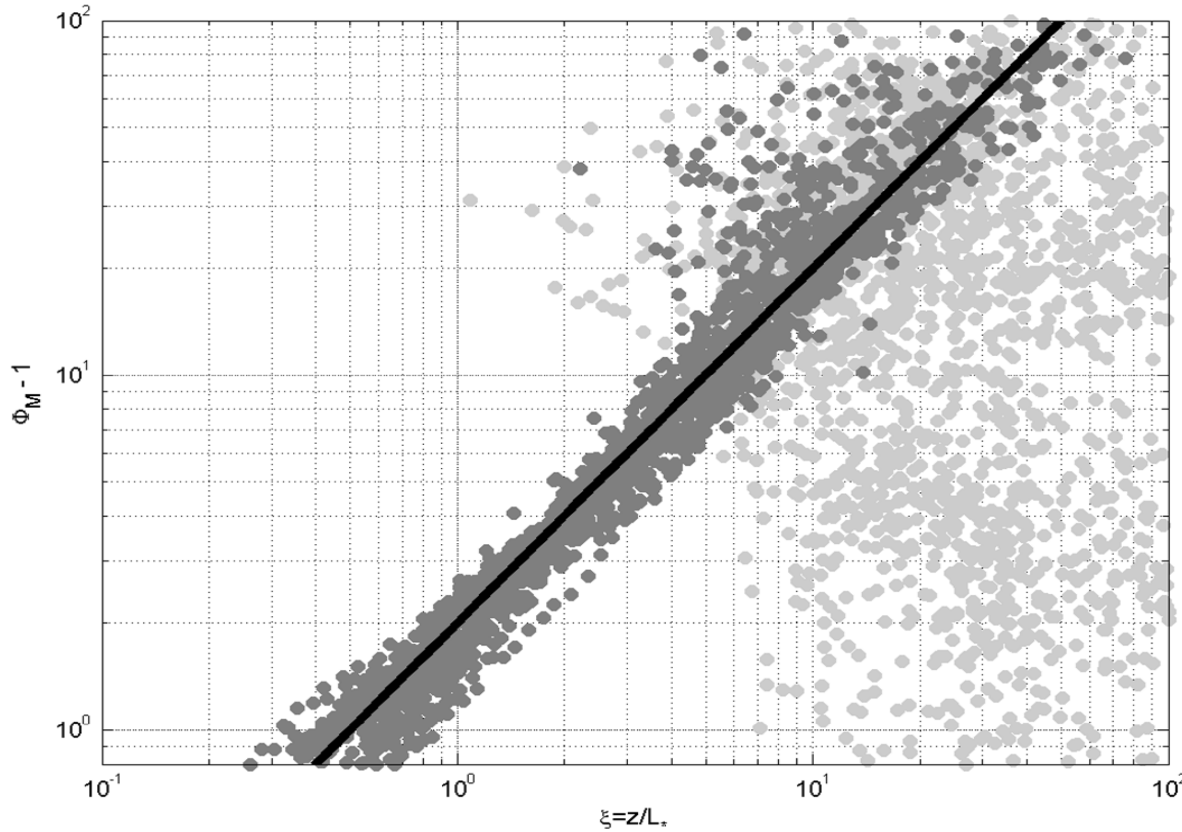
Flux Richardson number is limited: $\mathrm{Ri}_f = \dfrac{-\beta F_\theta}{\tau dU/dz} > \mathrm{Ri}_f^\infty \approx 0.2$

Hence asymptotically $\dfrac{dU}{dz} \to \dfrac{\tau^{1/2}}{\mathrm{Ri}_f^\infty L}$, and interpolating $\Phi_M = 1 + C_{U1}\xi$

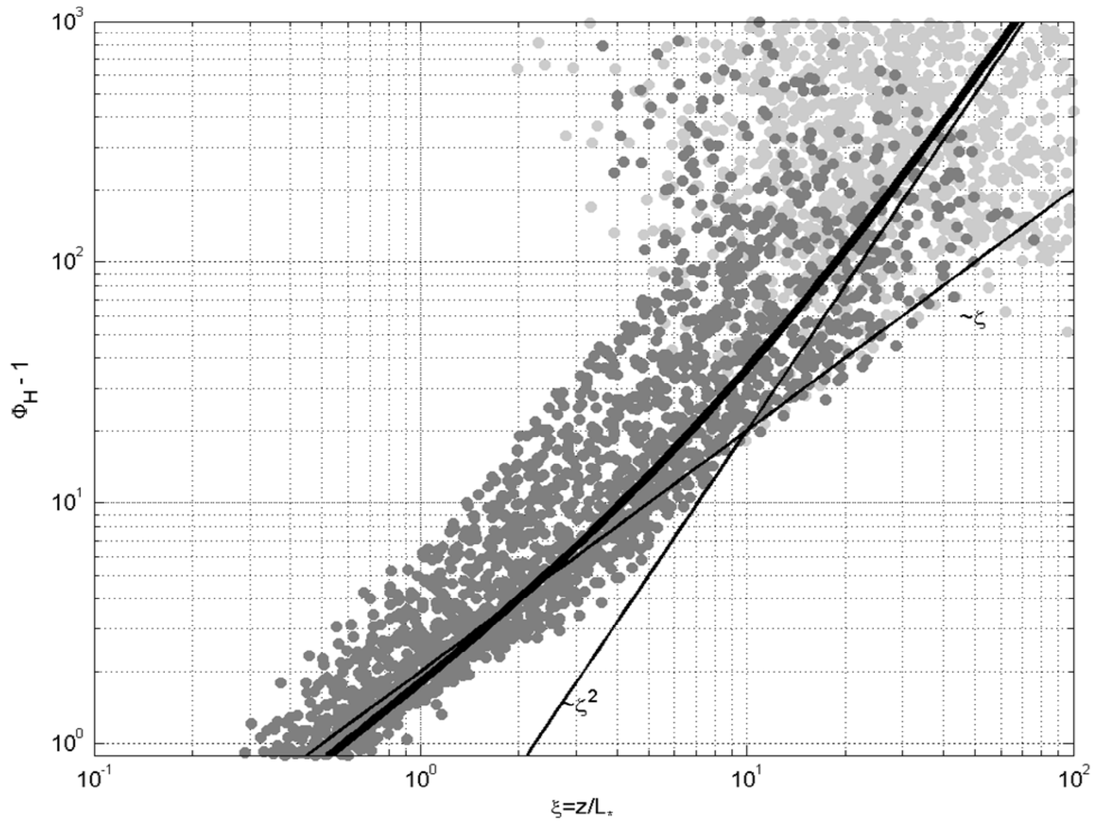
Gradient Richardson number becomes $\mathrm{Ri} \equiv \dfrac{\beta d\Theta/dz}{(dU/dz)^2} = \dfrac{k^2}{k_T}\dfrac{\xi\Phi_H(\xi)}{(1+C_{U1}\xi)^2}$

To assure no Ri-critical, ξ-dependence of Φ_H should be stronger then linear.

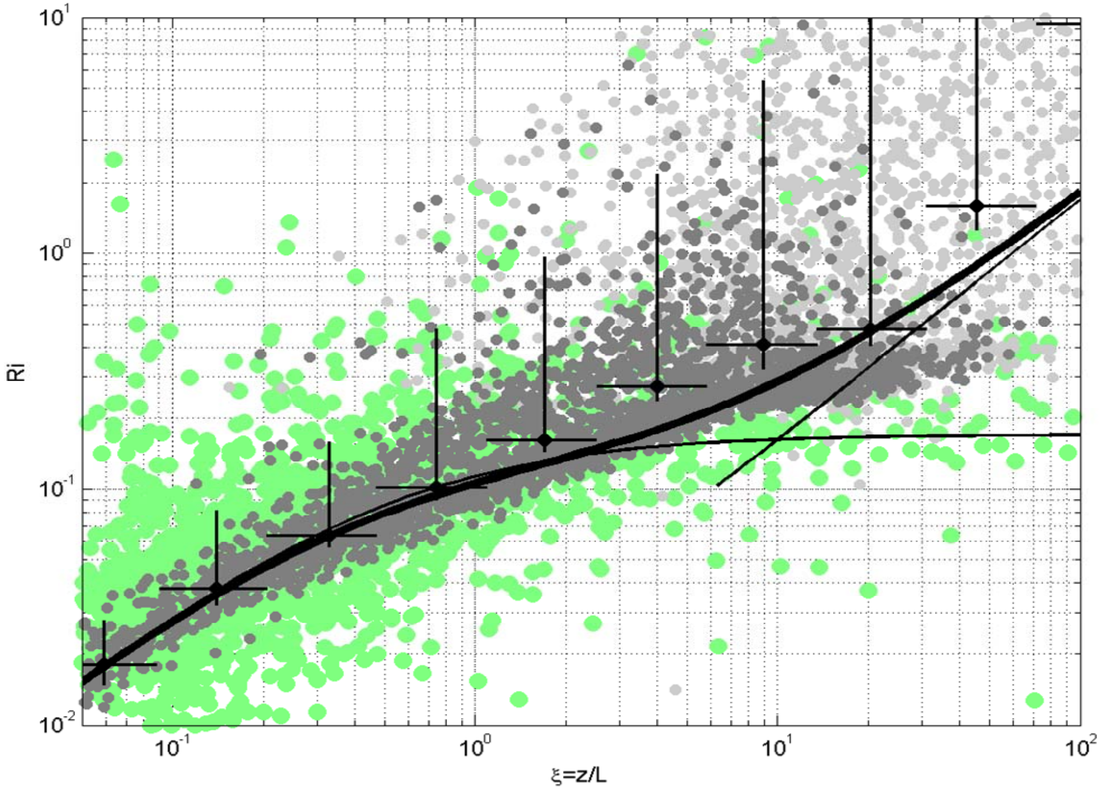
Including CN and LS ABLs: $\Phi_M = 1 + C_{U1}\dfrac{z}{L_*}$, $\Phi_H = 1 + C_{\Theta 1}\dfrac{z}{L_*}\xi + C_{\Theta 2}\left(\dfrac{z}{L_*}\right)^2$



Φ_M vs. $\xi = z / L_*$, after LES DATABASE64 (all types of SBL). Dark grey points for *z*<*h*; light grey points for *z*>*h*; the line corresponds to $C_{U1} = 2$.



Φ_H vs. $\xi = z / L_*$ (all SBLs). Bold curve is our approximation: $C_{\Theta 1} = 1.8$, $C_{\Theta 2} = 0.2$; thin lines are $\Phi_H = 0.2\xi^2$ and traditional $\Phi_H = 1+2\xi$.



Ri vs. $\xi = z / L$, after LES and field data (SHEBA - green points). Bold curve is our model with C_{U1}=2, $C_{\Theta 1}$=1.6, $C_{\Theta 2}$=0.2. Thin curve is Φ_H=1+2ξ.

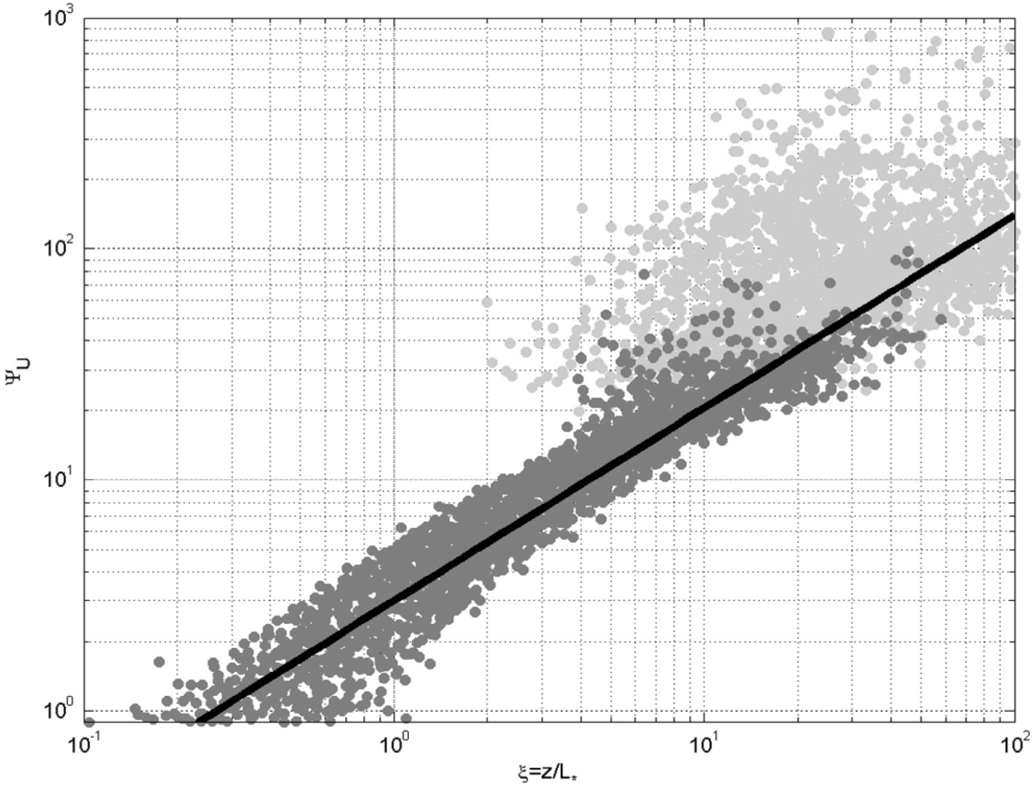
Mean profiles and flux-profile relationships

We consider wind/velocity and potential/temperature functions

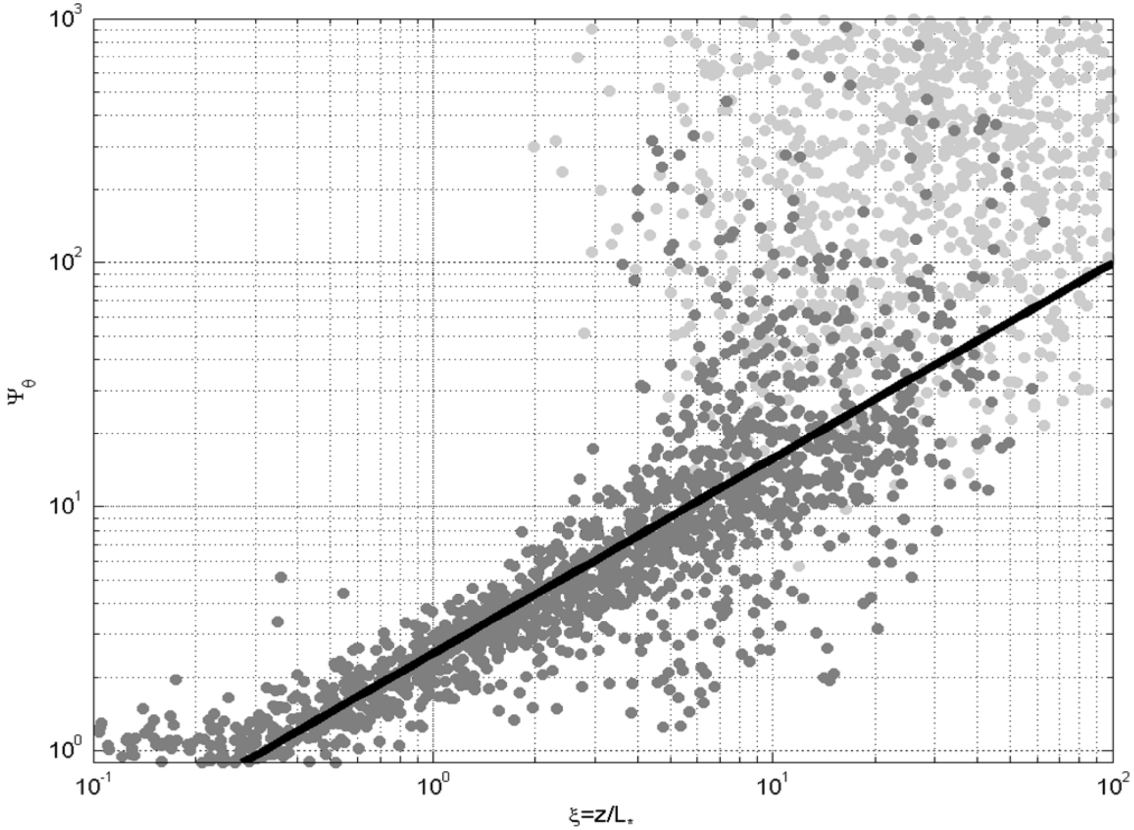
$$\Psi_U = \frac{kU(z)}{\tau^{1/2}} - \ln\frac{z}{z_{0u}} \quad \text{and} \quad \Psi_\Theta = \frac{k_T \tau^{1/2}\left[\Theta(z) - \Theta_0\right]}{-F_\theta} - \ln\frac{z}{z_{0u}}$$

Our analyses show that Ψ_U and Ψ_Θ are universal functions of $\xi = z / L_*$

$$\Psi_U = C_U \xi^{5/6}, \quad \Psi_\Theta = C_\Theta \xi^{4/5}, \text{ with } C_U\text{=3.0 and } C_\Theta\text{=2.5}$$



Wind-velocity function $\Psi_U = k\tau^{-1/2}U - \ln(z/z_{0u})$ vs. $\xi = z/L_*$, after LES DATABASE64 (**all types of SBL**). The line: $\Psi_U = C_U \xi^{5/6}$, C_U=3.0.



Pot.-temperature function $\Psi_\Theta = k\tau^{-1/2}(\Theta - \Theta_0)(-F_\theta)^{-1} - \ln(z/z_{0u})$ (**all types of SBL**). The line: $\Psi_\Theta = C_\Theta \xi^{4/5}$ with C_U=3.0 and C_Θ=2.5.

Analytical wind and temperature profiles (SBL)

$$\frac{kU}{\tau^{1/2}} = \ln\frac{z}{z_{0u}} + C_U\left(\frac{z}{L}\right)^{5/6}\left[1+\frac{(C_N N)^2+(C_f f)^2}{\tau}L^2\right]^{5/12}$$

$$\frac{k_T\tau^{1/2}(\Theta-\Theta_0)}{-F_\theta} = \ln\frac{z}{z_{0u}} + C_\Theta\left(\frac{z}{L}\right)^{4/5}\left[1+\frac{(C_N N)^2+(C_f f)^2}{\tau}L^2\right]^{2/5}$$

where C_N=0.1 and C_f=1. Given $U(z)$, $\Theta(z)$ and N, these equations allow determining τ, F_θ, and $L=\tau^{3/2}(-\beta F_\theta)^{-1}$, at the computational level z.

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Algorithm

Given τ, F_θ, surface fluxes are calculated using empirical dependencies

$$\frac{\tau}{\tau_*} = \exp\left[-\frac{8}{3}\left(\frac{z}{h}\right)^2\right], \quad \frac{F_\theta}{F_*} = \exp\left[-2\left(\frac{z}{h}\right)^2\right] \quad \text{(Figures above)}$$

The **equilibrium ABL height**, h_E, is determined diagnostically (Z. et al., 2006a):

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2 \tau_*} + \frac{N|f|}{C_{CN}^2 \tau_*} + \frac{|f\beta F_*|}{C_{NS}^2 \tau_*^2} \quad (C_R=0.6,\ C_{CN}=1.36,\ C_{NS}=0.51)$$

The **actual ABL height**, after prognostic equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U}\cdot\nabla h - w_h = K_h \nabla^2 h - C_t \frac{u_*}{h_E}(h - h_E) \quad (C_t = 1)$$

Given h, the free-flow Brunt-Väisälä frequency is $$N^4 = \frac{1}{h}\int_h^{2h}\left(\beta\frac{\partial\Theta}{\partial z}\right)^2 dz$$

Conclusions 1.1: mean profiles & surface fluxes

Background: Generalised scaling accounting for the free-flow stability,
No critical Ri (TTE closure)
Stable ABL height model

Verified against
LES DATABASE64 (4 ABL types: TN, CN, NS and LS)
Data from the field campaign SHEBA

Deliverable 1: analytical wind & temperature profiles in SBLs

Deliverable 2: surface flux scheme for use in operational models

Requested: (i) roughness lengths and (ii) ABL height

1.2 STRATIFICATION EFFECT ON THE ROUGHNESS LENGTH

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Reference (1.2)

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre, 2007: The roughness length in environmental fluid mechanics: the classical concept and the effect of stratification. Submitted to *Boundary-Layer Meteorology.*

Content (1.2)

- Roughness length and displacement height:

$$u(z) = \frac{u_*}{k}\left[\ln\frac{z - d_{0u}}{z_{0u}} + \Psi_u\left(\frac{z}{L}\right)\right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics: neutral-stability z_0 = level, at which $u(z)$ plotted vs. $\ln z$ approaches zero; $z_0 \sim 1/25$ of typical height of roughness elements, h_0

- Meteorology / oceanography: h_0 comparable with MO length $L = \frac{u_*^3}{-\beta F_{\theta s}}$

- Stability dependence of the actual roughness length, z_{0u}:

  $z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification

Surface layer and roughness length

Self similarity in the surface layer (SL) $5h_0 < z < 10^{-1}h$

Height-constant fluxes: $\tau \approx \tau|_{z=5h_0} \equiv u_*^2$

u_* and z serve as turbulent scales: $u_T \sim u_*,\ l_T \sim z$

Eddy viscosity $(k \approx 0.4)$ $K_M\ (\sim u_T l_T) = k u_* z$

Velocity gradient $\partial U / \partial z = \tau / K_M = u_* / kz$

Integration constant: $U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$

z_{0u} (redefined constant of integration) is "roughness length"

"Displacement height" d_{0u} $U = k^{-1} u_* \ln[(z - d_{u0}) / z_{u0}]$

Not applied to the roughness layer (RL) $0 < z < 5h_0$

Parameters controlling z_{0u}

Smooth surfaces: viscous layer → $z_{0u} \sim \nu / u_*$

Very rough surfaces: pressure forces depend on:

obstacle height h_0

velocity in the roughness layer $U_R \sim u_*$

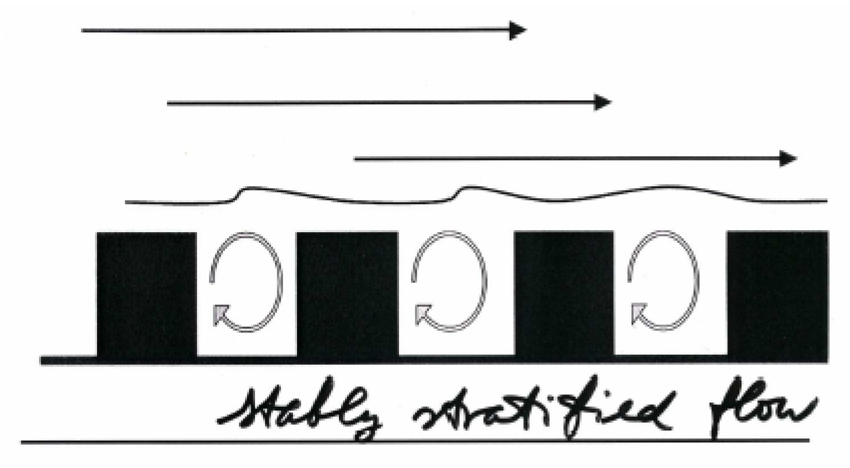
$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

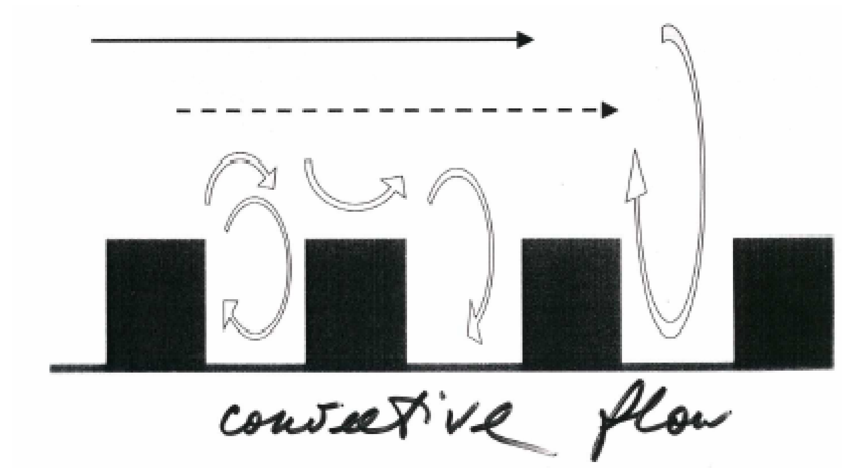
No dependence on u_*; surfaces characterised by $z_{0u} =$ constant

Generally $z_{0u} = h_0 f_0(\mathrm{Re}_0)$ where $\mathrm{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0

Stability Dependence of Roughness Length





For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L, the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL $(0 < z < 5h_0)$ with 90% of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(\mathrm{RL}) = K_M(h_0 + 0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ classical formula $z_{0u} \sim h_0$

Stable: $K_M = k u_* z (1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = k u_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} z^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$

Recommended formulation

Neutral ⇔ stable

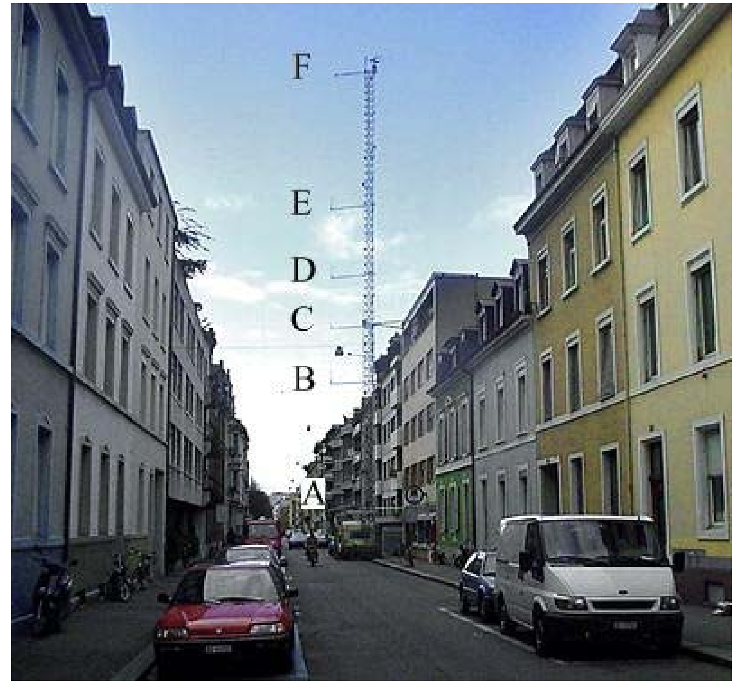
$$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

Neutral ⇔ unstable

$$\frac{z_{0u}}{z_0} = 1 + C_{US}\left(\frac{h_0}{-L}\right)^{1/3}$$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$

Experimental datasets



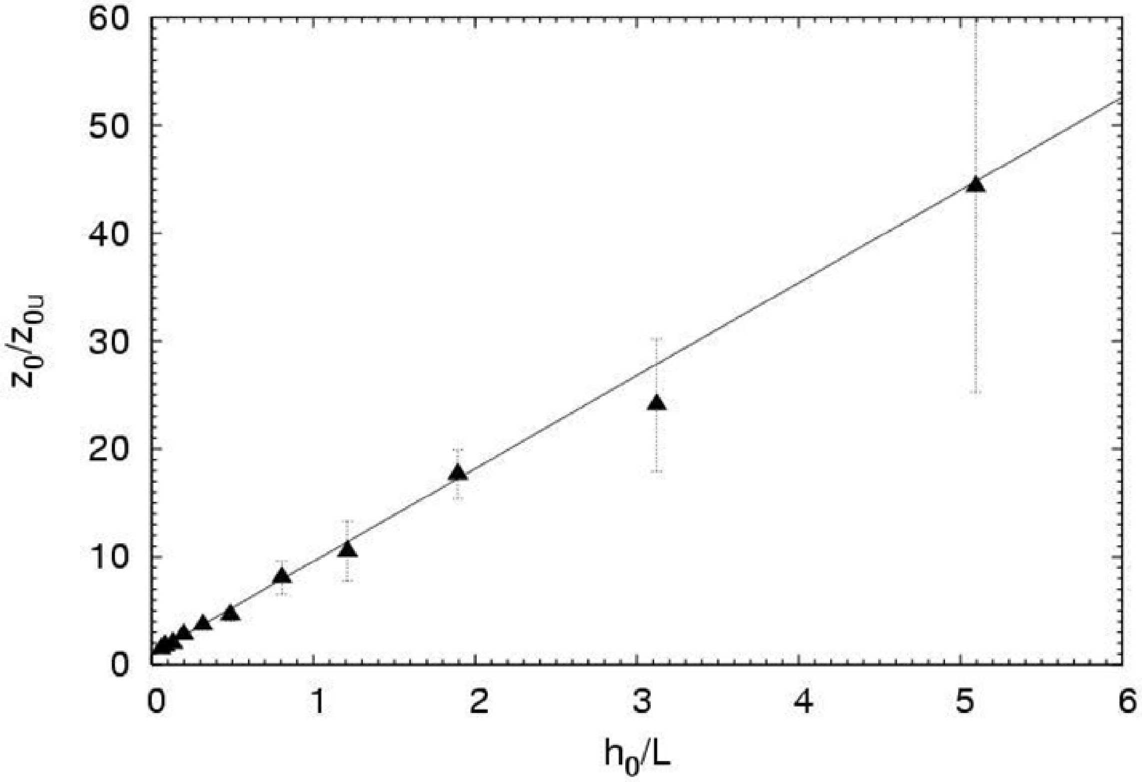
Sodankyla Meteorological Observatory, Boreal forest (FMI)

BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

h ≈ 13 m, measurement levels 23, 25, 47 m

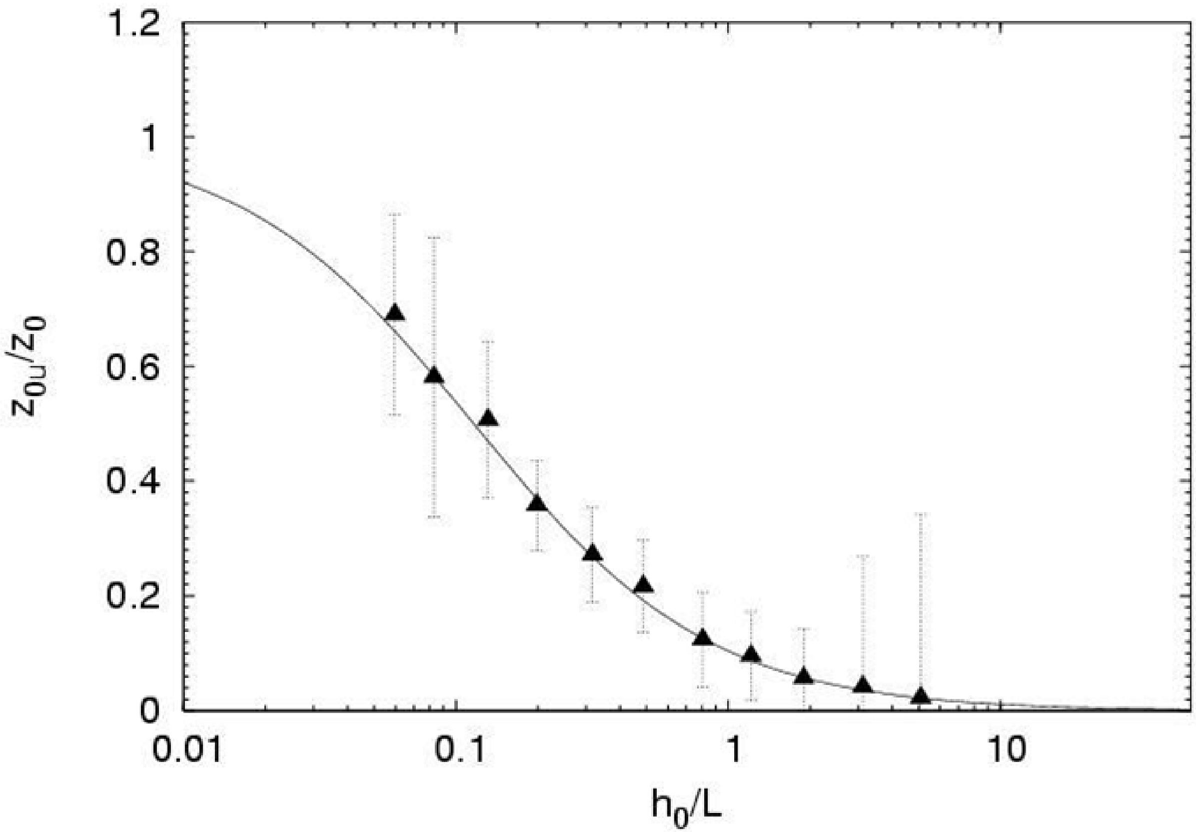
h ≈ 14.6 m, measurement levels 3.6, 11.3, 14.7, 17.9, 22.4, 31.7 m

Stable stratification



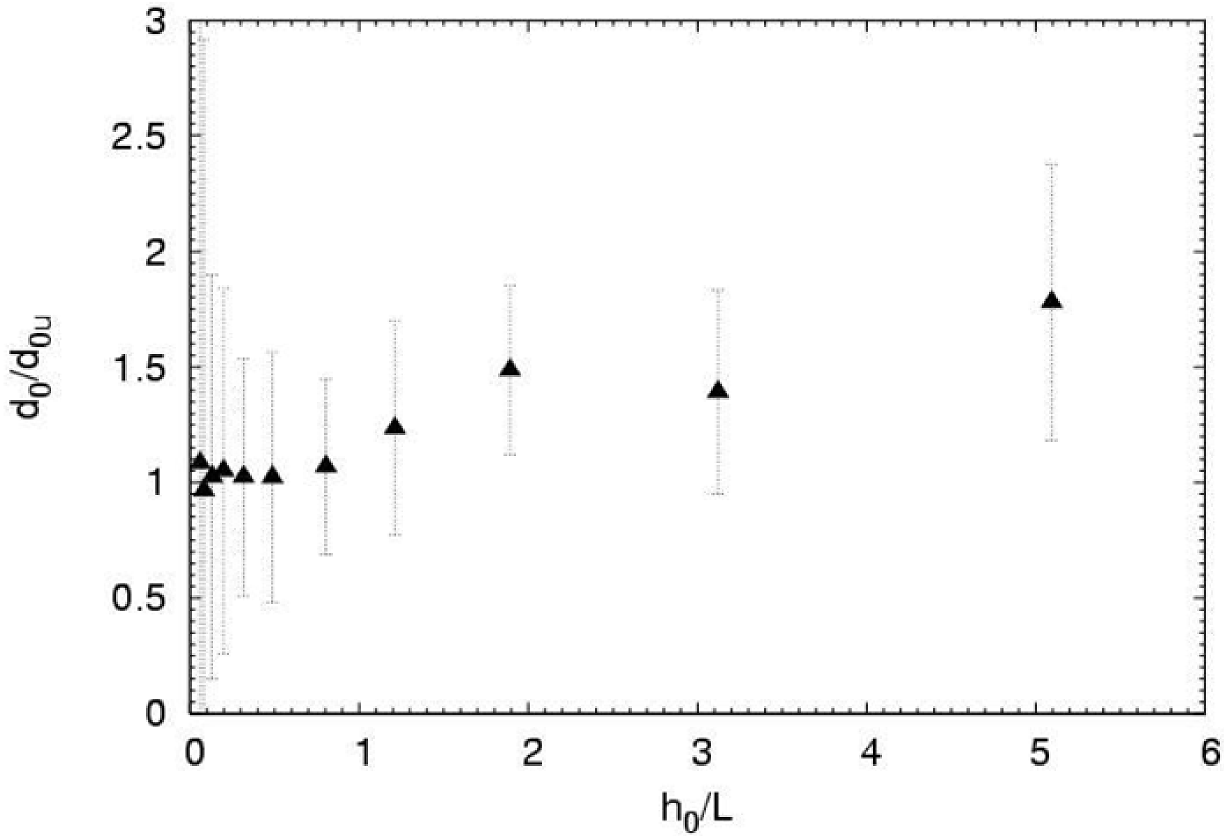
Bin-average values of z_0 / z_{0u} (neutral- over actual-roughness lengths) versus h_0/L in stable stratification for Boreal forest (h_0=13.5 m; z_0 =1.1±0.3 m). Bars are standard errors; the curve is $z_0 / z_{0u} = 1 + 8.13 h_0 / L$.

Stable stratification



Bin-average values of z_{0u}/z_0 (actual- over neutral-roughness lengths) versus h_0/L in stable stratification for boreal forest (h_0=13.5 m; z_0=1.1±0.3 m). Bars are standard errors; the curve is $z_{0u}/z_0 = (1+8.13h_0/L)^{-1}$.

Stable stratification



Bin-average values of the ratio $d_0 / d_{0,neutral}$ versus parameter h_0 / L h_c/L.

Unstable stratification

Convective eddies extend in the vertical causing $z_0 > z_{0u}$

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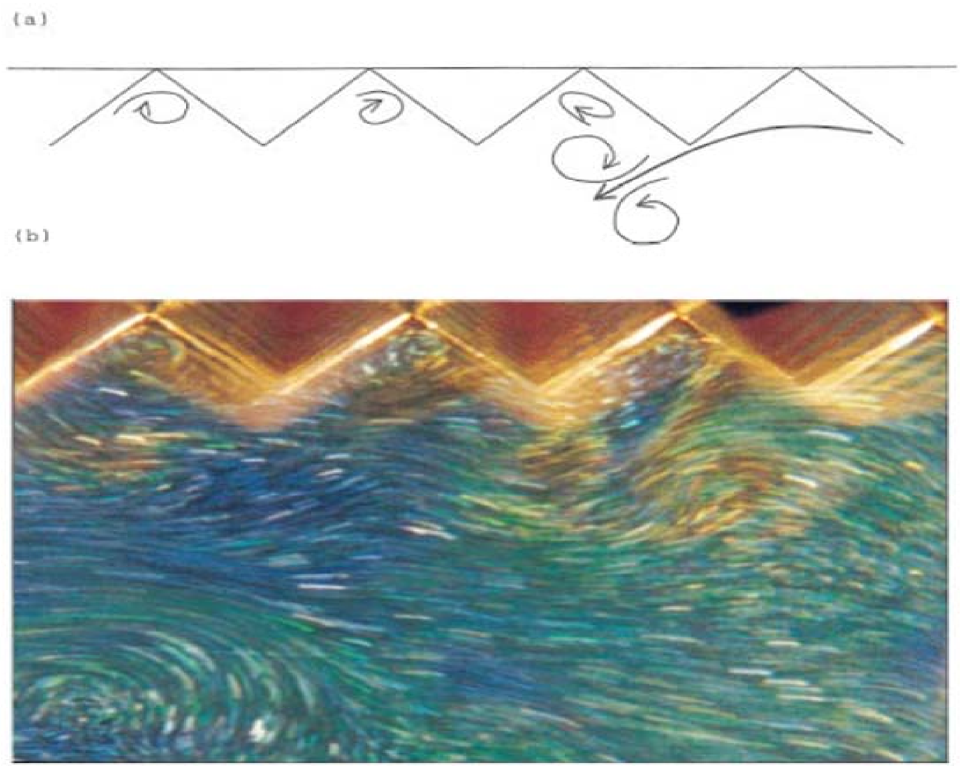
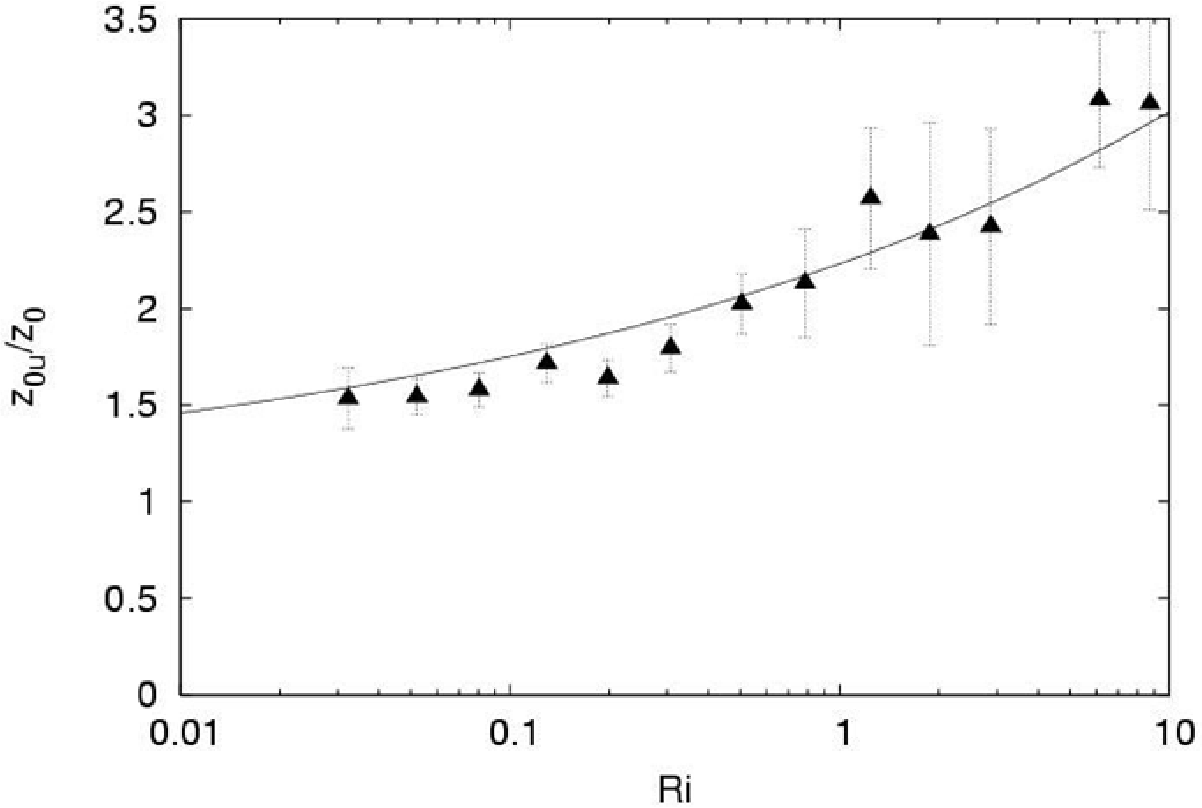
Y.-B. Du and P. Tong, Enhanced Heat Transport in Turbulent Convection over a Rough Surface

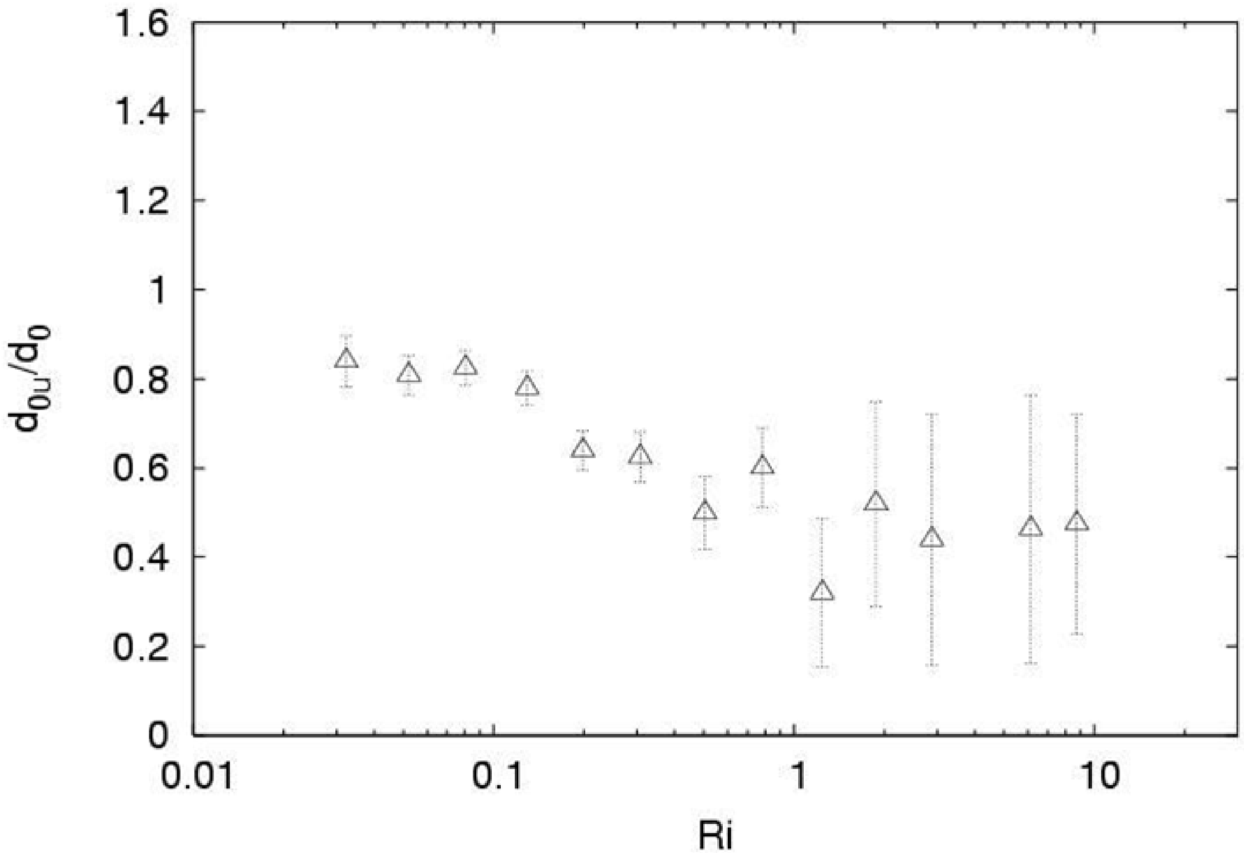


Unstable stratification



Bin-average values of z_{0u}/z_0 vs. Ri=$(g/\Theta_{31})(\Theta_{31}-\Theta_{18})h_0/U_{31}^2$, for the city of Basel (h_0~14.6 m; $z_0 \approx 1.2\pm0.4$) in unstable stratification. Bars are standard errors; the curve is $z_{0u}/z_0 = 1+1.23\ \mathrm{Ri}^{3/14}$.

Unstable stratification

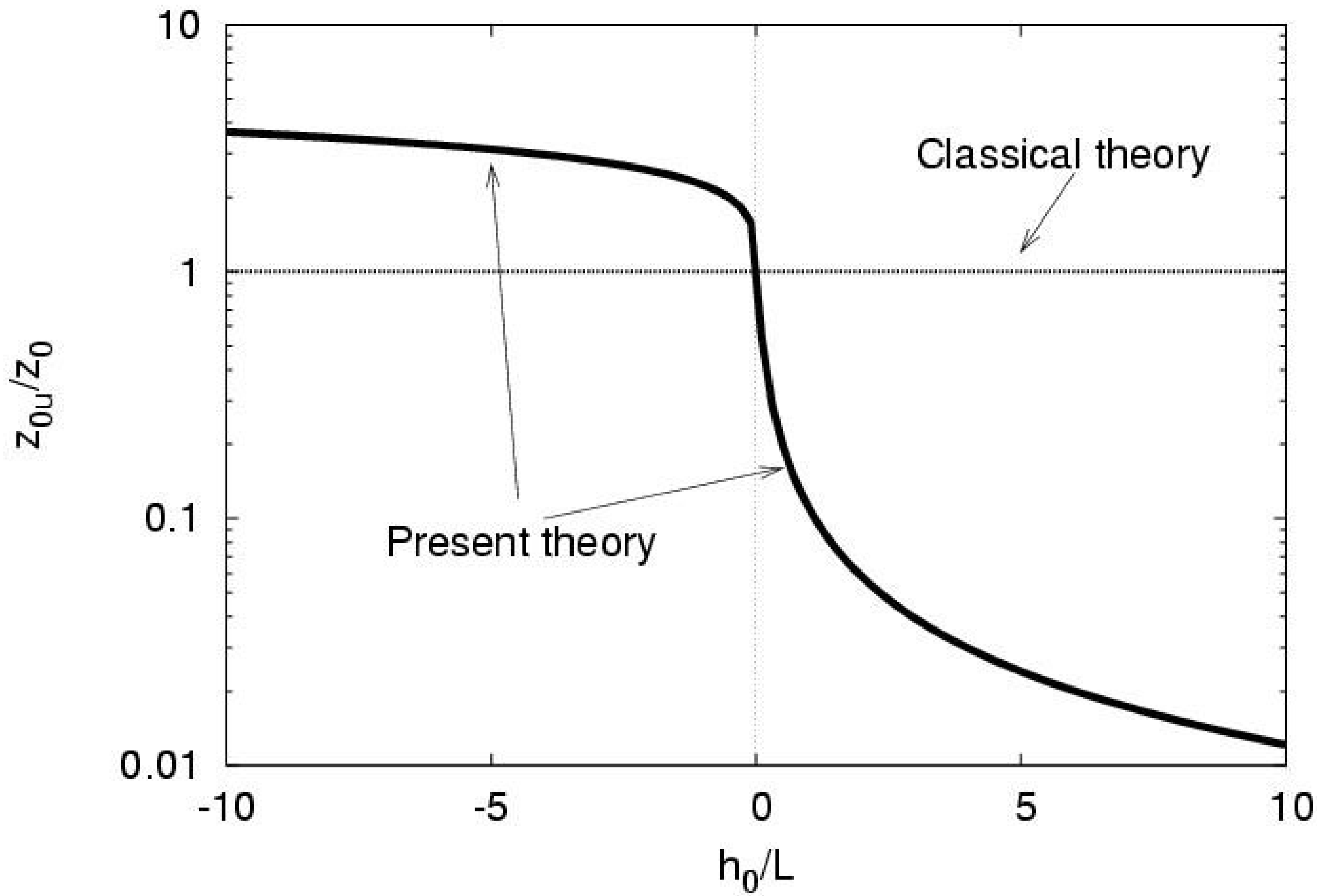


Actual over neutral displacement height, $d_0 / d_{0,neutral}$, versus $Ri=[(g/T_{ref})(\theta_{31m}- \theta_{18m})h_c/U_{31m}]$.

STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the "meteorological interval" -10 $< h_0/L <$10 after new theory and experimental data

Solid line: z_{0u}/z_0 versus h_0/L **Dashed line:** traditional formulation $z_{0u} = z_0$



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Conclusions: 1.2 Roughness length

- **Traditional concept:** roughness length fully characterised by geometric features of the surface
- **New theory and data:** essential dependence on hydrostatic stability especially strong in stable stratification
- **Applications:** to urban and terrestrial-ecosystem meteorology
- **Practically sound:** urban air pollution episodes in very stable stratification

1.3 NEUTRAL and STABLE ABL HEIGHT

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Factors controlling PBL height

Basic factors:

- Deepening due shear-generated turbulence
- Swallowing by earth's rotation and negative buoyancy forces:
  (i) flow-surface interaction, (ii) free-flow stability atmosphere

Additional factors:

- baroclinic shears (enhances deepening)
- large-scale vertical motions (both ways))
- temporal and horizontal variability

Strategy:

Basic regimes → theoretical models → general formulation

Scaling analysis

Ekman (1905): $h_E \sim \sqrt{K_M / |f|}$; K_M in three basic regimes:

$$h_E^2 \sim \frac{K_M}{|f|}, \qquad K_M \sim u_T l_T \sim \begin{cases} u_* h_E & \text{for TN} \\ u_* L_N & \text{for CN} \\ u_* L & \text{for NS} \end{cases}$$

$l_T \sim$ h_E in TN $\quad$ $L_N = u_* N^{-1}$ in CN $\quad$ $L = -u_*^3 F_{bs}^{-1}$ in NS

Basic formulations

$$h_E \sim \begin{cases} u_* |f|^{-1} & \text{Rossby, Montgomery (1935) TN} \\ u_* |fN|^{-1/2} & \text{Pollard et al. (1973) CN} \\ u_*^2 |fB_s|^{-1/2} & \text{Zilitinkevich (1972, 74) NS} \end{cases}$$

Dominant role of the smallest scale

$$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N|f|}{(C_{CN} u_*)^2} + \frac{|fB_s|}{(C_{NS} u_*^2)^2}, \qquad C_R, C_{CN}, C_{NS} = \text{constant}$$

Four parameters u_*, f, N, B_s; hence two dimensionless numbrers:

$$\mu = u_* |fL|^{-1} \text{ and } \mu_N = N/|f|$$

More generally, h_E dependents also on

- geostrophic shear $\Gamma = |\partial \mathbf{u}_g / \partial z|$ (increases h_E: Z & Esau, 2003)
- vertical velocity w_h ($\pm w_h t_{PBL}$, $t_{PBL} \sim h_E / u_*$: Z & Baklanov, 2002).

Hence, additional (usually unavailable) parameters:

$$\mu_\Gamma = \Gamma / N \quad \text{and} \quad \mu_w = w_h / u_*$$

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How to verify *h*-equations?

Stage I: TN $h_E = C_R u_* / f$ transitions TN → CN and TN → NS

$$\left(\frac{u_*}{f h_E}\right)^2 = \begin{cases} C_R^{-2} + C_{CN}^{-2}\mu_N & \text{TN - CN} \\ C_R^{-2} + C_{NS}^{-2}\mu & \text{TN - NS} \end{cases}$$

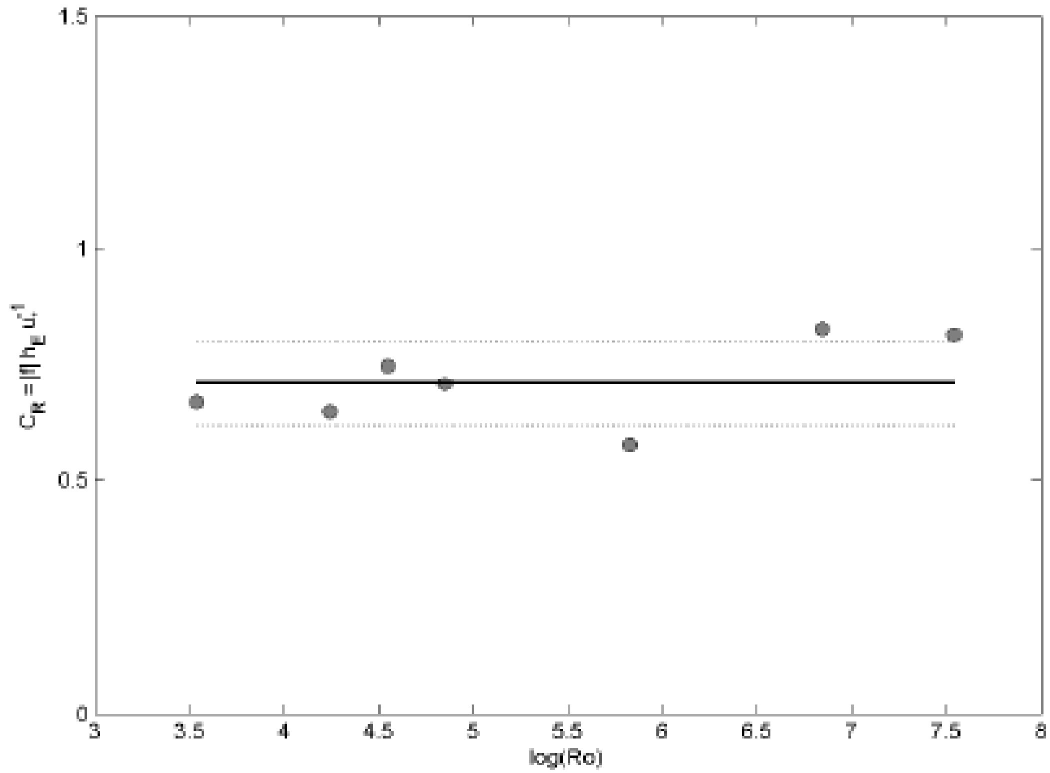
to determine constants C_R, C_{CN}, C_{NS} from selected high-quality data

---

Stage II: Substitute constants in $$\frac{1}{h_E^2} = \frac{f^2}{(C_R u_*)^2} + \frac{N|f|}{(C_{CN} u_*)^2} + \frac{|fB_s|}{(C_{NS} u_*^2)^2}$$

and verify against all available data

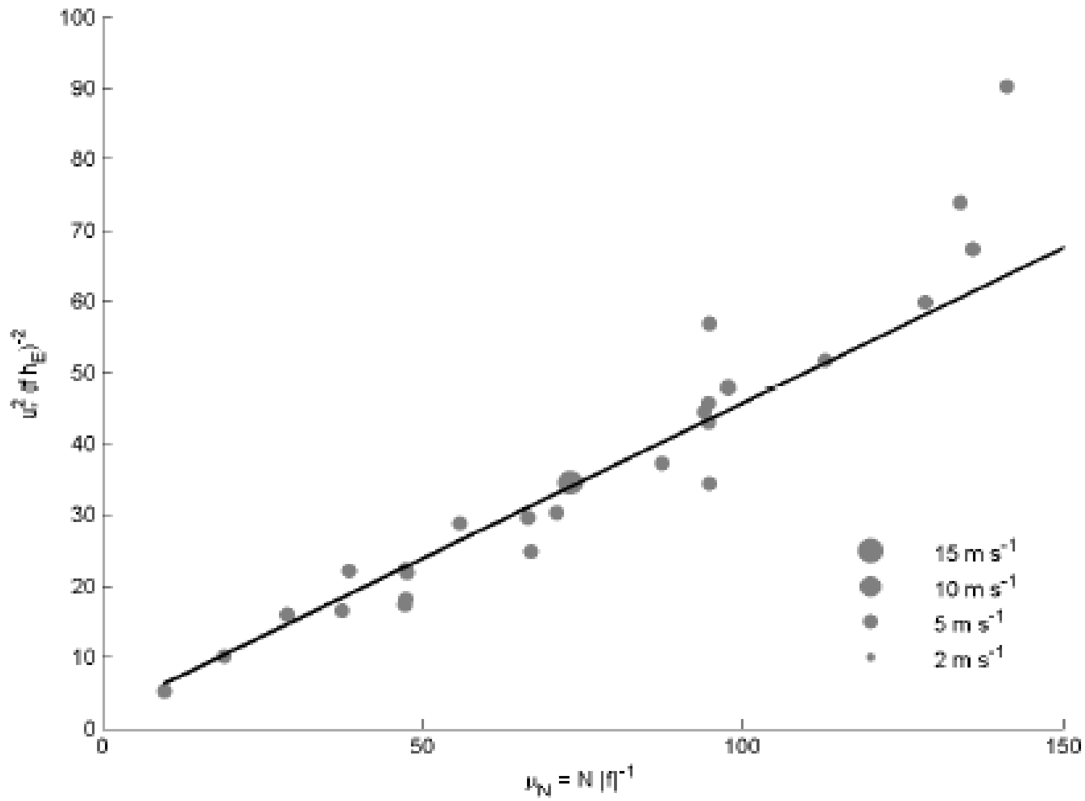
Stage I: Truly neutral ABL



Stage I **TN ABL**: C_R vs. Ro=$U_g(|f|z_{0u})^{-1}$ after LES

Bold line: C_R=0.7±0.1. Dotted line: standard deviation

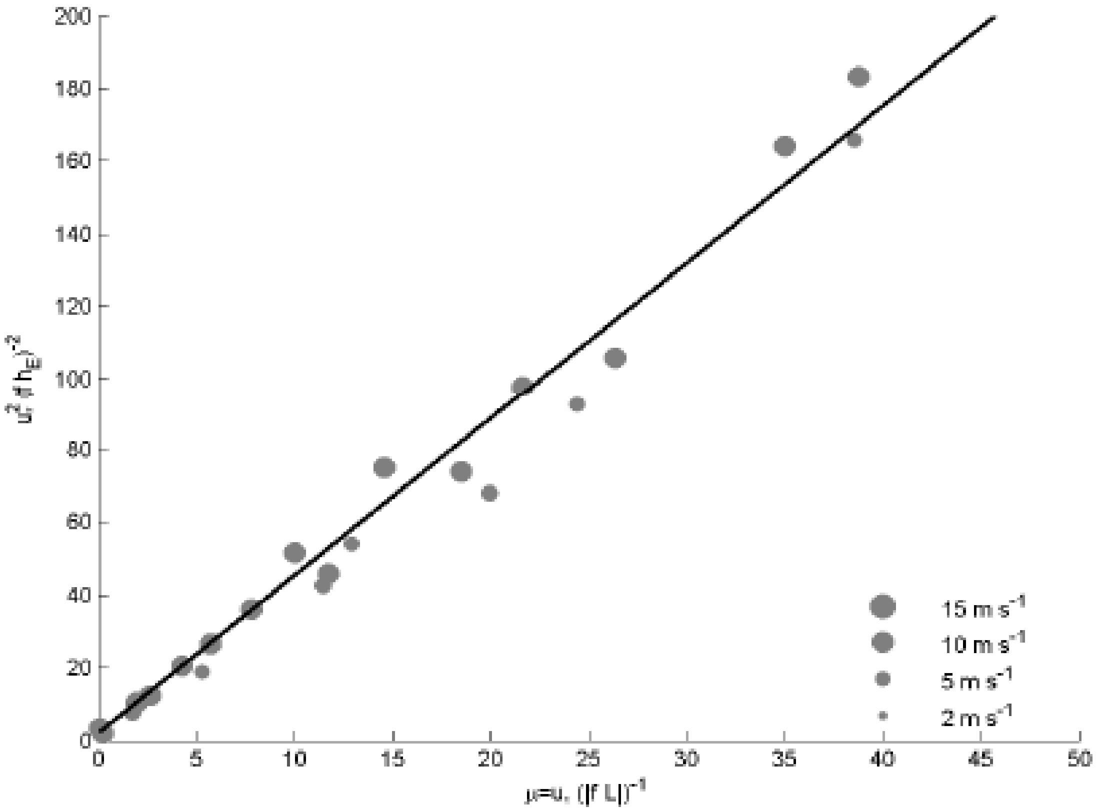
Stage I: Transition TN→CN ABL



Stage I **Transition TN• CN**: $u_*^2 (fh_E)^{-2}$ vs. $\mu_N = N / |f|$, after LES:

Theory: $u_*^2 (fh_E)^{-2} = C_R^{-2} + C_{CN}^{-2} \mu_N$. Empirical C_R=0.6, C_{CN}=1.36

Stage I: Transition TN→NS ABL



Stage I **Transition TN • NS**: $u_*^2(fh_E)^{-2}$ vs. $\mu = u_* \mid fL \mid^{-1}$, after LES.

Theory: $u_*^2(fh_E)^{-2} = C_R^{-2} + C_{NS}^{-2}\mu$, empirical C_R=0.6, C_{NS}=0.51

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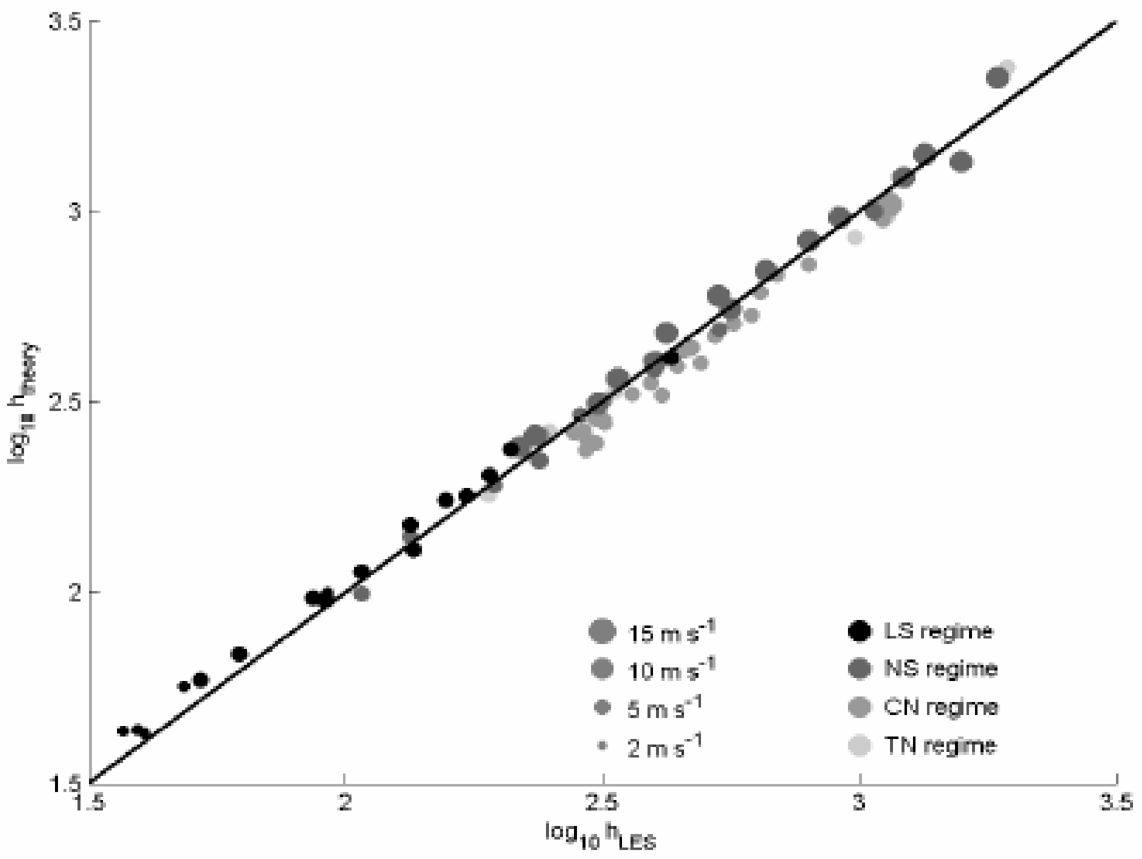
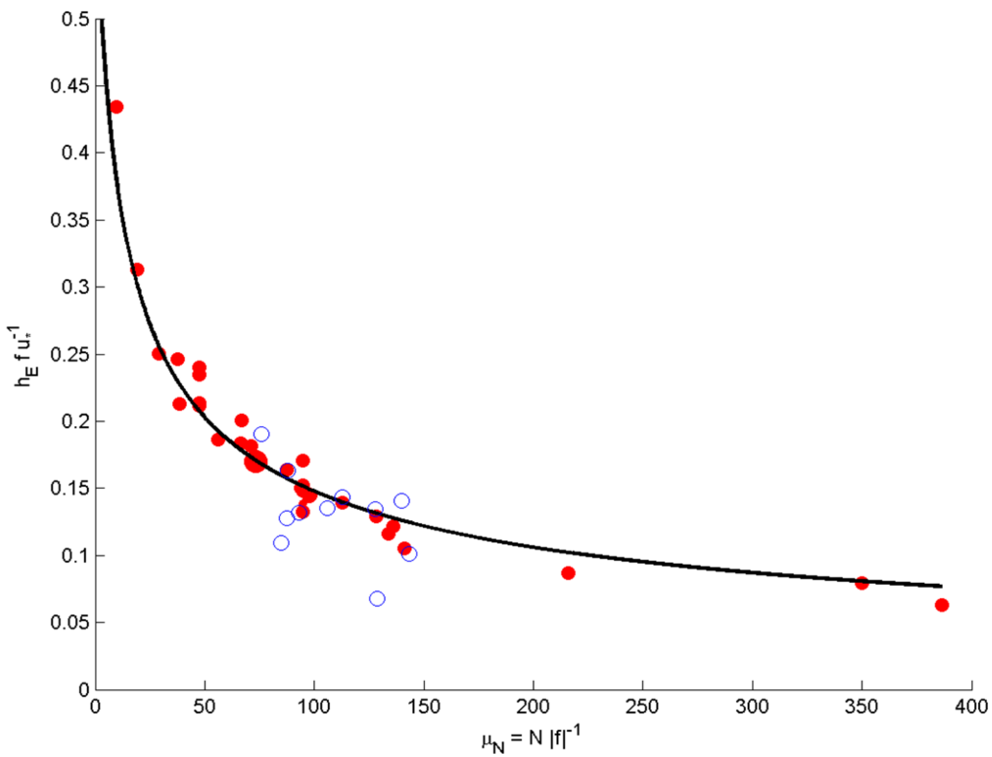
Stage II: General case



Stage II: Correlation: h_{theory} vs. h_{LES} after all available LES data

The height of the conventionally neutral (CN) ABL



Z & Esau, 2002, 2007: the effect of free-flow stability (N) on CN ABL height, h_E, (LES – red, field data – blue, theory – curve). Classical theory overlooks it and overestimates h_E up to an order of magnitude.

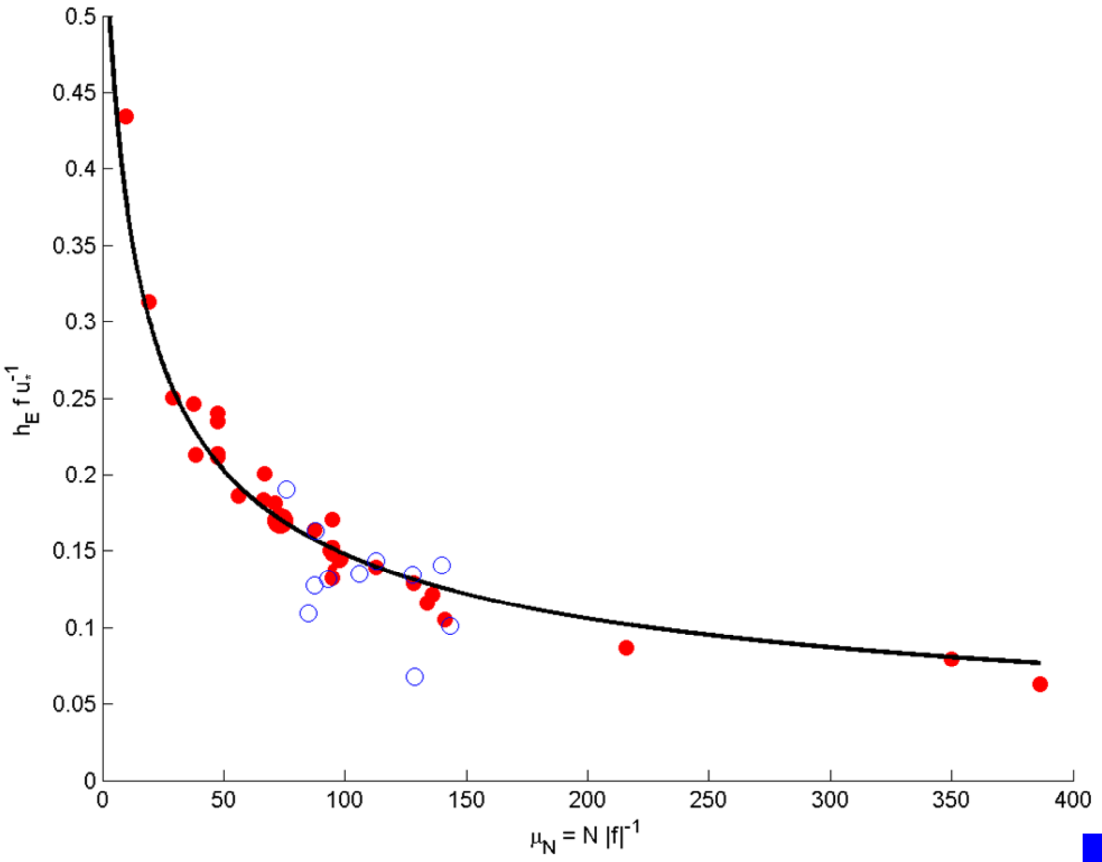
Conclusions: 1.3 SBL height

- h_E, depends on many factors → multi-limit analysis / complex formulation
- difficult to measure: baroclinic shear (Γ), vertical velocity (w_h), h_E itself
- hence necessity to use LES, DNS and lab experiments
- baroclinic ABL: substitute $u_T = u_*(1 + C_0\Gamma/N)^{1/2}$ for u_* in the 2nd term of

$$\frac{1}{h_E^2} = \frac{f^2}{C_R^2\tau_*} + \frac{N|f|}{C_{CN}^2\tau_*} + \frac{|f\beta F_*|}{C_{NS}^2\tau_*^2} \qquad (C_R=0.6,\ C_{CN}=1.36,\ C_{NS}=0.51)$$

- account for vertical motions: $h_{E-\mathrm{corr}} = h_E + w_h t_T$, where $t_T = C_t h_E / u_*$
- generally prognostic (relaxation) equation (Z. and Baklanov, 2002):

$$\frac{\partial h}{\partial t} + \vec{U}\cdot\nabla h - w_h = K_h\nabla^2 h - C_t\frac{u_*}{h_E}(h - h_E) \qquad (C_t = 1)$$



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