Energetics of turbulence, Ri-critical, and new closure model for stable stratification

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References

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Content

Traditional approach

Richardson (1920): Ri

Kolmogorov (1941): TKE budget + closure hypotheses → $\tau = K_M\, dU/dz,\ F_b = -K_H\, db/dz$

Ri_c follows from (i) TKE budget and (ii) perturbation analysis (Ri_c =0.25)

Not applied in practice: to get rid of Ri_c turbulence closures use correction functions

Proposed

Turbulent potential and total energies: TTE=TKE+TPE

No critical Ri but a threshold between two turbulent regimes

- **strong, chaotic**
- **weak, intermittent (wave-dominated)**

Key question

Buoyancy	$b=(g/\rho_0)\rho= \beta\theta$	$(\beta=g/T_0)$
Brunt-Väisälä frequency	$N^2 = db/dz = \beta d\theta/dz$	
Shear	$S=dU/ds$	
1920: Richardson number	$\mathrm{Ri}=(N/S)^2$	

Can local shear maintain turbulence at large Ri?

'No' from classical energy- and perturbation-analyses (Richardson, 1920; Taylor, 1931; Prandtl, 1930, 1942; Chandrasekhar, 1961; Miles, 1961, 1986;…)

'Yes' from overwhelming majority of experiments, LES and DNS

Perturbation analysis

Infinitesimal perturbations are indeed stable at $Ri > Ri_c$ ($Ri_c \sim 0.25$)

But finite perturbations could be unstable…

Mechanism → Internal waves – orbital motions – shear instability – turbulent patches (Phillips 1966, Miropolsky, 1981)

Ri>>1 → Turbulence in
deep ocean (Thorpe, 2005)
free atmosphere (Lawrence et al., 2004; Tokovinin, 2005)

Spectral aspect → short-wave perturbations always stable: stratification shifts stable modes to lover frequencies (Sun, 2006)

Problems in currently used closure schemes

To afford sufficient turbulent mixing at large Ri (needed for realistic modelling) turbulence closures are equipped with

- Ri-dependencies of $K_{\{M,H\}}/K_{\{M,H\}\mathrm{neutral}}$ and $\mathrm{Pr_T} \equiv K_M / K_H$ preventing appearance of $\mathrm{Ri_c}$
- and/or non-zero background turbulent diffusivities (e.g., Shir and Bornstein, 1977 – free atmosphere; Kantha and Klayson, 2000 – deep ocean)

Turbulent potential energy

$$\rho' = \frac{\partial \overline{\rho}}{\partial z} \delta z$$ density fluctution

$$\delta E_P = \frac{g}{\rho_0} \int_z^{z+\delta z} \rho' \, \mathrm{d}z = \frac{1}{2} \frac{b'^2}{N^2} = \frac{1}{2} \frac{\beta \theta'^2}{N^2}$$ potential energy fluctuation

Budget equations for

TKE, $E_K = \tfrac{1}{2}\overline{u_i'u_i'}$

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \tau S + F_b - D_K, \quad \text{where} \quad \tau = \overline{u_i'w'}$$

TPE, $E_P = \tfrac{1}{2} N^{-2}\overline{b'^2}$

$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -F_b - D_P, \quad \text{where} \quad F_b = \overline{b'w'}$$

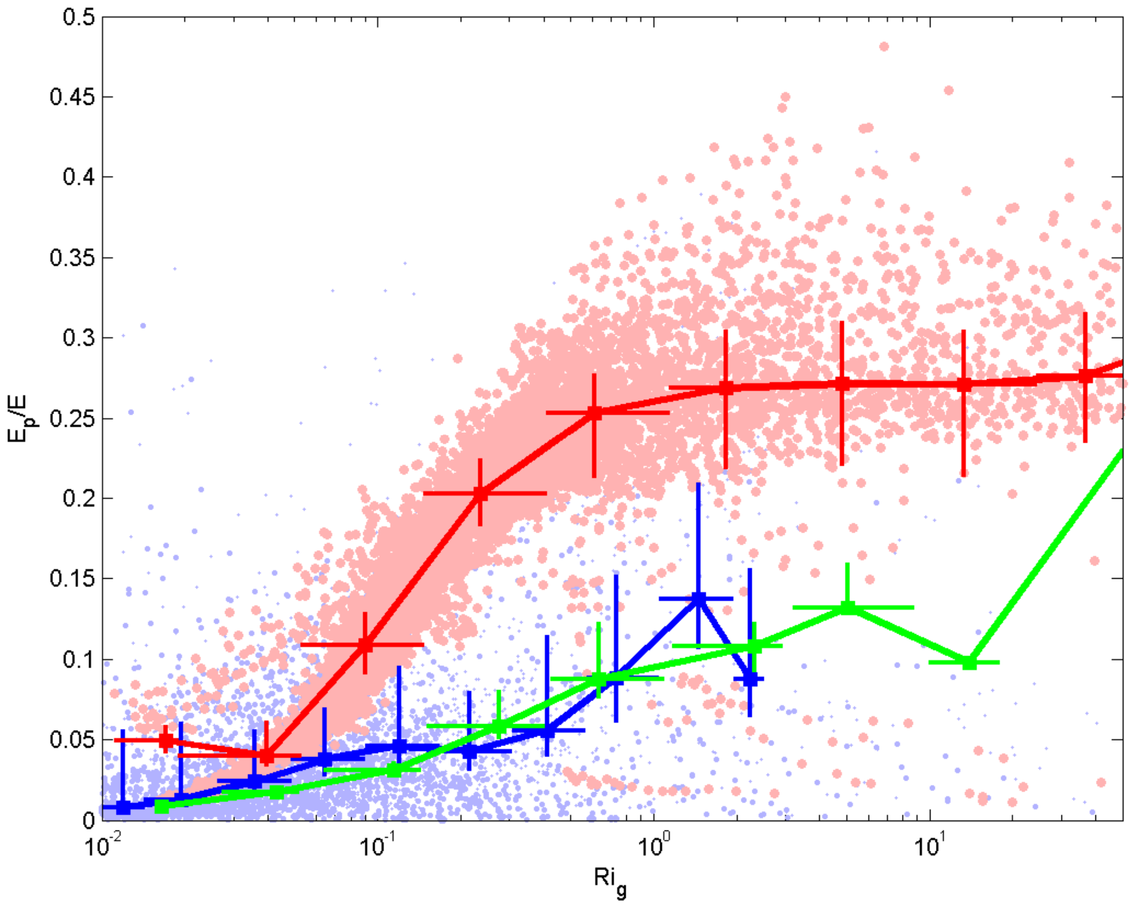
TTE, $E_T = E_K + E_P$

$$\frac{DE_T}{Dt} + \frac{\partial \Phi_T}{\partial z} = \tau S - D_T$$

does not include F_b!

Role of TPE (TPE/TTE = $\mathrm{Ri}_f = -\beta F_\theta / \tau S < 1$)

TPE / TKE vs. Ri: lab (Ohya, 2001), LES (Esau)



for LES

transition at Ri=0.25

Classical (but erroneous) energy analysis

From TKE equation:

At large Ri ($\mathrm{Ri}>\mathrm{Ri}_c$), F_b passes a threshold, after which the TKE production, $P=\tau S$, is insufficient to compensate its consumption → TTE decays.

However

The TKE budget equation is not closed. It only shows that ***flux Richardson number***, $\mathrm{Ri}_f = -F_b/(\tau S)$ cannot exceed unity.

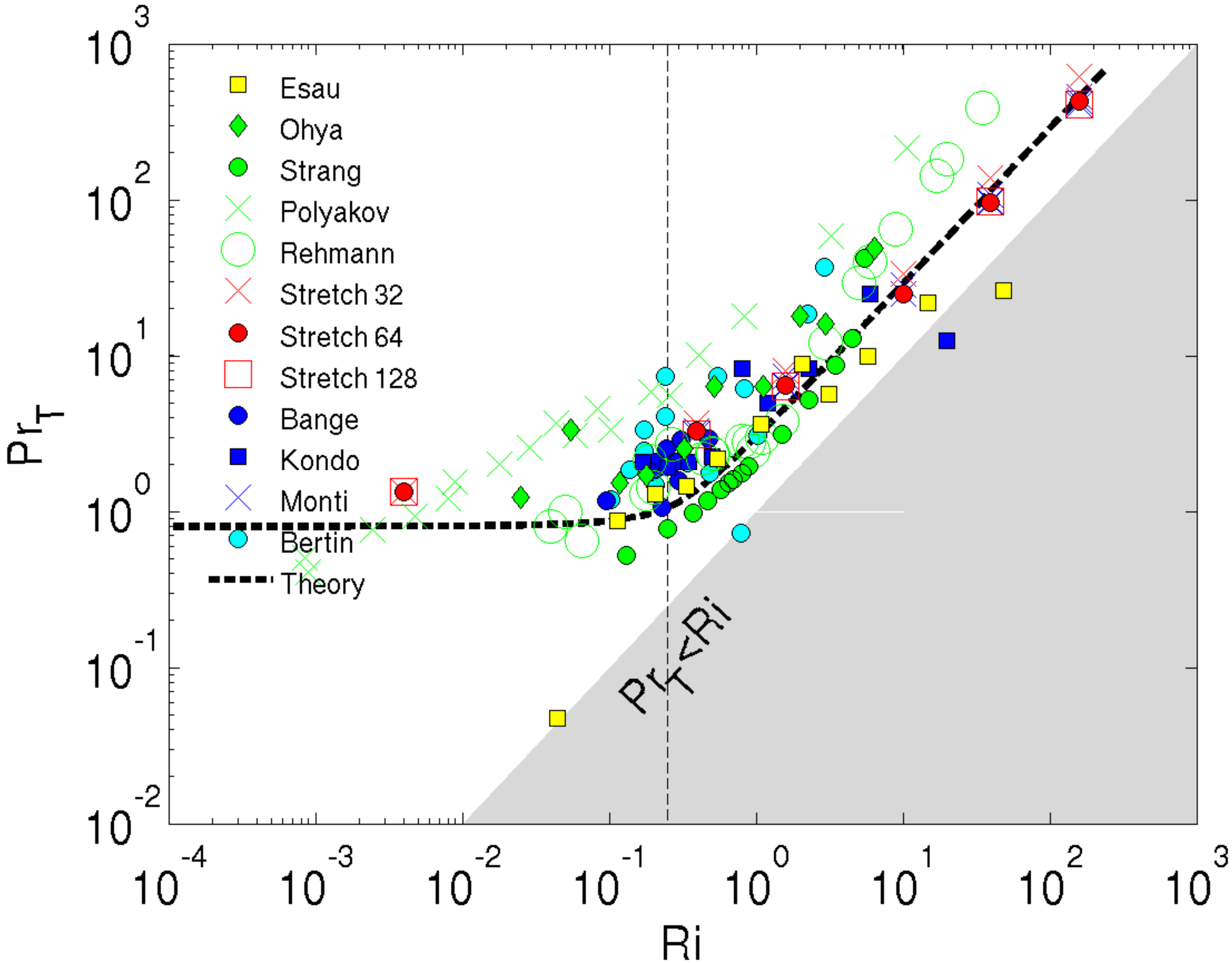
But Ri_f is an internal turbulent parameter (F_b and τ depend on each other)
Inequality $\mathrm{Ri}_f<1$ does not say anything about maintenance or degeneration of turbulence at large Ri.

Traditional derivation of Ri_c assumes

$\tau = K_M\, dU/dz$, $F_b = K_H\, db/dz$, with $\mathrm{Pr}_T = K_M/K_H$ =*constant*

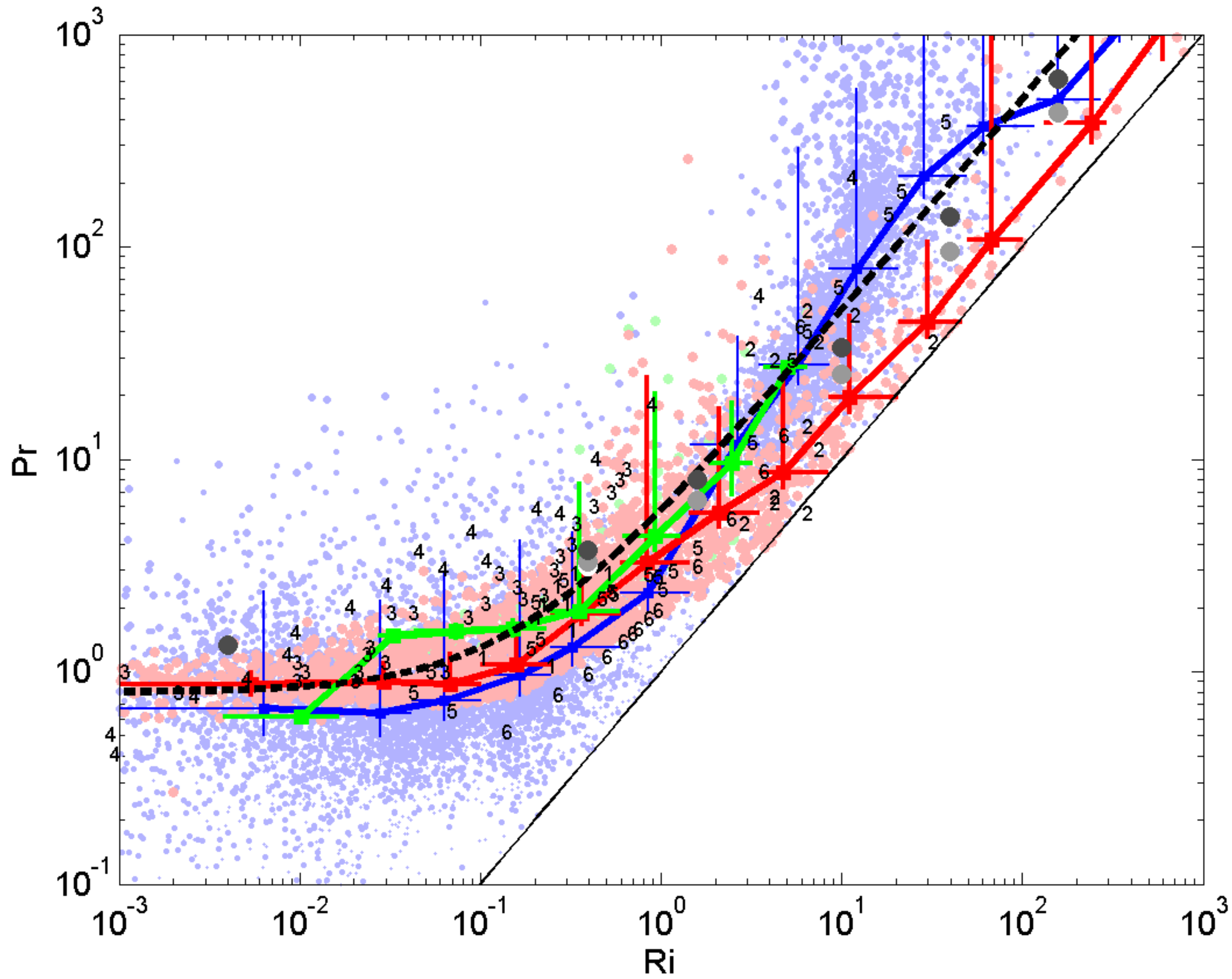
which contradicts to experimental data (next slides)

Ri dependence of turbulent Prandtl number



transition at Ri=0.25

Ri dependence of the turbulent Prandtl number



Similar dependence after recent data

Blue: atmosphere (SHEBA & CASES-99),

green: laboratory,

red: LES (NERSC),

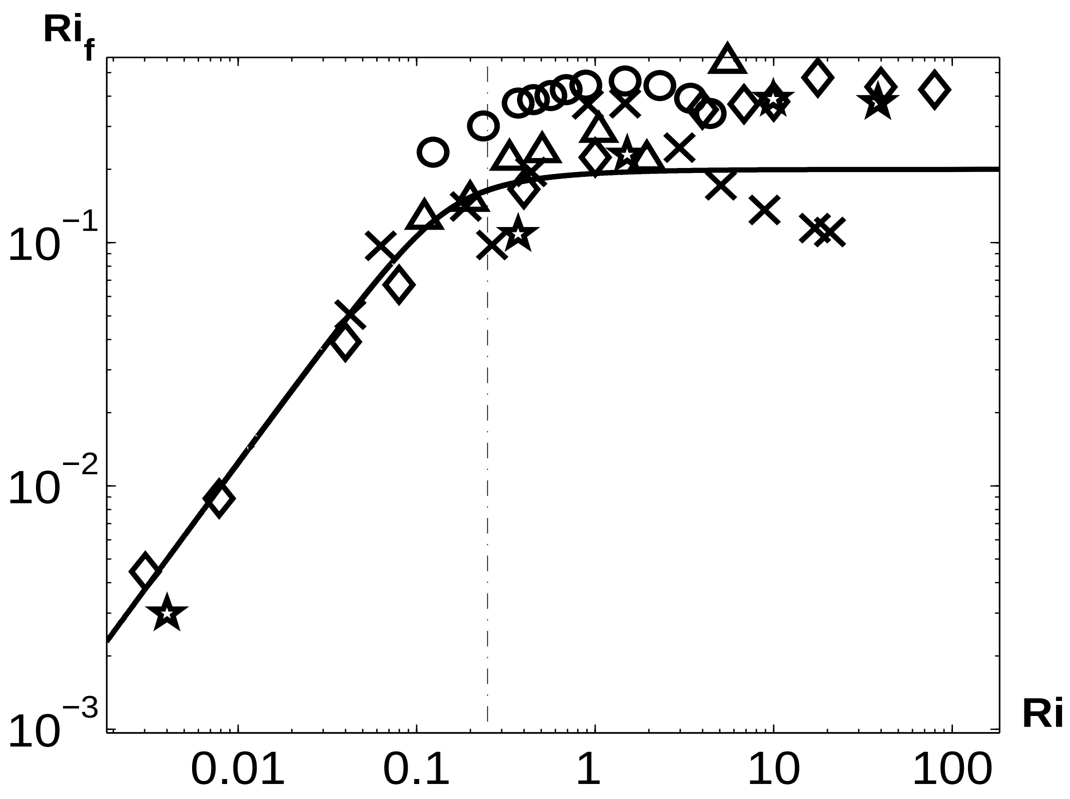
grey: DNS;

curve: $Pr_T = 0.8 + 5\ Ri$

transition at Ri=0.25

Ri-dependence of the flux Richardson number

Theory (for homogeneous turbulence) – solid line; **Laboratory**: circles – Strang & Fernando (2001), diamonds – Ohya (2001), crosses – Rehmann & Koseff (2004); **LES** (Esau, 2006) – triangles; **DNS** Stretch et al., 2001) – stars



transition at Ri=0.25

Proposed hierarchy of models

Budget equations for the basic statistical moments:

Turbulent energies: TKE and TPE (or TTE and either TKE or TPE)
Vertical turbulent fluxes of potential temperature, F_θ, and momentum $\tau_{\{x, y\}}$

Closure hypotheses:

Kolmogorov's dissipation rates (generalised – for "dissipation" of $\tau_{\{x, y\}}$)

The "pressure ↔ Θ-gradient" correlation term in the F_θ -equation is ~ $-\beta\theta^2$
(spectral analysis and data) → **crucial role of $\beta\theta^2$** (counter gradient flux)

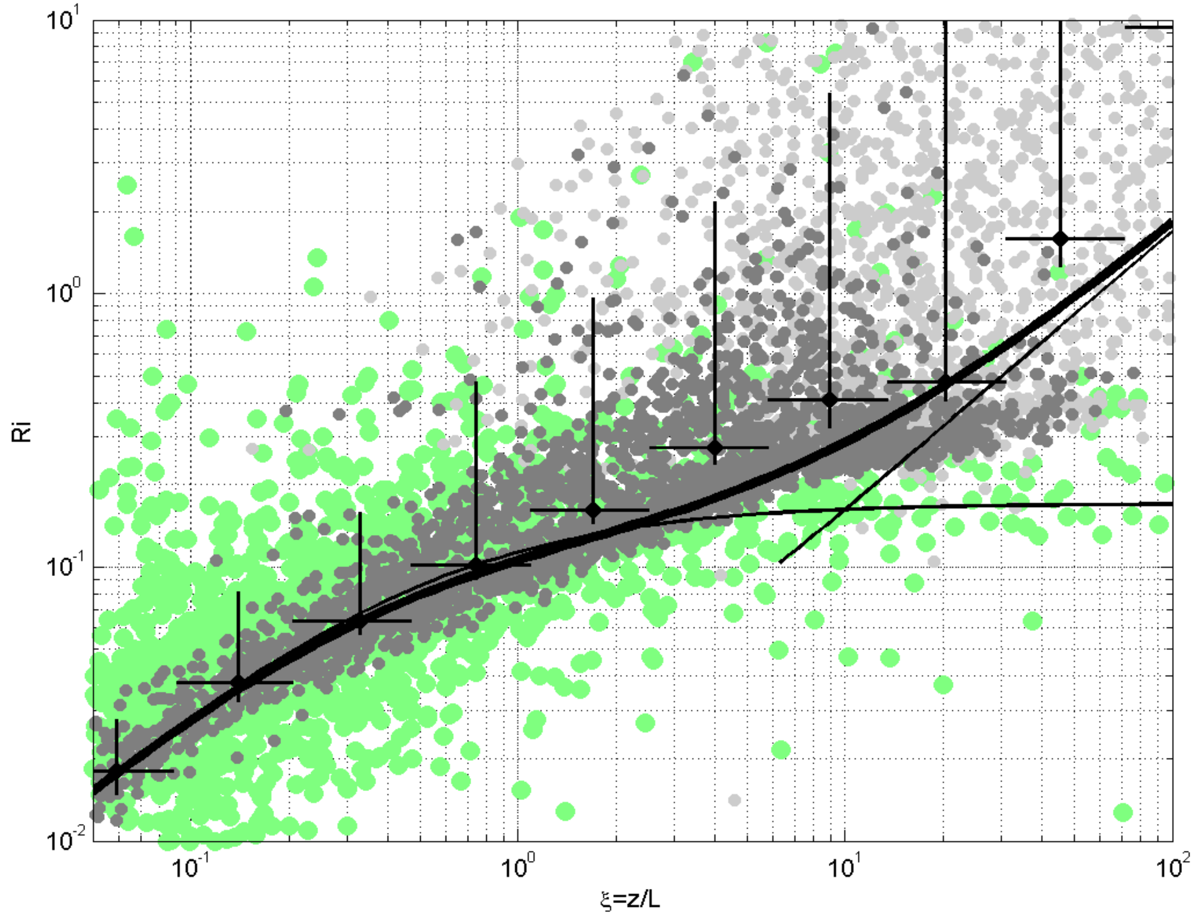
Generalised "return-to-isotropy" → redistribution of TKE between E_x, E_y, E_z
accounting for the stability effect on the x → z energy transfer

Mixing length, $l = z\Phi(\mathrm{Ri}_f)$ asymptotes → $\Phi \sim (1\text{-}\mathrm{Ri}_f / \mathrm{Ri}_{f,\mathrm{crit}})^n$
experimental data → n=4/3 (next slides)

Part I: Local (algebraic) closure for the steady-state homogeneous regime of turbulence

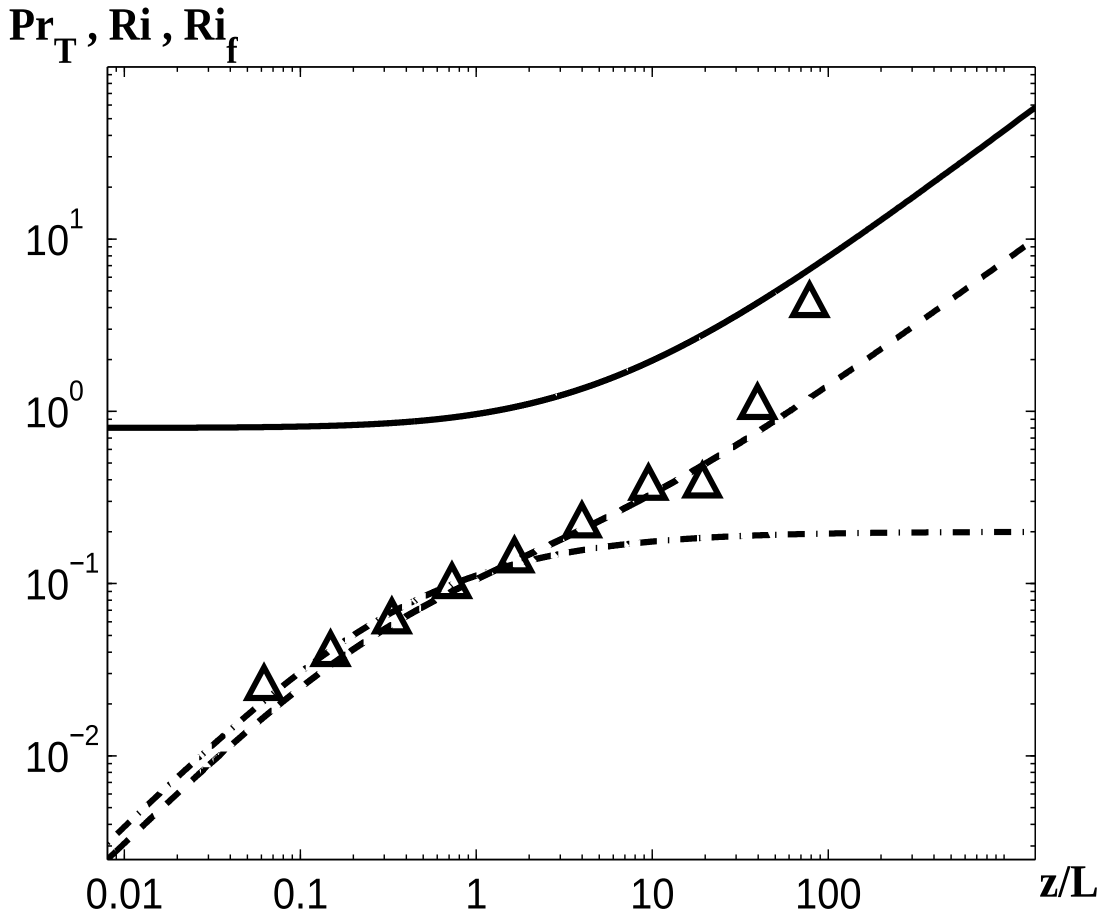
Ri versus z/L in stably stratified ABLs

LES: dark grey $z<h$, light grey $z>h$; SEBA field data: green; Bin-averaged: black with error bars; Bold line: analytical approximation: $L=\tau^{3/2}(-\beta F_\theta)^{-1}$



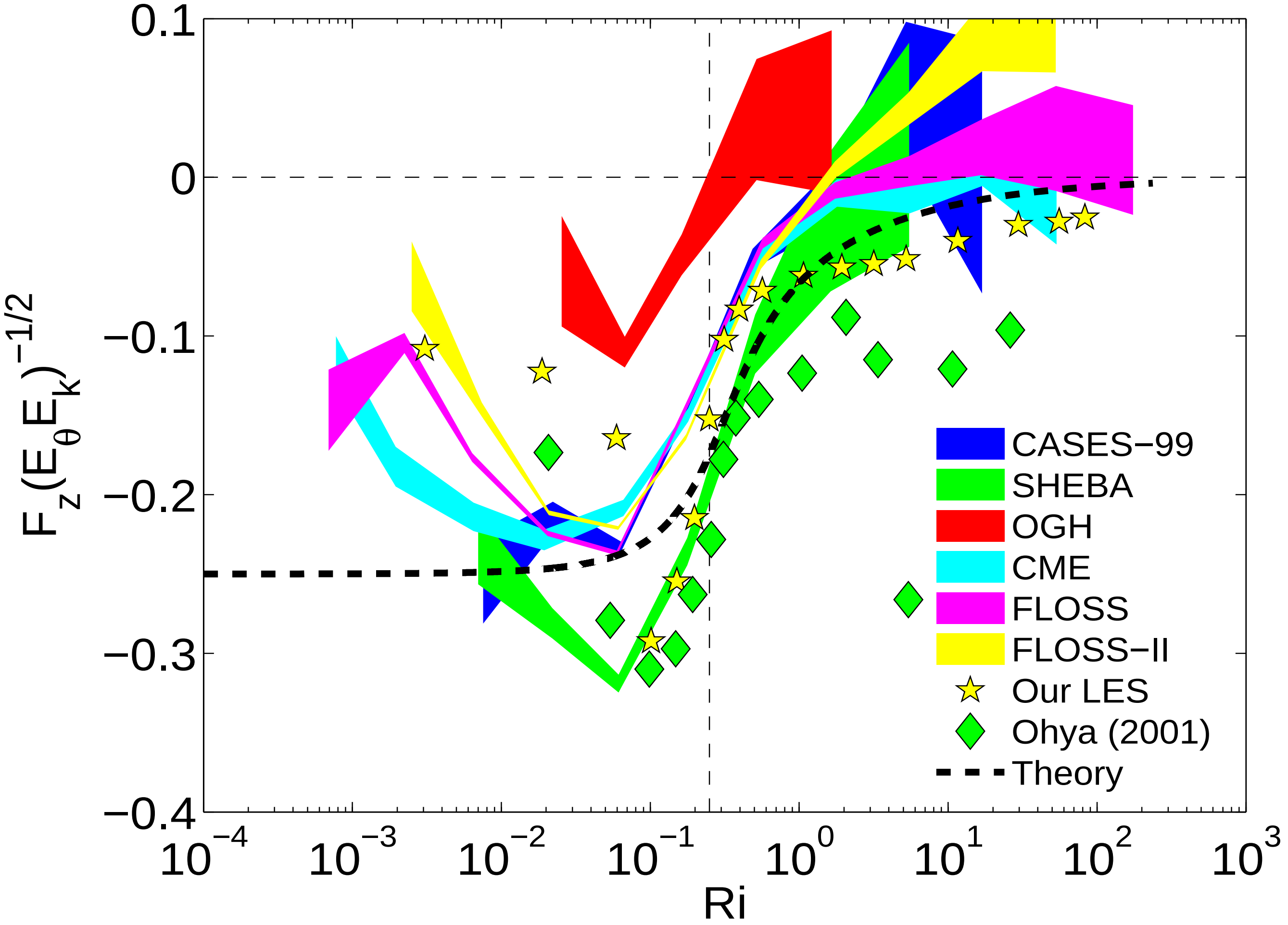
Ri versus z/L in stably stratified ABLs

triangles: LES; dashed line: $l/z=(1-\mathrm{Ri}_f/\mathrm{Ri}_{f,\mathrm{crit}})^{4/3}$; L is MO length



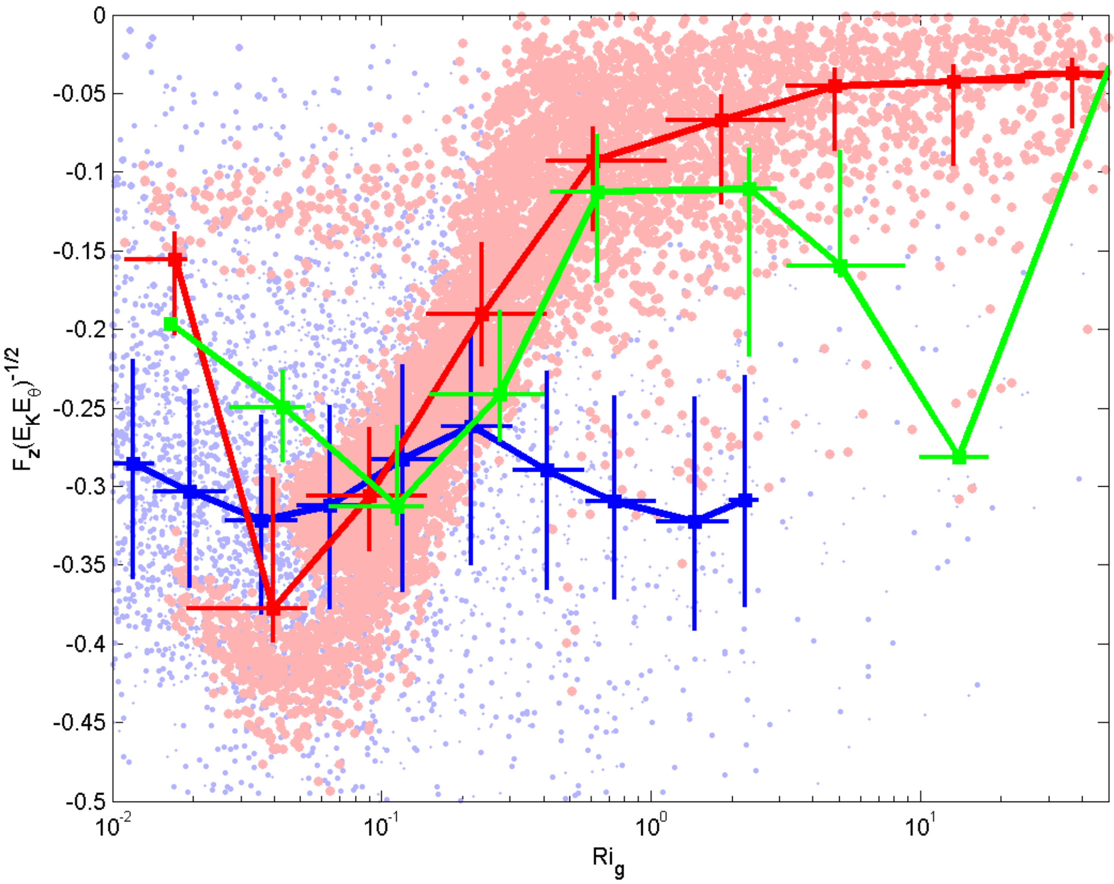
Normalised turbulent heat flux versus Ri

analysed by Mauritsen: transition at Ri=0.25 (dashed line: theory)



Normalised turbulent heat flux vs. Ri

analysed by Esau: LES (red), lab (green, Ohya, 2001), blue (Sheba)

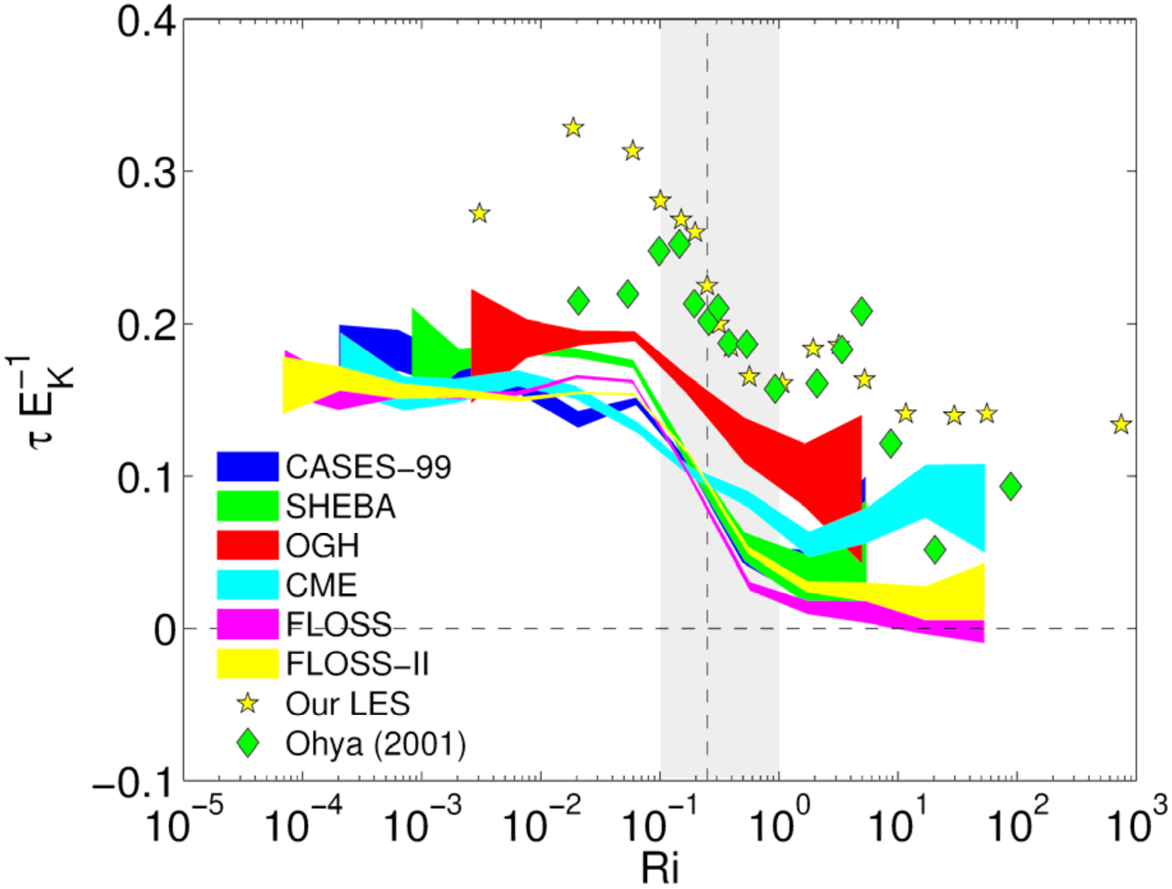


for LES

transition at Ri~0.25

Normalised flux of momentum vs. Ri

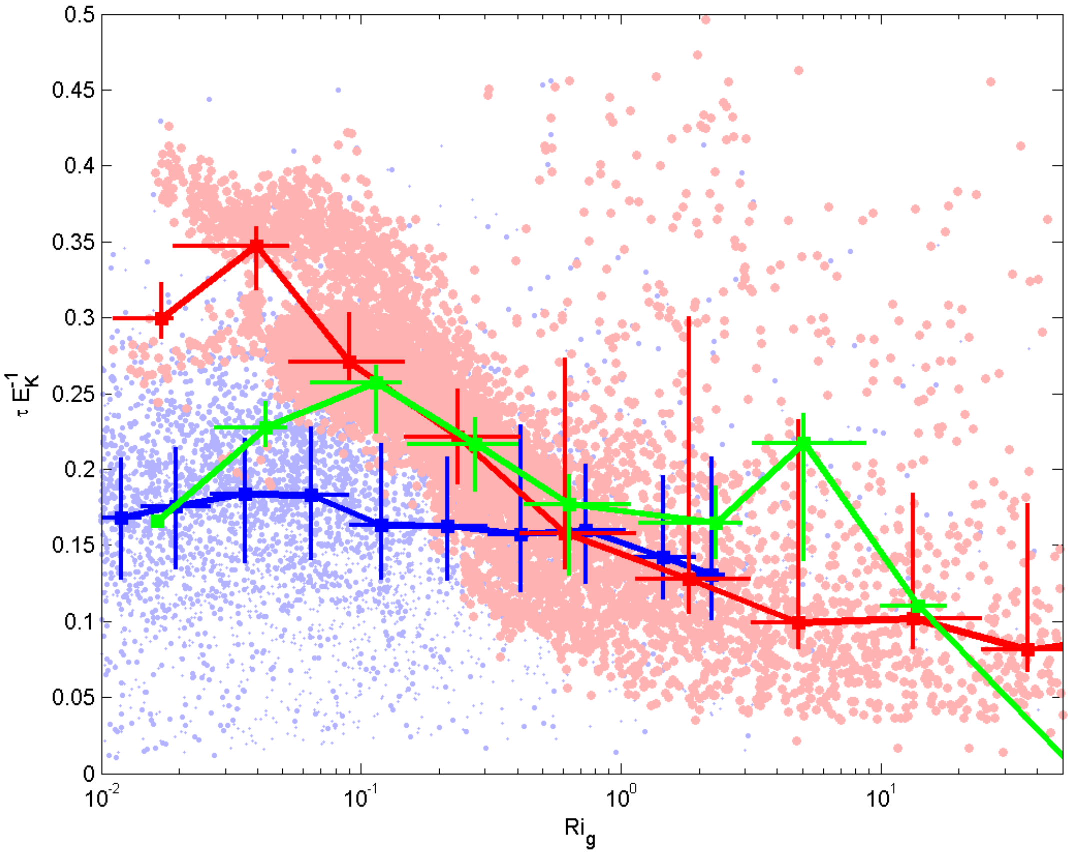
analysed by Mauritsen: transition at Ri=0.25



transition at Ri=0.25

Normalised flux of momentum vs. Ri

analysed by Esau: LES (red), lab (green, Ohya, 2001), blue (Sheba)



for LES

transition at Ri~0.25

Sharp transition – between two regimes

Anisotropy and normalised heat flux (also other dimensionless parameters) turn into constants in the two alternative regimes: (i) near-neutral, (ii) very stable, with **sharp transition** in a narrow interval of Ri around Ri=0.25

Coincidence of with **classical hydrodynamic instability threshold**

However, this threshold separates not **turbulent** and **laminar** regimes, as classical concept says, but two different turbulent regimes: **strong, chaotic** (Pr_T of order unity) and **weak, intermittent** (wave-induced momentum transfer, Pr_T linearly increases with Ri)

Part II: Minimal non-local closure (future work)

Five budget equations (for TKE, TPE and turbulent fluxes) and one relaxation equation for the mixing length accounting for

- temporal evolution,
- transport by mean wind,
- vertical turbulent transport (at 1^{st} stage through turbulent diffusion)

Conclusions: physics

- The TKE equation do not fully characterise energy transformations. Equally important are the **TPE or TTE equations** (TTE = TKE+TPE)

- TTE **(invariant) conserves along motion** in the absence of shear and dissipation. TPE-equation allows developing advanced turbulence closures

- It proves that velocity shear generates turbulence in any stratification, and **disapproves the critical Ri concept**. Field, lab, LES and DNS data show maintaining of turbulence at Ri>100 – two orders larger than Ri_c=0.25

- Ri-dependences disclose **sharp transition** at Ri~0.25 (linear instability limit), which separates "strong, chaotic" and "weak, intermittent" regimes

- The model is extended to **turbulent diffusion of passive scalars**

Conclusions: modelling

L'vov, Procaccia and Rudenko, 2007: Energy conservation an second-order statistics in SBLs. Submitted to *BLM* (Wezmann institute of Science, Israel) → detailed analysis of the momentum-flux budget

Mauritsen et al., 2006: TTE closure model for SBLs. *JAS* in press (Stockholm University) → a model employing TTE and TPE budgets and empirical Ri-dependencies of τ/E_K and $F_z(E_K E_\theta)^{-1/2}$, **consistent with Z et al. non-local scaling and experimental & LES data for mean profiles and ABL height, including conventionally neutral and long lived stable ABLs**

↓

Application to wind energy

Revised high resolution modelling of the wind field in the lover 200-300 m to refine the wind energy potential, especially for coastal zones

Future: unsolved problems for convective flows

Decomposition: mean, turbulent, semi-organised (cells / rolls)

Nature and theory of rolls: aspect ratio? energetics?

Parameterization

Application to turbulent transport: advanced closure

Application to wind energy (contribution from rolls)

END