STRATIFICATION EFFECT ON THE ROUGHNESS LENGTH

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Reference (1.2)

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre, 2007: The roughness length in environmental fluid mechanics: the classical concept and the effect of stratification. Submitted to *Boundary-Layer Meteorology.*

Content

- Roughness length and displacement height

$$u(z) = \frac{u_*}{k}\left[\ln\frac{z-d_{0u}}{z_{0u}} - \Psi_u\left(\frac{z-d_{0u}}{L}\right) + \Psi_u\left(\frac{z_{0u}}{L}\right)\right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics: neutral-stability z_0 = level, at which $u(z)$ plotted vs. $\ln z$ approaches zero; z_0 ~ (1/25) h_0 the typical height of roughness elements.

- Meteorology / oceanography: h_0 comparable with MO length $L = \frac{u_*^3}{-F_b}$ where $F_b = \beta F_\theta$, $\beta = g / T_0$ and $F_\theta = \overline{w'\theta'}$

- Stability dependence of the actual roughness length, z_{0u}:

  $z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification

Surface layer and roughness length

Self similarity in the surface layer (SL) 2-5 $h_0 < z < 10^{-1} h$

Height-constant fluxes: $\tau \approx \tau|_{SL} \equiv u_*^2$

u_* and z serve as turbulent scales: $u_T \sim u_*,\ l_T \sim z$

Eddy viscosity $(k \approx 0.4)$ $K_M\ (\sim u_T l_T) = k u_* z$

Velocity gradient $\partial U / \partial z = \tau / K_M = u_* / kz$

Integration constant: $U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$

z_{0u} (redefined constant of integration) is “roughness length”

“Displacement height” d_{0u} $U = k^{-1} u_* \ln\left[(z - d_{0u}) / z_{0u}\right]$

Not applied to the roughness layer (RL) $0 < z < 2\text{-}5\, h_0$

Parameters controlling z_{0u}

Smooth surfaces: viscous layer → $z_{0u} \sim \nu / u_*$

Very rough surfaces: pressure forces depend on:

obstacle height h_0

velocity in the roughness layer $U_R \sim u_*$

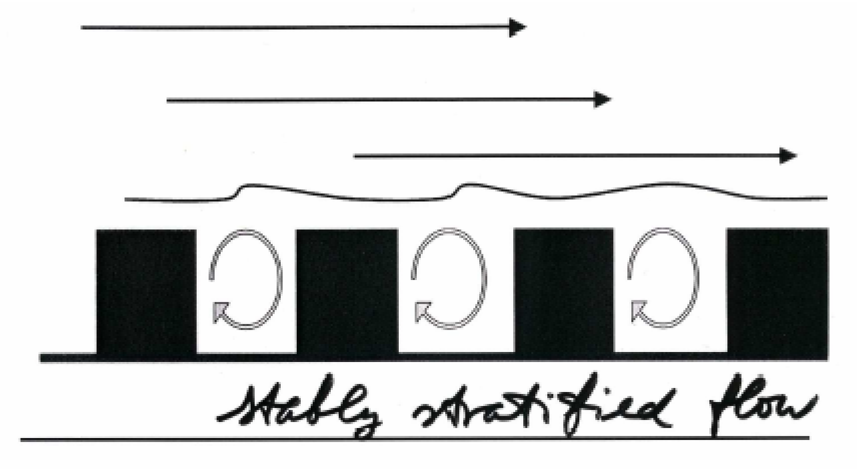
$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

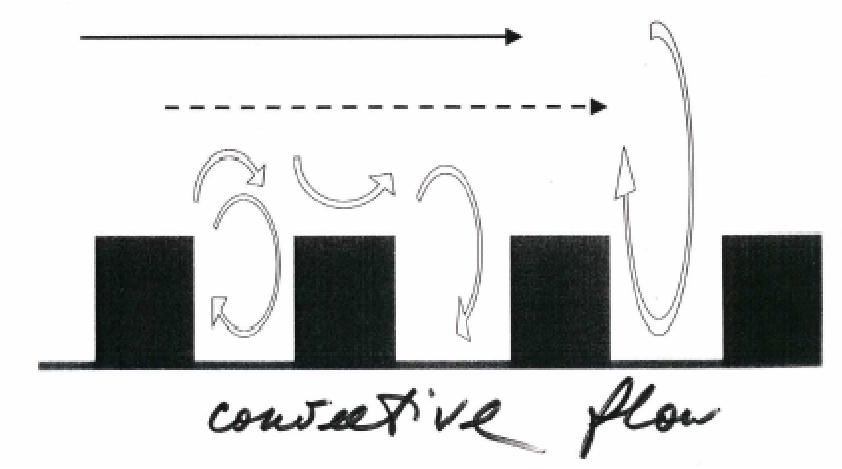
No dependence on u_*; surfaces characterised by $z_{0u} =$ constant

Generally $z_{0u} = h_0 f_0(\mathrm{Re}_0)$ where $\mathrm{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0

Stability Dependence of Roughness Length





For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L, the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL ($0 < z < 2\text{-}5h_0$) with 90 % of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(RL) = K_M(H_0 \sim 2-5h_0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ classical formula $z_{0u} \sim h_0$

Stable: $K_M = ku_* z(1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = ku_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} h_0^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$

Where $C_u \approx 2$ (Zilitinkevich and Esau, 2007) and $C_U \approx 1.7$ (Kader and Yaglom, 1990) are empirical constants.

Recommended formulation

Neutral⇔ stable

$$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

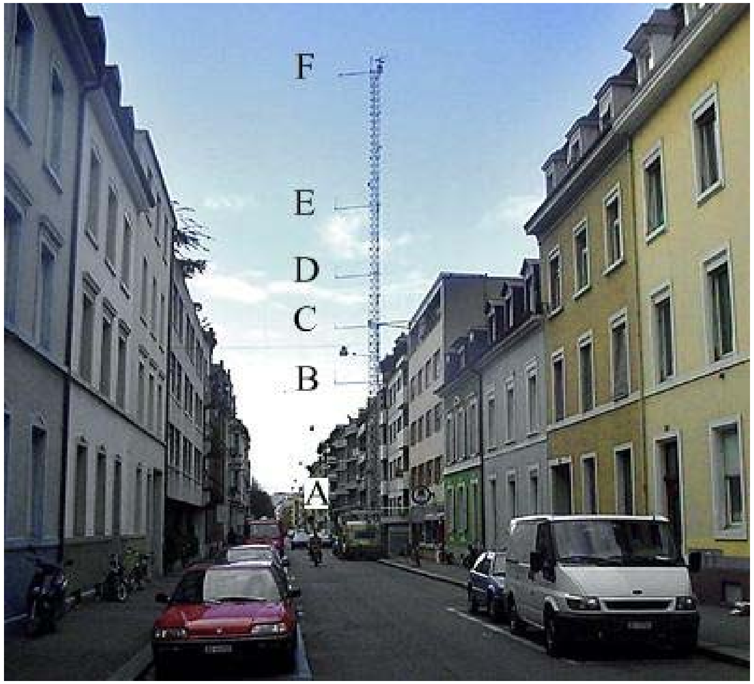
Neutral⇔ unstable

$$\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$$

Experimental datasets

Sodankyla Meteorological Observatory, Boreal forest (FMI)

h0 ≈ 13 m, measurement levels 23, 25, 47 m



BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

h0 ≈ 14.6 m, measurement levels 17.9, 22.4, 31.7 m

Sodankyla site:

- Scots Pine forest
- h_0 = 13.5 m
- z_0 = 1.1 m ; d_0 = 10 m
- EC measurements at 23, 25 and 47 m
- Long fetch wind direction sector

Sperrstrasse site:

- urban canopy
- h_0 = 14.6 m
- z_0 = 1.2 m ; d_0 = 12 m
- EC measurements at 17.9, 22.4 and 31.7 m
- Cross canyon wind direction sector

Calculation of the effective values of roughness length and displacement height

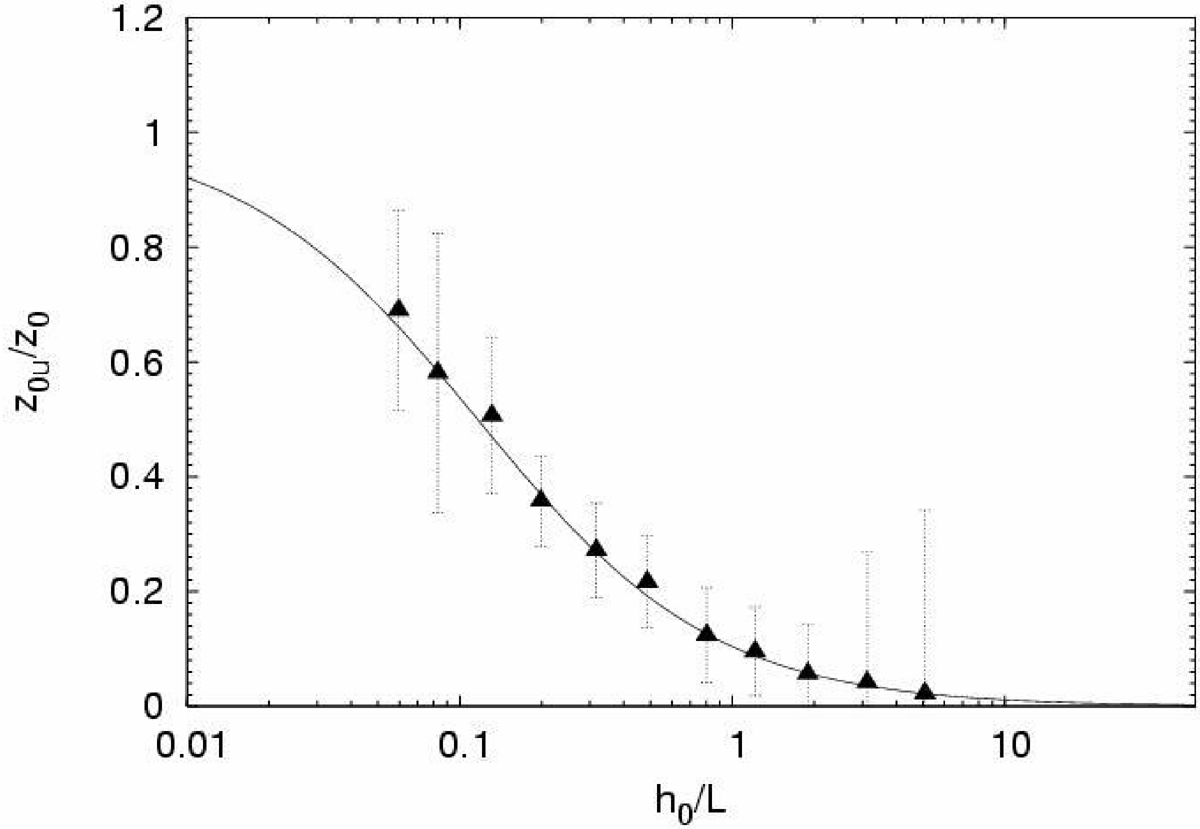
For stable stratification → the log-linear velocity profile:

$$U(z) = k^{-1} u_* \left[\ln[(z - d_{0u}) / z_{0u}] + C_u (z - d_{0u}) / L \right]$$

For unstable stratification → the -1/3 power law velocity profile:

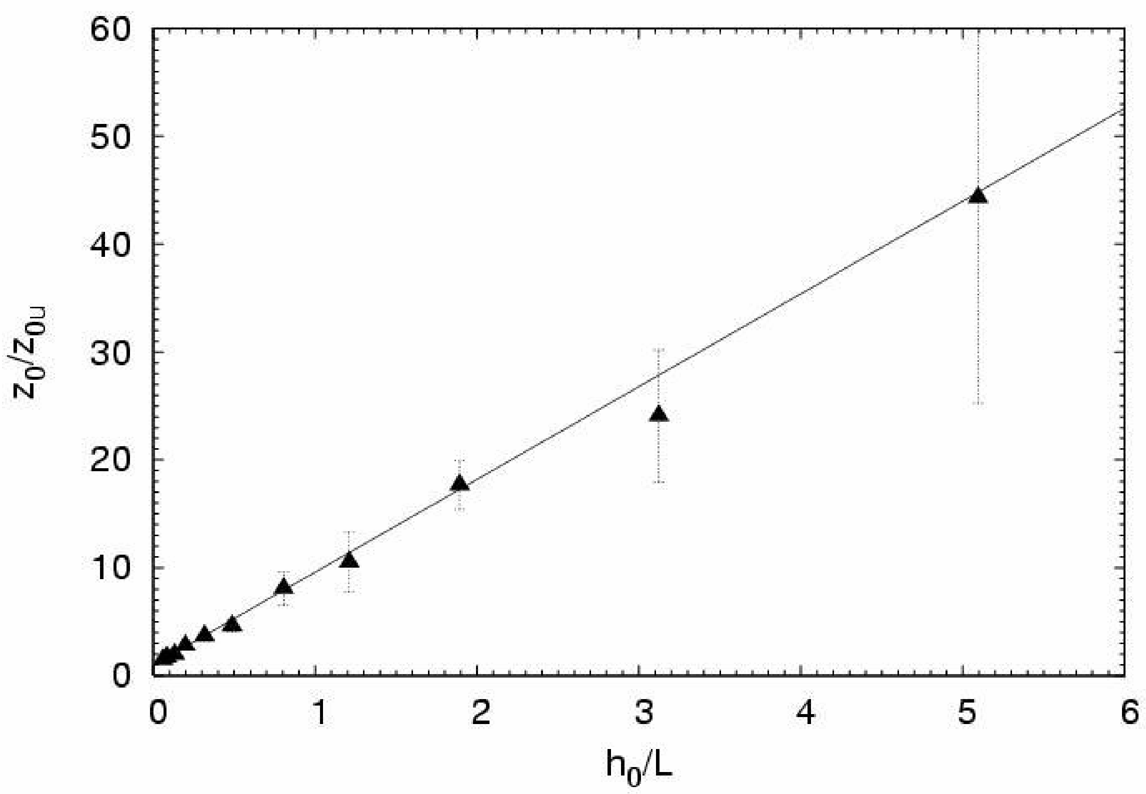
$$U(z) = 3 C_U u_* \left[\left(\frac{z_{0u}}{-L} \right)^{-1/3} - \left(\frac{z - d_{0u}}{-L} \right)^{-1/3} \right]$$

Stable stratification



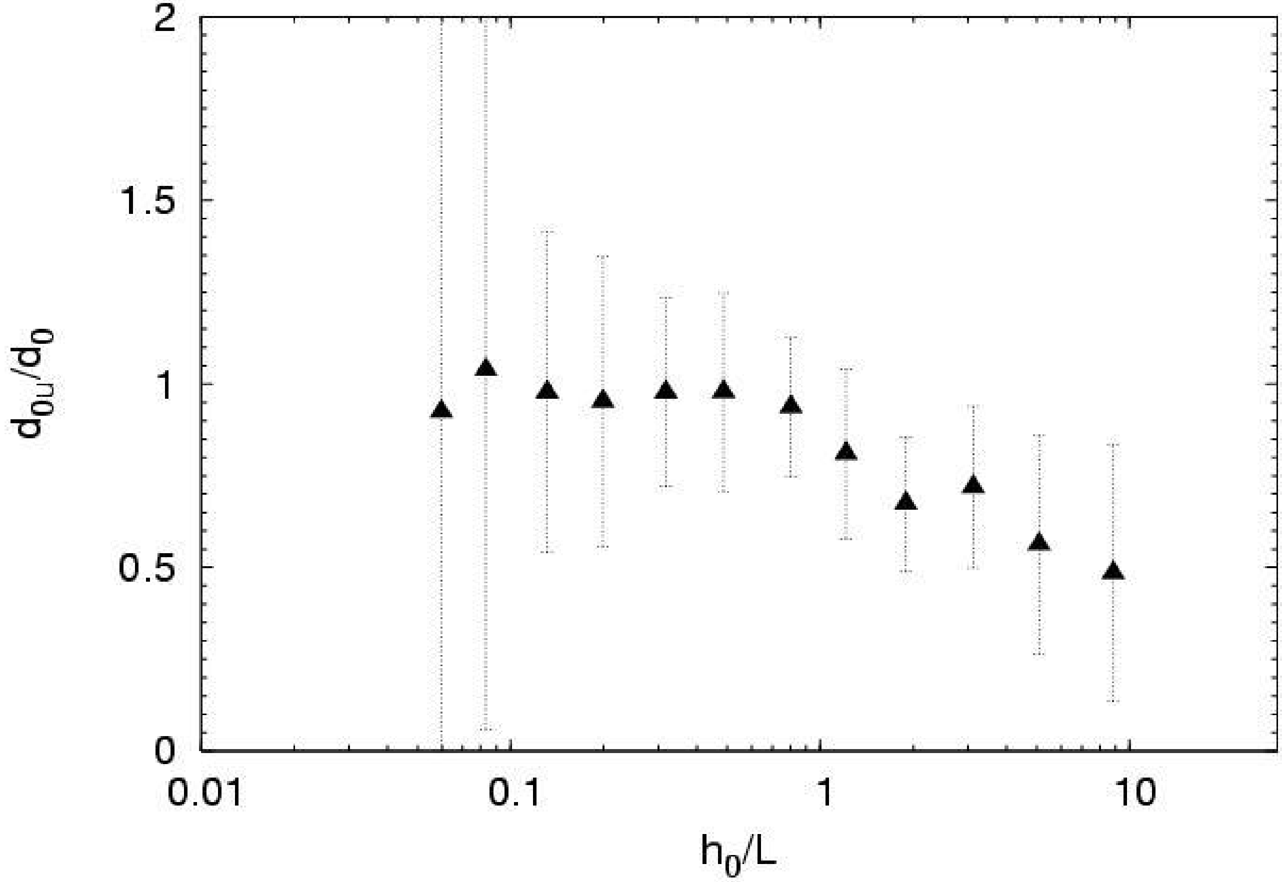
$$\boxed{\frac{z_{0u}}{z_0} = \left[1 + 8.13\frac{h_0}{L}\right]^{-1}}$$

Stable stratification



$$\boxed{\frac{z_0}{z_{0u}} = 1 + 8.13\,\frac{h_0}{L}}$$

Stable stratification



For unstable stratification plotting z_{0u}/z_0 versus h_0/L would suffer from artificial self-correlation.

Then we used a Richardson Number as a stratification parameter

$$Ri = (g/\Theta_{32})(\Theta_{18} - \Theta_{32})h_0/U_{32}^2$$

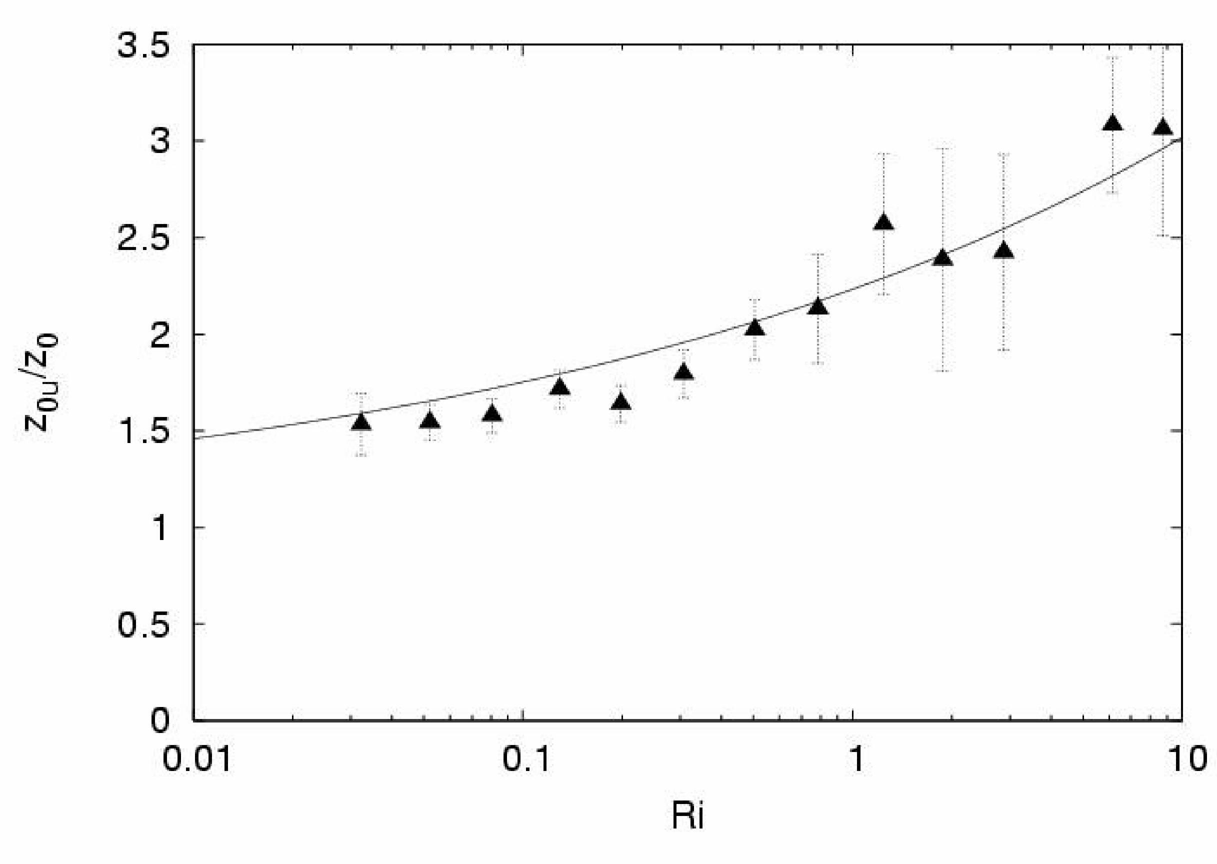
Replacing the temperature difference and the mean wind velocity with the free convective forms for the potential temperature and velocity profiles, in combination with $z_{0u} \sim h_0(-h_0/L)$, we have

$$Ri \sim (h_0/L)^{14/9}$$

And then $$\frac{z_{0u}}{z_0} = 1 + C_* Ri^{3/14}$$

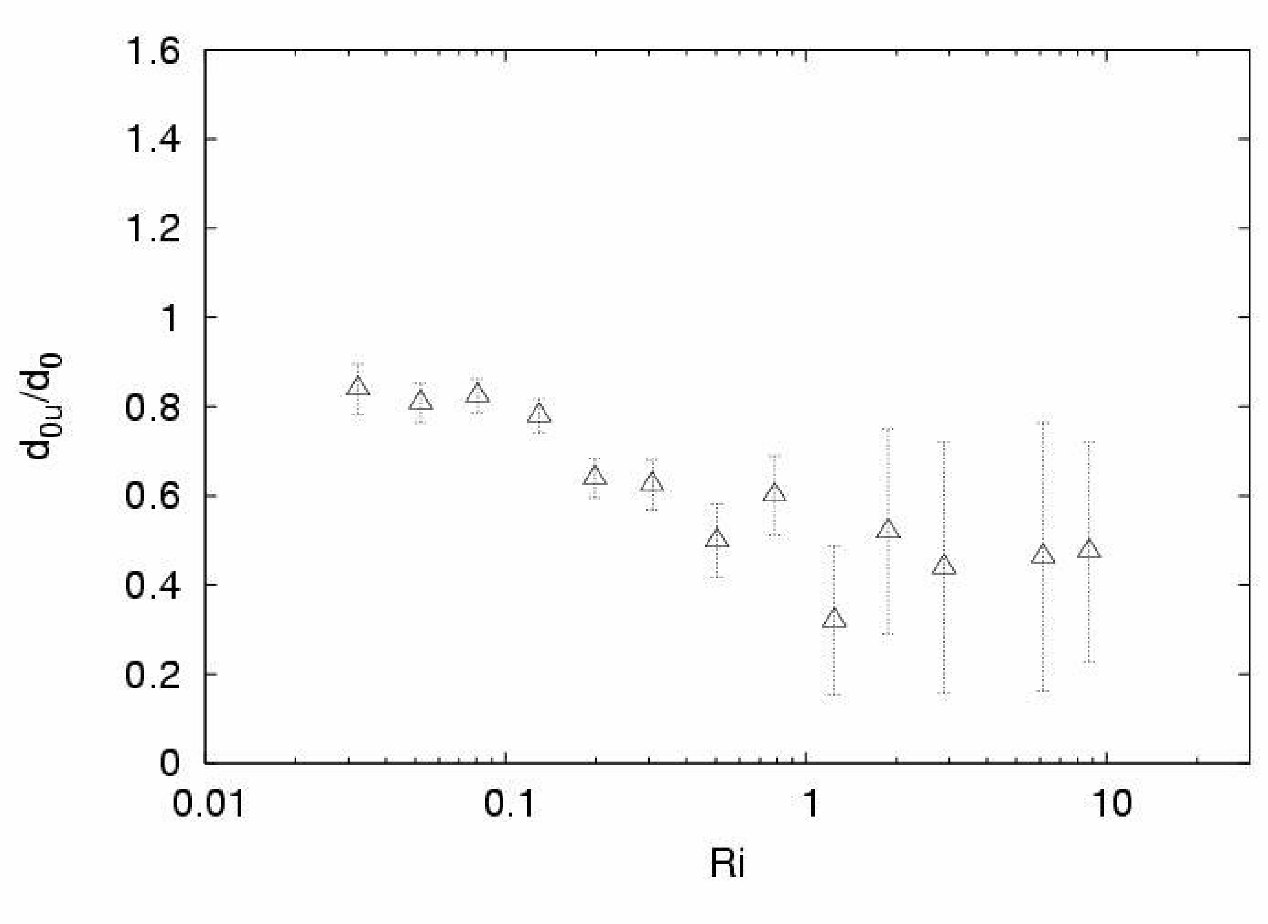
where C_* is a new empirical coefficient

Unstable stratification



$$\boxed{\frac{z_{0u}}{z_0} = 1 + 1.23 Ri^{3/14}}$$

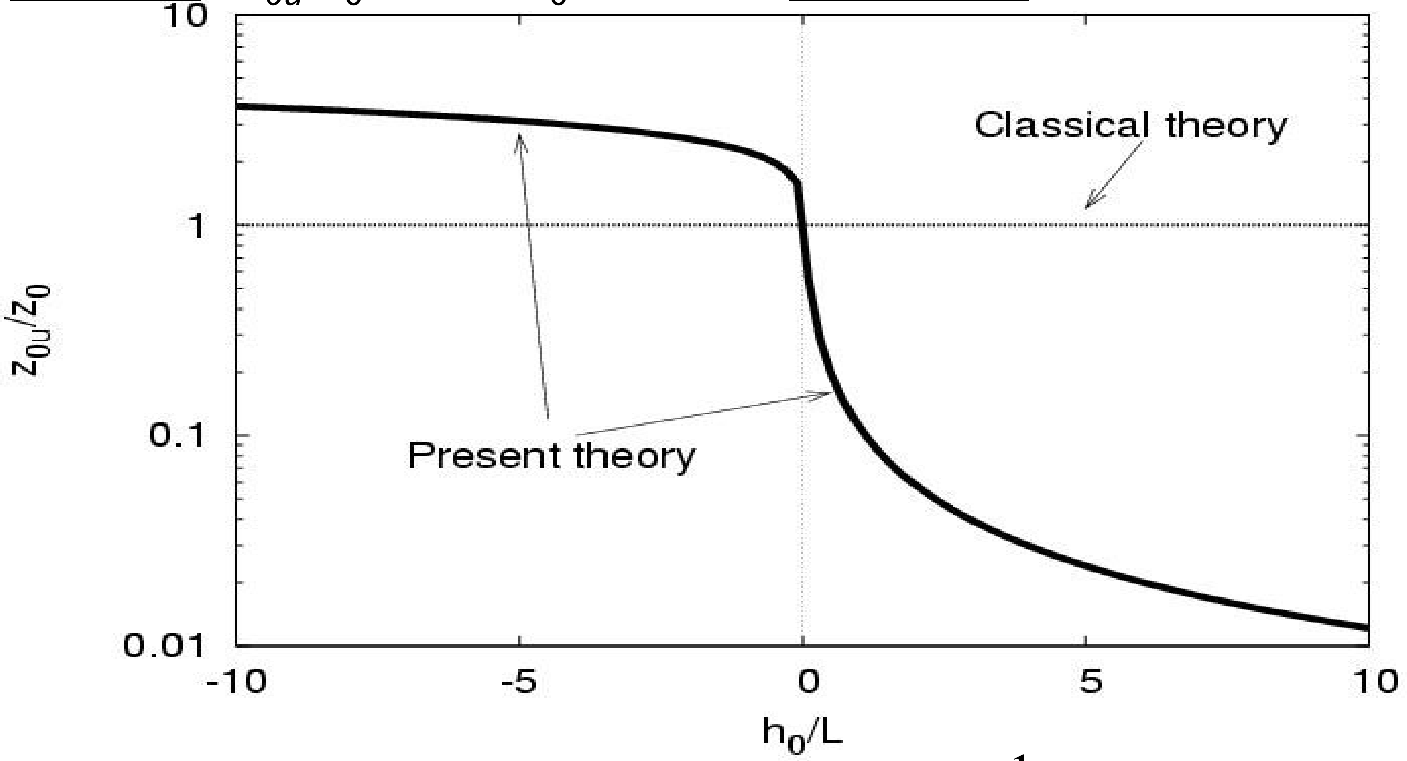
Unstable stratification



STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the “meteorological interval” $-10 < h_0/L < 10$ after new theory and experimental data

Solid line: z_{0u}/z_0 versus h_0/L **Dotted line:** traditional formulation $z_{0u} = z_0$



Neutral ⇔ stable

$$\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$$

Neutral ⇔ unstable

$$\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$

Conclusions: 1.2 Roughness length

- **Traditional concept:** roughness length fully characterised by geometric features of the surface

- **New theory and data:** essential dependence on hydrostatic stability especially strong in stable stratification

- **Applications:** to urban and terrestrial-ecosystem meteorology

- **Practically sound:** urban air pollution episodes in very stable stratification