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An Adaptive Unscented Kalman Filter

as an alternative to simplified EKF

Han The

given by Jelena Bojarova (theory) and
Nils Gustafsson (practical application)

Article:

An Adaptive UKF Algorithm for the State and Parameter Estimations of a Mobile robot.

Song Qi, Han Jian-Da.

Acta Automatica Sinica, 34, 72-79. (2008)

Non-linear models

$x_t = f(x_{t-1}) + \eta_{t-1}$
$y_t = h(x_t) + \varepsilon_t$

$Y_t = \{y_1, y_2, \ldots, y_t\}$

Deterministic data assimilation

$x^f_t = E(x_t|Y_{t-1});\ B^f_t = var(x_t|Y_{t-1});$
$x^a_t = E(x_t|Y_t);\ B^a_t = var(x_t|Y_t);$

Dynamic update

$B^f = var(f(x_{t-1})|Y_{t-1}) + Q_{t-1}$
$x^a_t = x^f_t + K_t (y_t - h(x^f_t)) =$
$= x^f_t + cov(x_t, h(x_t)) var(y_t - h(x^f_t))^{-1} (y_t - h(x^f_t))$
$B^a_t = B^f_t - K_t (\ F_t\ )^{-1} (K_t)^T$

Extended Kalman Filter

The first-order approximation

$$x_t = x^f_t + \delta x^a_{t-1} \approx f(x^a_{t-1}) + \frac{\partial f(x^a_{t-1})}{\partial x_{t-1}} \delta x^a_{t-1}$$

$$h(x_t) = h(x^f_t) + \frac{\partial h(x^f_t)}{\partial x_t} \delta x^f_t$$

$B^f_t = F B^a_{t-1} F^T + Q;$
$var(h(x_t)) = H B^f_t H^T$
$cov(x_t, h(x_t)) = B^f_t H^T$

Unscented Kalman filter

The second-order approximation

Ensemble Transform Kalman Filter

B^a_{t-1} → select: $Z^a_{t-1,i} = X_{t-1,i} - x^a_{t-1}$ → apply: $f(X_{t-1,i})$ → B^f_t → select: $Z^f_{t,i} = X_{t,i} - x^f_t$ → apply: $h(X_{t,i})$ → B^a_t

$Z^a_{t-1,i}$ from eig $(B^a_{t-1})_i$
$Z^f_{t,i}$ from eig $(B^f_t)_i$

$B^a_{t-1} = Z^a_{t-1} (Z^a_{t-1})^T$
$B^f_t = Z^f_t (Z^f_t)^T$
$Z^a_t = Z^f_t C$

An Adaptive Unscented Kalman Filter

$$w^m_0 = \frac{\lambda}{n+\lambda}$$

$$w^c_0 = \frac{\lambda}{n+\lambda} + (n-\alpha^2+\beta)$$

$$w^c_i = w^m_i = \frac{1}{2(n+\lambda)}$$

$$\lambda = n(\alpha^2-1)$$

Initialisation

$$x^a_0 = E(X_0)$$

$$B_0 = E((X_0 - x^a_0)(X_0 - x^a_0)^T)$$

Forecast

$$X^{f*}_{t,i} = f(x^a_{t-1} \pm ((n+\lambda)B^a_{t-1})^{1/2}_i)$$

$$x^f_t = \sum_{0\le i\le 2n} w^m_i \, X^{f*}_{t,i}$$

$$B^f_t = \sum_{0\le i\le 2n} w^c_i \, (X^{f*}_{t,i} - x^f_t)(X^{f*}_{t,i} - x^f_t)^T + Q_t$$

$$X^f_{t,i} = f(x^f_t \pm ((n+\lambda)B^f_t)^{1/2}_i)$$

Analysis

$$\gamma_{t,i} = h(X^f_{t,i})$$

$$y^f_t = \sum_{0\le i\le 2n} w^m_i \, \gamma_{t,i}$$

$$B_{yy,t} = \sum_{0\le i\le 2n} w^c_i \, (\gamma_{t,i} - y^f_t)(\gamma_{t,i} - y^f_t)^T + R$$

$$B_{xy,t} = \sum_{0\le i\le 2n} w^c_i \, (X^f_{t,i} - x^f_t)(\gamma_{t,i} - y^f_{t,i})^T$$

$$K_t = B_{xy,t}(B_{yy,t})^{-1}$$

$$B^a_t = B^f_t - K_t (B_{yy,t})^{-1}(K_t)^T$$

$$x^a_t = x^f_t + K_t(y_t - y^f_t)$$

An adaptive filtering

- Kalman filter is optimal only in the case of linear Gaussian systems
- Stable linear Gaussian system converge to a steady state solution
- At steady state solution innovation variance $(B_{yy,t} + R)$ is constant provided that the observation error variance R is constant

One way to optimize behavior of the system is to force the innovation variance vary less

$$v_t = B_{yy,t} = (H(B^{*f}_t) + Q_t)H^T + R$$

In the adaptive unscented Kalman filter scheme, all diagonal entries of Q_t are
corrected each assimilation step to minimize

$$V_t = \mathrm{trace}(\Delta S_t) = \mathrm{tr}((S_t - S^o_t))2$$

$S^o_t = \sum_{t-N+1 \leq i \leq t} v_i v_i^T$ (long term innovation variance),

$S_t = B_{yy,t}$ (estimated innovation variance from the ensemble spread)

$$\dot{q}^m_t = -\eta \frac{\partial V_t}{\partial q^m_t}$$

Result : innovation is too large in comparison to the spread of the ensemble; the model does not predict the observation well; model is wrong; increase model error and come closer to the next observation;

innovation is small in comparison to the model spread; the model predict the observation well; model is right; reduce model error and filter away observational noise stronger.