Surface parameterization

We use prognostic equations for soil temperature and soil moisture in two layers T_s, T_d, w_s, w_d. There is also a third layer with climatological temperatures and and soil moisture T_{cli}, w_{cli}. Furthermore we have a prognostic equation for snow depth H_{sn}.

The depth of the surface layer and the "deeper" layer are denoted D_1 and D_2 respectively. The numerical values are:

$D_1 = 7.2 \cdot 10^{-2}$ m and $D_2 = D_3 = 6 \cdot D_1 = 43.2 \cdot 10^{-2}$ m

Note that for sea $T_s = T_{sea} =$ constant i.e. the surface scheme is only used if $F_{land} + F_{ice} > 0$.

The equation for the upper layer temperature:

$$\frac{\partial T_s}{\partial t} = \frac{1}{\rho_s\, c_s\, D_1} \sum_i \Phi_i + \frac{\kappa_0\,(1 - k_{sn}\, F_{sn})\,(T_d - T_s)}{0.5\, D_1\,(D_1 + D_2)}$$

$$\sum_i \Phi_i = \Phi_r + \Phi_s + \Phi_q \;\; ; \;\; F_{sn} = min(H_{sn}/H_{snc}, 1)$$

T_s and T_d= temperatures in upper and second layer, H_{sn}, H_{snc} = snow height in water equivalent, ρ_s = soil density, c_s = soil heat capacity, κ_0 = heat diffusivity for snowfree soil, k_{sn} = tuning constant, F_{sn} = fraction of snow cover, Φ_r, Φ_s, Φ_q = net absorbed flux density ($J\, s^{-1}\, m^{-2}$) due to radiation, sensible and latent heat respectively.

Values: $H_{snc} = 0.015$, $\rho_s \cdot c_s = 2.7 \cdot 10^6$, $\kappa_0 = 7.5 \cdot 10^{-7}$, $k_{sn} = \frac{2}{3}$

The second term models the heat conduction into the soil.

1 The surface layers

D1	Ts(t)	Ws(t)
D2	Td(t)	Wd(t)
D3	Tclim=Const.	Wclim=Const.

$$D_1 = 7.2 \cdot 10^{-2} \text{ m and } D_2 = D_3 = 6 \cdot D_1 = 43.2 \cdot 10^{-2} \text{ m}$$

The equation for the second layer temperature:

$$\frac{\partial T_d}{\partial t} = -\frac{(T_d - T_s)\,\kappa_0}{0.5\,D_1\,(D_1 + D_2)} + \frac{(T_{scli} - T_d)\,\kappa_0}{D_2\,D_3}$$

Here we have only heat conduction, forcing the second layer temperature. Note the restoring term from climatology.

The equation for the upper layer soil moisture:

$$\frac{\partial w_s}{\partial t} = (1 - F_{sn})\,\Phi_q + P_{rn} + M_{sn} + \frac{(w_d - w_s)\,\lambda}{0.5\,D_1\,(D_1 + D_2)}$$

Here we have source terms due to precipitation (P_{rn}) and snow melting (M_{sn}), and $\lambda = 1. \cdot 10^{-7}$ is the diffusivity for conduction of soil water.

The equation for the second layer soil moisture:

$$\frac{\partial w_d}{\partial t} = -\frac{(w_d - w_s)\,\lambda}{0.5\,D_1\,(D_1 + D_2)} + \frac{(w_{scli} - w_d)\,\lambda}{D_2\,D_3}$$

Again we have a restoring force from climatology, which is important to keep the moisture "on the track". Note that both w_s and w_d are less than or equal to the saturation value $w_{smax} = 0.02$ m. The rest is regarded as runoff. The degree of saturation or *surface wetness* G is (over land) given by:

$$G = F_{sn} + (1 - F_{sn})[a_1 + (1 - a_1)\,(w_s/w_{smax})^{a_2}]$$

Here $a_1 = 0.05$ and $a_2 = 8$.
G is used to determine the lower boundary value of q_s over land in the vertical diffusion:

$$q_s = min\,[G \cdot q_{sat}(T_s) + (1 - G) \cdot q_n\,,\; q_{sat}(T_s)]$$

The equation for the snow height:

$$\frac{\partial H_{sn}}{\partial t} = F_{sn}\,\Phi_q/\rho_{h2O} \; + \; P_{sn} - M_{sn}$$

The computation of M_{sn} is done using the preliminary value of T_s, denoted $\tilde{T}_s$, estimated from the equation for the surface temperature according to:

$$M_{sn} = min\,[\frac{\rho_s\,c_s\,D_1\,(\tilde{T}_s - T_{melt})}{2\,\Delta t\,L_i\,\rho_{h2O}}\,,\; H_{sn}]$$

2 Numerical treatment:

To prevent numerical instability the surface temperature equation is treated implicitely according to the following:

$$\frac{\partial T_s}{\partial t} = f(T_s)$$

$$f^{\tau+1} \simeq f^{\tau-1} + \frac{\partial f^{\tau-1}}{\partial T_s} \left({T_s}^{\tau+1} - {T_s}^{\tau-1}\right)$$

A centered implicit time step then yields:

$$\frac{{T_s}^{\tau+1} - {T_s}^{\tau-1}}{2\,\Delta t} = \frac{f^{\tau+1} + f^{\tau-1}}{2}$$

Finally we get:

$${T_s}^{\tau+1} = {T_s}^{\tau-1} + \frac{2\,\Delta t\, f^{\tau-1}}{(1 - \Delta t \cdot \partial f^{\tau-1}/\partial T_s)}$$

This means that we must estimate the partial derivatives with respect to T_s of the terms Φ_r, Φ_s and Φ_q. The first derivative is estimated in RADIA:

$$\frac{\partial \Phi_r}{\partial T_s} = 4\,\epsilon_s\, b_a\, {T_s}^3$$

The other two derivatives are estimated in VDIFF and transferred to SURF.