Turbulence

Turbulent mixing is a 3-dimensional process, but horizontal mixing between gridsquares is negligible. (There is a process called *horizontal diffusion* which is present in the models, but it is needed for numerical reasons only).

1 Vertical diffusion

This is a very important physical process in the numerical models. The reasons for this is as following:

- The fluxes from the ground and sea are in some situations very large, and the mixing of temperature (e.g. static energy) and humidity, especially in the boundary layer, strongly effects the condensation processes.
- This process is the main sink of momentum in the model,(in HIRLAM we have at present no gravity wave drag parameterization) and this effects the cross-isobaric flow in the boundary layer, which is important for the cyclone development.

2 The closure problem

Look at the last term in the equation for $\overline{u}$:

$$\frac{\partial \overline{u}}{\partial t} = \cdots + \frac{1}{\rho} \frac{\partial}{\partial z} (-\rho \, \overline{u'w'})$$

It is possible to derive an equation for the time derivative of $\overline{u'w'}$, which, in that case, will contain terms of the type $\overline{u'w'v'}$. To compute the non-resolved terms in this way is called *second order closure.* A somewhat simpler way to do this is to lump some terms together and derive an equation for the time history of the *turbulent kinetic energy.* We have in HIRLAM developed schemes like that for the vertical diffusion, but it is not operational yet.

Here we will consider the current scheme (Louis, 1979), which is based on *first order closure.* By that we mean that the unresolved terms are directly expressed in the resolved i.e. $\overline{u}, \overline{w}$ etc.

We call that K-theory. This means that a variable Ψ which means u, v, q or $S = c_p T$ (S is conservative like potential temperature, Θ) has a time derivative due to vertical diffusion ($VDIFF$) according to:

$$\left(\frac{\partial \Psi}{\partial t}\right)_{VDIFF} = \frac{1}{\rho} \frac{\partial}{\partial z} J_\Psi$$

Here J_Ψ is the *flux* of the variable Ψ and it is assumed that this can be modelled according to:

$$J_\Psi = \rho K_\psi \frac{\partial \Psi}{\partial z}$$

where K_ψ is the so called *exchange coefficient*. To this equation we must have two boundary conditions:

$J_\Psi = 0 \quad ; \quad p = 0$

The boundary condition at the surface is based on a *drag coefficient* formulation:

$J_\Psi = \rho C_\Psi |\mathbf{V}| (\Psi(z) - \Psi_s) \quad ; \quad p = p_s$

Here z is the height of the lowest model level.

3 The condition at the surface:

The drag coefficient C_Ψ is written as:

$$C_\Psi = \left(\frac{k}{ln(z/z_0)}\right)^2 f_\Psi (R_i z/z_0)$$

Here f_Ψ is formulated differently in stable and unstable situations:

$f_\Psi = 1 - \frac{a_\Psi b R_i}{1 + 3 b c [k/ln(z/z_0)]^2 \sqrt{1 + z/z_0 - R_i}} \qquad R_i \leq 0$

$$\left.\begin{array}{l} f_{ms} = \frac{1}{1+2\,b\,R_i/\sqrt{1+dR_i}} \\ \\ f_{ss} = f_{qs} = \frac{1}{1+3\,b\,R_i\,\sqrt{1+dR_i}} \end{array}\right\} \quad R_i > 0$$

The constants are: $a_m = 2$, $a_s = a_q = 3$, $b = c = d = 5$;
The Richardson number R_i in our discretizised system (Δ means differences between levels of the model) can be written:

$$R_i = \frac{\Delta\Phi_v\,(c_p\,\Delta T_v + \Delta\Phi_v)}{c_p\,T_v\,|\Delta\mathbf{V}|^2}$$

Here T_v is the virtual temperature $T_v = T(1 + (1/\epsilon - 1)q)$

4 The condition in the free atmosphere:

The echange koefficient K_ψ mentioned above is given by:

$$K_\Psi = l^2\,|\frac{\partial\mathbf{V}}{\partial z}|\,f_\Psi\,(R_i,\,\frac{\Delta z}{z})$$

where f_Ψ is slightly modified as compared to the surface case. The mixing length l is formulated according to the so called Blackadar (1962) formula:

$$l = \frac{kz}{1 + kz/\lambda(z)}$$

$$\lambda(z) = \begin{cases} \lambda_b\,; \quad z < H \\ \\ \lambda_0 + (\lambda_\beta - \lambda_0)\,exp(-\frac{(z-H)}{H})\,; \quad z \geq H \end{cases}$$

Here λ_0 is the asymptotic mixing length for the free atmosphere. Current parameter values are:

λ_0=30 m, λ_β=300 m and H=1000 m

5 Shallow convection

To enhance the vertical mixing in moist air, mainly to avoid too shallow and too moist boundary layers, a modification to the Richardson number is introduced. This is called *shallow convection*, but is merely a modification of the exchange coefficients. This modification is not used for the vertical diffusion of momentum.

6 Numerical solution of the diffusion equation

The vertical diffusion for a varible Ψ can formally be written:

$$\Psi^{\tau+1} - \Psi^{\tau-1} = 2\,\Delta t \,\frac{\partial}{\partial z}\, K_\Psi \,\frac{\partial \Psi}{\partial z}$$

The crucial thing is at what time level the right hand side is evaluated. Since vertical diffusion is a very rapid process we would need a very short time step to describe it properly. That is, however too expensive and, in order to use a larger timestep, the equation is treated *implicitely* i.e. the derivative of Ψ is evaluted as:

$$\Psi = A\Psi^{\tau+1} + (1-A)\Psi^{\tau-1} \qquad A = 1.5$$

Note that the exchange coefficient K_Ψ is computed at $\tau - 1$. This can create some problems since the exchange coefficients can vary over orders of magnitudes.

The discretized version of this equation leads to a tri-diagonal system, which is easy to solve (Richtmeyer and Morton).

7 Surface fluxes

To compute the surface fluxes we need values at the surface of temperature and humidity (wind is zero by definition). Since each gridsquare can contain land and/or sea, the fluxes at the surface is computed as a weighted value. Thus we store both a land surface temperature and the temperature for the sea in each gridsquare. The same for the specific humidity, the value of the sea is taken as $q_{sat}(T_{sea})$.

Note that the sea temperature is constant during the forecast, while the land surface variables have a time dependency (i.e. determined by the surface scheme). We have only one value at the lowest model level, but different surface values (land/sea), and we can thus compute different diagnostically interpolated surface layer values over the different fractions of the gridsquare. (2 m temperature and humidity, 10 m wind, see below).

Note that the surface roughness length over sea is computed diagnostically using the Charnock formula:

$$z_0(sea) = max[\beta\, {u_*}^2/g\, ,\, 1.5 \cdot 10^{-5}]\, ; \quad \beta = 0.018$$

8 Diagnostic calculation of near surface parameters

Since 2-m temperature and humidity as well as 10 m winds are not on model levels, and these parameters are of general interest, we compute interpolated values between the surface and the lowest model level. This is done using the Monin-Obukhov similarity theory.

Stable case:

$$u(z) = (u_*/k)\, ln(z/z_0)\, +\, u_n\, (1 - exp(-b_m\, k^{-1}\, u_*\, {u_n}^{-1}\, z\, L^{-1}))$$

$$v(z) = (v_*/k)\, ln(z/z_0)\, +\, v_n\, (1 - exp(-b_m\, k^{-1}\, v_*\, {v_n}^{-1}\, z\, L^{-1}))$$

$$T(z) = T_s\, +\, (\Theta_*/k)\, ln(z/z_0)\, +\, \Delta\Theta_n\, (1 - exp(-b_h\, k^{-1}\, \Theta_*\, {\Theta_n}^{-1}\, z\, L^{-1}))$$

$$q(z) = q_s\, +\, (q_*/k)\, ln(z/z_0)\, +\, \Delta q_n\, (1 - exp(-b_h\, k^{-1}\, q_*\, {q_n}^{-1}\, z\, L^{-1}))$$

Here u_*, v_* is friction velocity i.e. $-\overline{u'w'}, -\overline{v'w'}$, $\Theta_* = \overline{u'\theta'}/u_*$ and $L = -\frac{{u_*}^2\, T_0}{k\, g\, \Theta_*}$ is the Obukhov length. Von Karmans constant is denoted as k and Δ means differences between lowest model layer and surface values (=0 for wind). $b_m = b_h = b_q \simeq 4$.

Unstable case:

$$u(z) = (u_*/k)\,[ln(z/z_0)\; - \;(ln(\frac{1+x^2}{2}) + 2\,(ln(\frac{1+x}{2}) - 2\,tan^{-1}x + \frac{\pi}{2})]$$

$$v(z) = (v_*/k)\,[ln(z/z_0)\; - \;(ln(\frac{1+x^2}{2}) + 2\,(ln(\frac{1+x}{2}) - 2\,tan^{-1}x + \frac{\pi}{2})]$$

$T(z) \cong\; c_\theta\,\Theta_*\,k^{-1}\,[ln(z/z_0)\; - \;2\,(ln(\frac{1+y}{2})]$

$q(z) =\; c_q\,q_*\,k^{-1}\,[ln(z/z_0)\; - \;2\,(ln(\frac{1+y}{2})]$

Where $x = (1 - 15\,z\,L^{-1})^{\frac{1}{4}}$, $x = (1 - 9\,z\,L^{-1})^{\frac{1}{2}}$ and $c_\theta = c_q = 1$