Towards improved treatment of lake stratification in FLake

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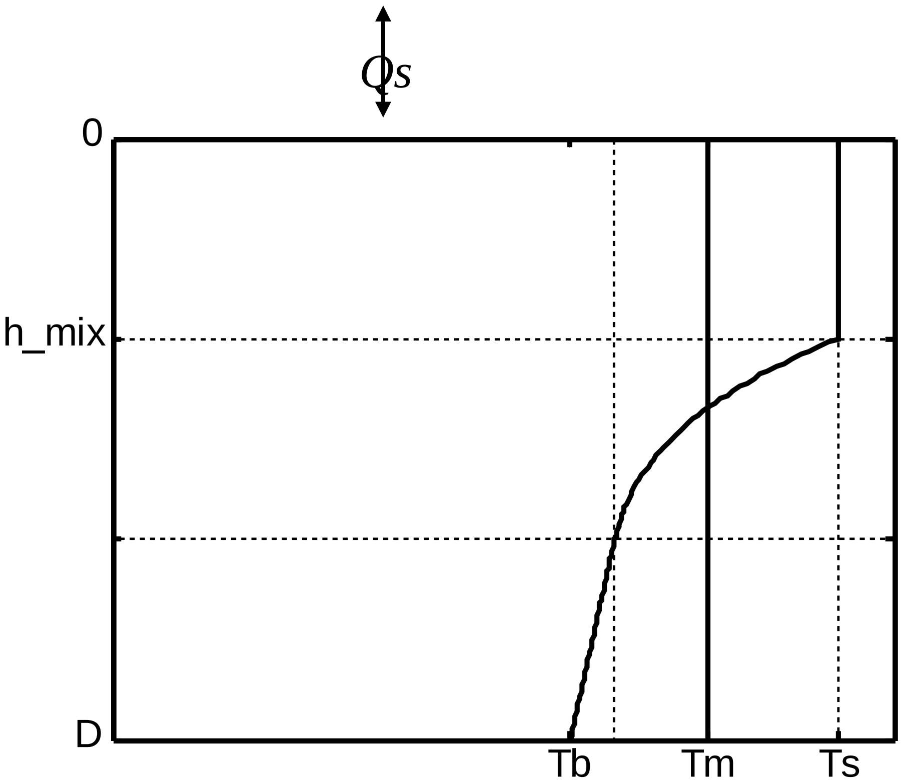
Outline

- Current treatment of stratification in FLake: hidden problems
- An alternative parameterization for the restratification parameter
- How to extend FLake on 3 layers:
  - The universal self-similarity function for the thermocline
  - A parameterization for the thermocline thickness

Background



The key task of a 1D lake model is establishing of relationship between Qs and Ts

If the lake is mixed, Ts = Tm, and

H dTs/dt = (Qs - Qb) ~ Qs

(where H is the lake depth)

Lakes stratify, especially the deep ones, -

a proper description of heat transfer in stratified media improves prediction of Ts and Qs

FLake utilizes one of the simplest parameterization for stratification

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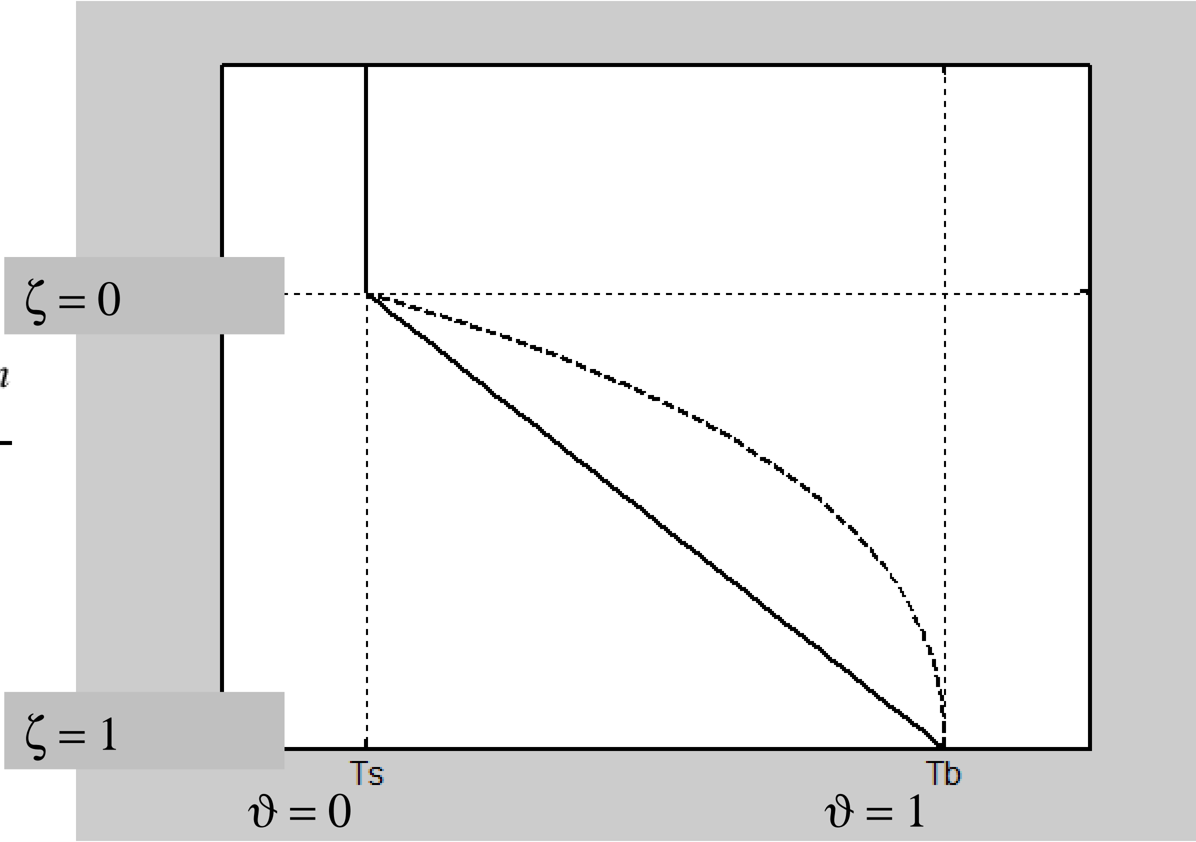
A time-variable self-similarity profile in a stratified lake

$$C_\vartheta \equiv \int_0^1 \vartheta(\zeta)\mathrm{d}\zeta$$

$$\frac{\mathrm{d}C_\vartheta}{\mathrm{d}t} = \mathrm{sign}(\dot{h})\frac{C_\vartheta^{max} - C_\vartheta^{min}}{t_*}$$

$\vartheta max \approx 0.8$

$\vartheta min \approx 0.5$



Profile relaxation parameter C_{rc} (c_relax_C)

$$C_\vartheta \equiv \int_0^1 \vartheta(\zeta)\mathrm{d}\zeta$$

$$\frac{\mathrm{d}C_\vartheta}{\mathrm{d}t} = \mathrm{sign}(\dot{h})\frac{C_\vartheta^{max} - C_\vartheta^{min}}{t_*}$$

$$\boxed{t_* = \frac{\Delta h^2 \bar{N}}{C_{rc} U_*^2}}$$

where $\dot{h} = \mathrm{d}h_{mix}/\mathrm{d}t$ is the entrainment rate, $\Delta h = (D - h_{mix})$ is the thickness of the lower stratified layer (thermocline), $\bar{N} = \Delta h^{-1} \int_{h_{mix}}^{h_{mix}+\Delta h} N\mathrm{d}z$ is the mean buoyancy frequency

$U_* = max(w_*, u_*)$ is the mixing velocity scale

C_{rc} is a dimensionless constant

Profile relaxation parameter C_{rc} (c_relax_C)

$$t_* = \frac{\Delta h^2 \bar{N}}{C_{rc} U_*^2}$$

- The relaxation time scale represents the time necessary for the temperature profile to completely change its shape from one self-similar limit to another, and scales with the thermal diffusion time across the thermocline layer of depth Δh,

$$t_* \propto (K_z \Delta h^{-2})^{-1},$$

- The thermal diffusion coefficient in the stratified media is the ratio of the TKE e to the stratification strength $\bar{N}$,

$$K_z = e\bar{N}^{-1},$$

- the turbulent kinetic energy e scales with the mixing rate at the top of the thermocline

$$e \propto U_*^2,$$

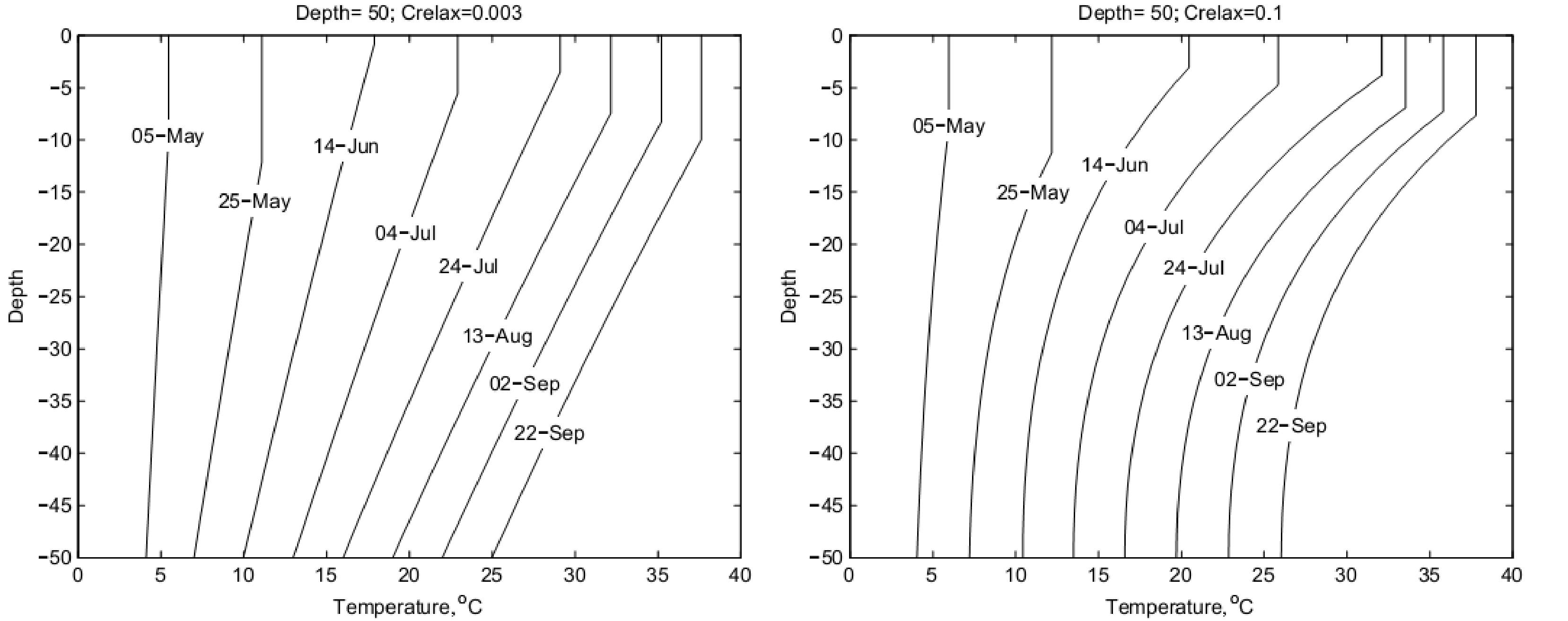
where $U_* = max(w_*, u_*)$.

Older versions of FLake: $C_{rc} = 0.1$

Current version of FLake: $C_{rc} = 0.003$

Temperature profile changes in a 50m-deep lake

$C_{rc} = 0.003$ $C_\vartheta \approx C_\vartheta^{min}$ all through the most of the summer



the model:

e relaxation time is too slow in deep lakes

reality:

e relaxation time is independent of lake depth

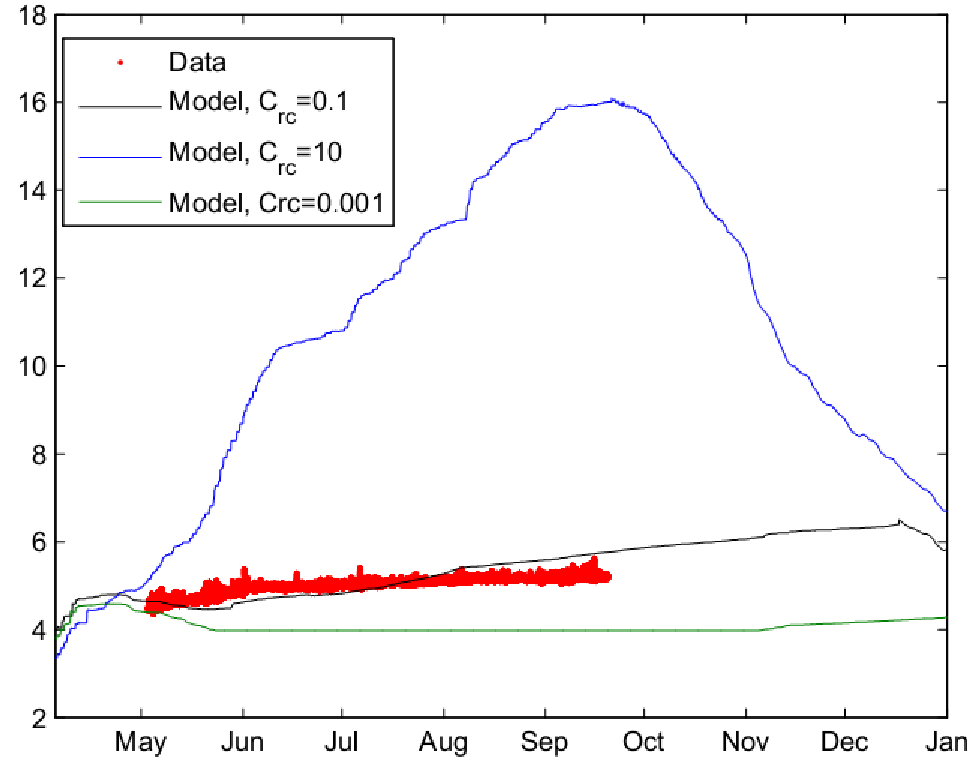
er versions of FLake: $C_{rc} = 0.1$

ent version of FLake: $C_{rc} = 0.003$

How a wrong choice for Crc affects the key output parameters of the model?

Lake bottom temperatures at different Crc, a 50m-deep lake ('Lake Stechlin')

- Crc has to be tuned based on the lakes depth
- The *surface temperatures* are, probably, not strongly affected by the Crc, especially in shallow lakes
- But, the predictions for the *bottom temperatures*, *stratification* (*N*) and *mixed layer depth* can be completely wrong:



An alternative formulation for Crc

Here is one Δh too much

$$t_* = \frac{\Delta h^2 \bar{N}}{C_{rc} U_*^2}$$

- The relaxation time scale represents the time necessary for the temperature profile to completely change its shape from one self-similar limit to another, and scales with the thermal diffusion time across the thermocline layer of depth Δh,

$$t_* \propto (K_z \Delta h^{-2})^{-1},$$

- ~~The thermal diffusion coefficient in the stratified media is the ratio of the TKE e to the stratification strength $\bar{N}$~~,

$$\cancel{K_z = e\bar{N}^{-1},} \quad K_z \propto l\sqrt{e} \text{ and } l \propto \Delta h$$

- the turbulent kinetic energy e scales with the mixing rate at the top of the thermocline

$$e \propto U_*^2,$$

where $U_* = max(w_*, u_*)$.

$$\boxed{t_* = C_{rc} \frac{\Delta h}{U_*}}$$

Based on several validation model runs:

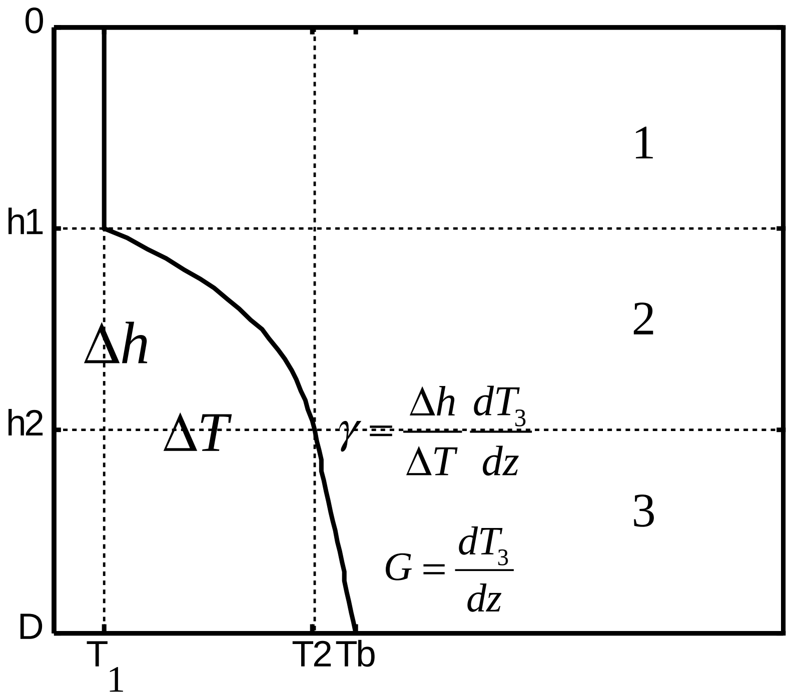
$$C_{rc} \approx 10$$

Intermediate conclusions

- When using the current version of FLake, the relaxation parameter Crc has to be tuned to lake depth

- Alternatively, the proposed parameterization for the profile relaxation time can be adopted. The new relaxation constant is about 10, subject to validation on lakes of different depths.

A concept for three-layer model



1. Introducing a quiescent layer below the thermocline (hypolimnion);
2. The simplest parameterization for the temperature profile in the hypolimnion: linear, $T_3(z)=G*z$
3. A relationship is needed between G and the shape of the temperature profile in the thermocline $T_2 = f(\gamma)$
4. Two new unknown variables arise:
   - Temperature gradient in the hypolimnion G
   - Thickness of the thermocline h_2

Self-similar temperature profile in the thermocline at variable temperature gradient below

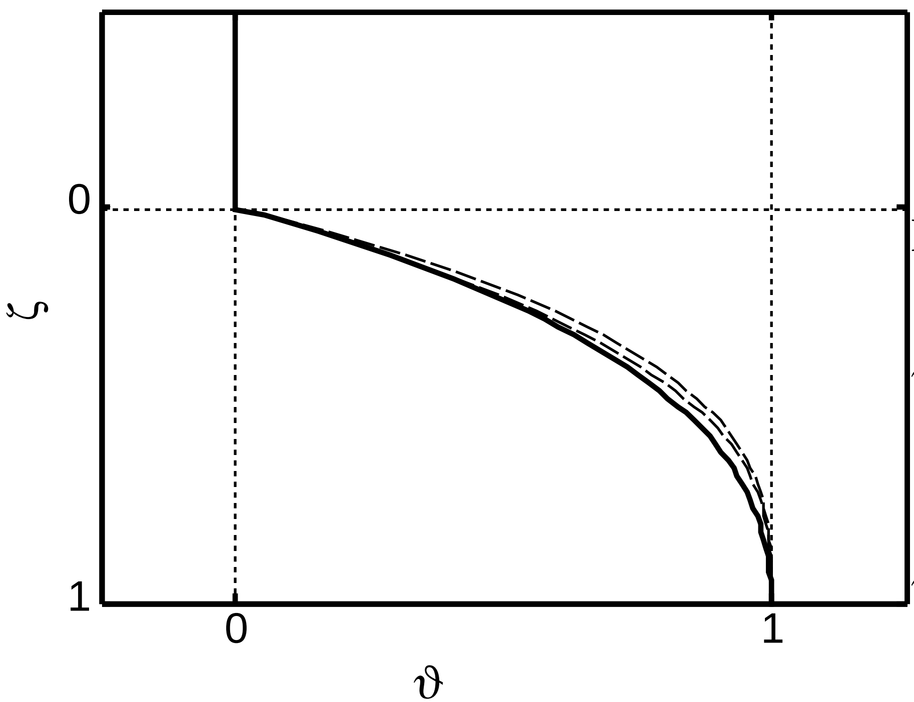
$$\gamma = \frac{\Delta h}{\Delta T}\frac{dT_3}{dz}$$

$$\vartheta(\zeta) = \zeta \exp\left[(1-\zeta)(1-\gamma)\right]$$

At $\gamma = 0$:

$$\vartheta(\zeta) = \zeta \exp(1-\zeta)$$

$[d\vartheta/d\zeta|_{\zeta=0} = e]$



Dashed lines:

$\vartheta = 1-(1-\zeta)^3$ $[d\vartheta/d\zeta|_{\zeta=0} = 3]$

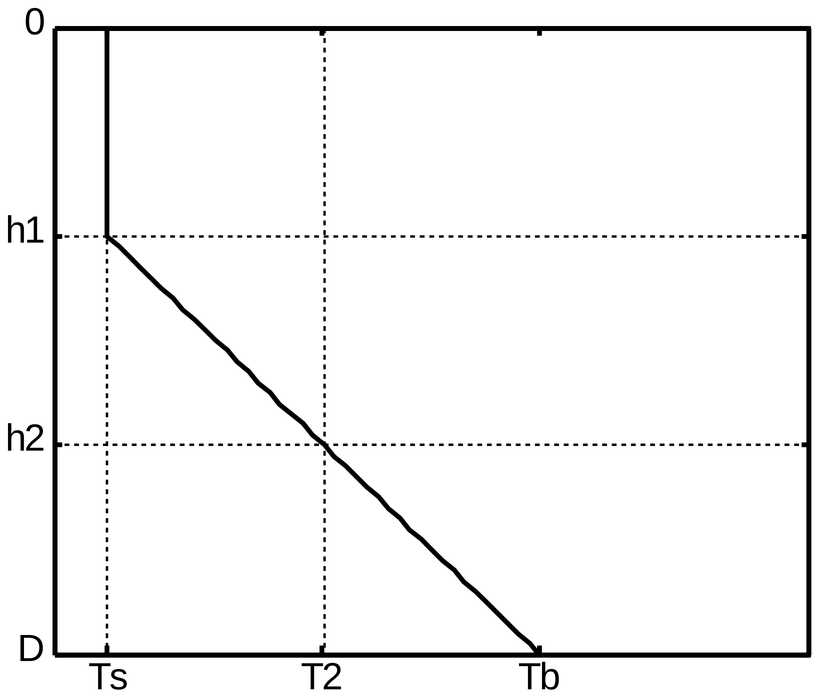
(Arsenyev and Felsenbaum 1977)

$\vartheta = 8/3\zeta - 2\zeta^2 + \zeta^4/3$; $[d\vartheta/d\zeta|_{\zeta=0} = 8/3]$

(Kitaigorodski and Miropolski 1970)

$t\ \gamma = 1: \quad \vartheta = \zeta$

$$\vartheta(\zeta) = \zeta \exp\left[(1-\zeta)(1-\gamma)\right]$$

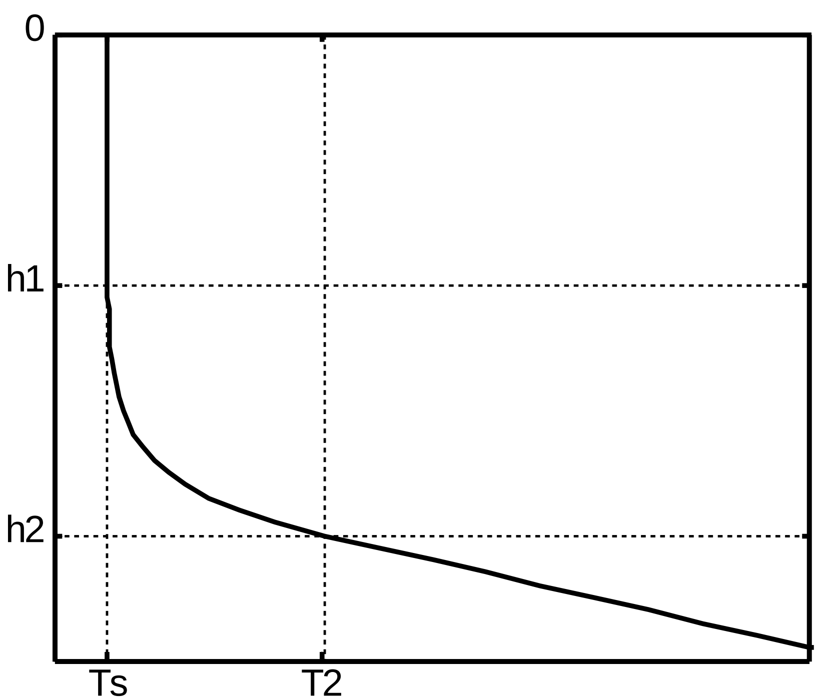


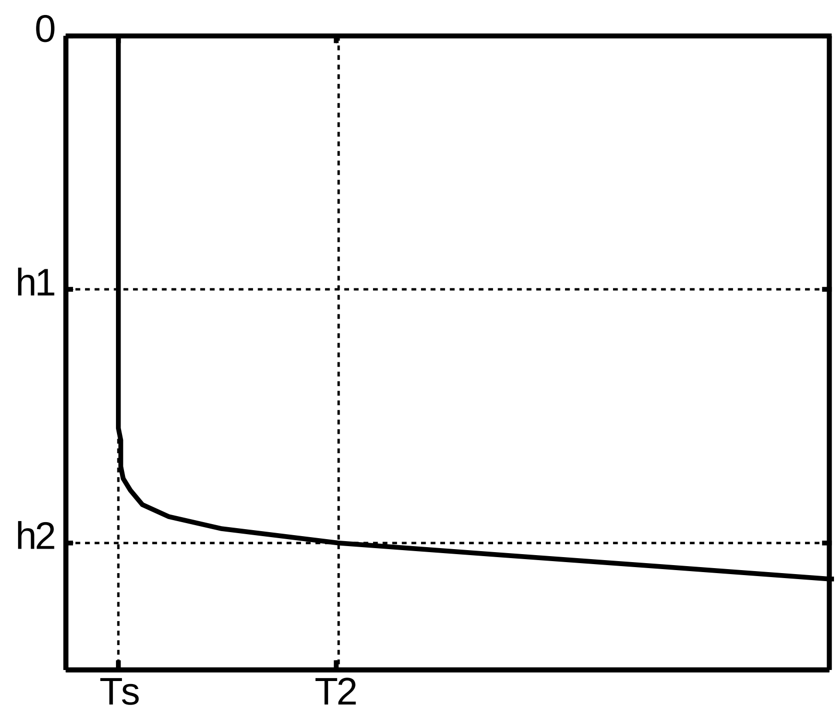
The classic 1st order (linear) entrainment layer approximation Lilly (1960ties-now)

At $\gamma \to \infty$:

$$\vartheta(\zeta) = \zeta \exp\left[\left(1-\zeta\right)\left(1-\gamma\right)\right]$$

$$C_\vartheta \equiv \int_0^1 \vartheta(\zeta)\,\mathrm{d}\zeta \to 0$$

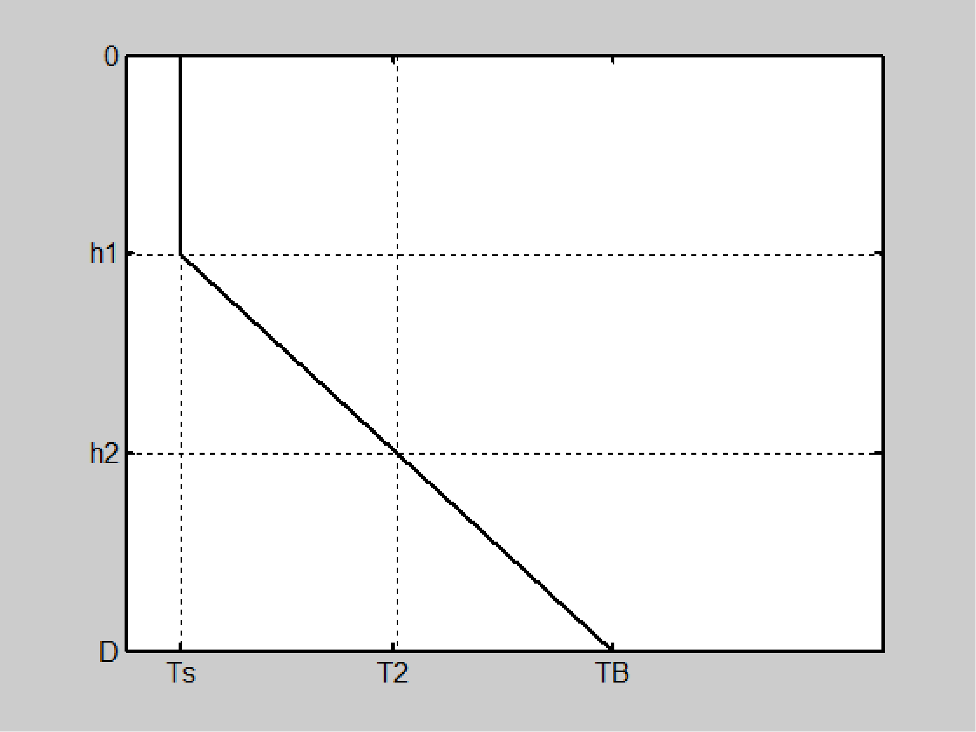




$$\vartheta(\zeta) = \zeta \exp\left[(1-\zeta)(1-\gamma)\right]$$

The formula captures the main features of the thermocline in the whole range of background stratification $0 < \gamma < \infty$

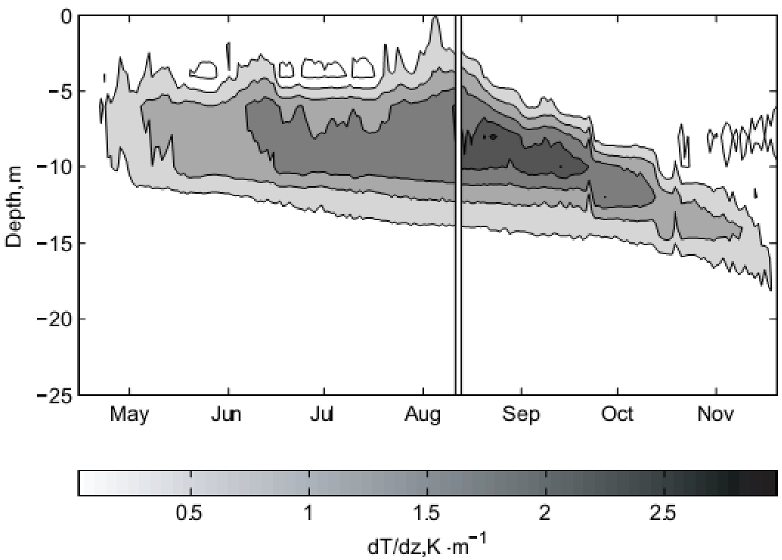
It seems like an exact solution of the heat equation at certain constraints. The corresponding problem statement is not found yet.



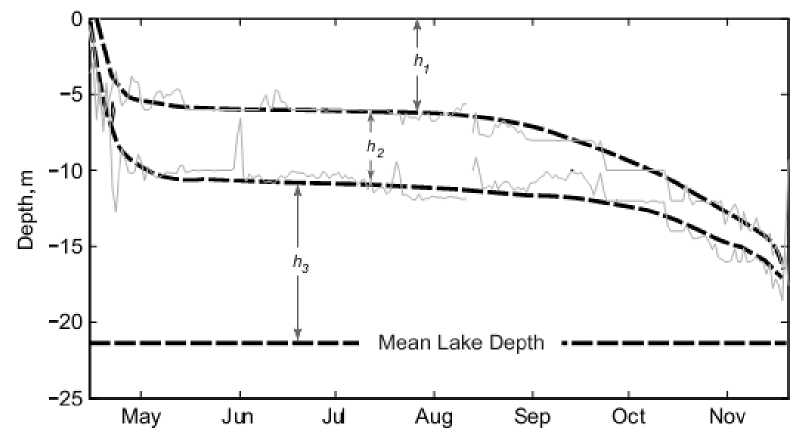
A parameterization for the thermocline thickness h_2

- h_1 remains at about 5-10 m during the stratified period. It tends to be smaller in shallow lakes, but becomes independent of the lake depth in deeper lakes.
- In deep lakes: h_2 is of the same order as h_1.
- When h_1 grows during the autumn cooling, h_2 becomes small.

Vertical temperature gradient dT/dz



Thickness of the thermocline, h_2



Previous parameterizations:
$h_2 \sim 0.3h_1$, or $h_2 \propto h_1 \mathrm{Ri}_0^{-1}$ (Deardorff et al., 1980), where

$$\mathrm{Ri}_0 = \frac{g' h_0}{w_*^2}$$

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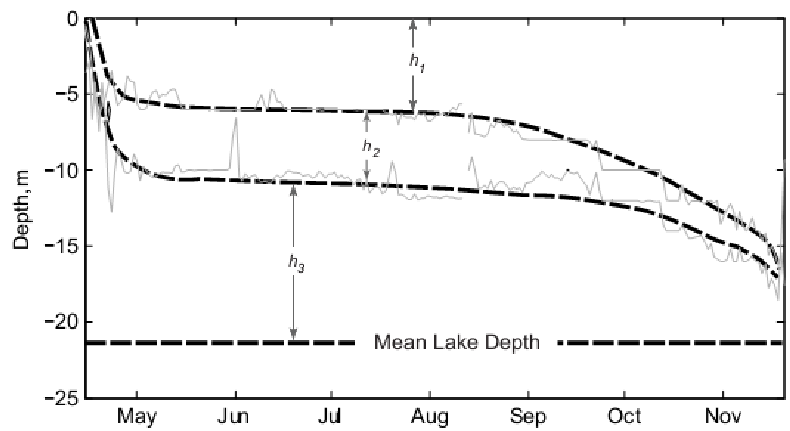
- h_1 remains at about 5-10 m during the stratified period. It tends to be smaller in shallow lakes, but becomes independent of the lake depth in deeper lakes.

- In deep lakes: h_2 is of the same order as h_1.

- When h_1 grows during the autumn cooling, h_2 becomes small.

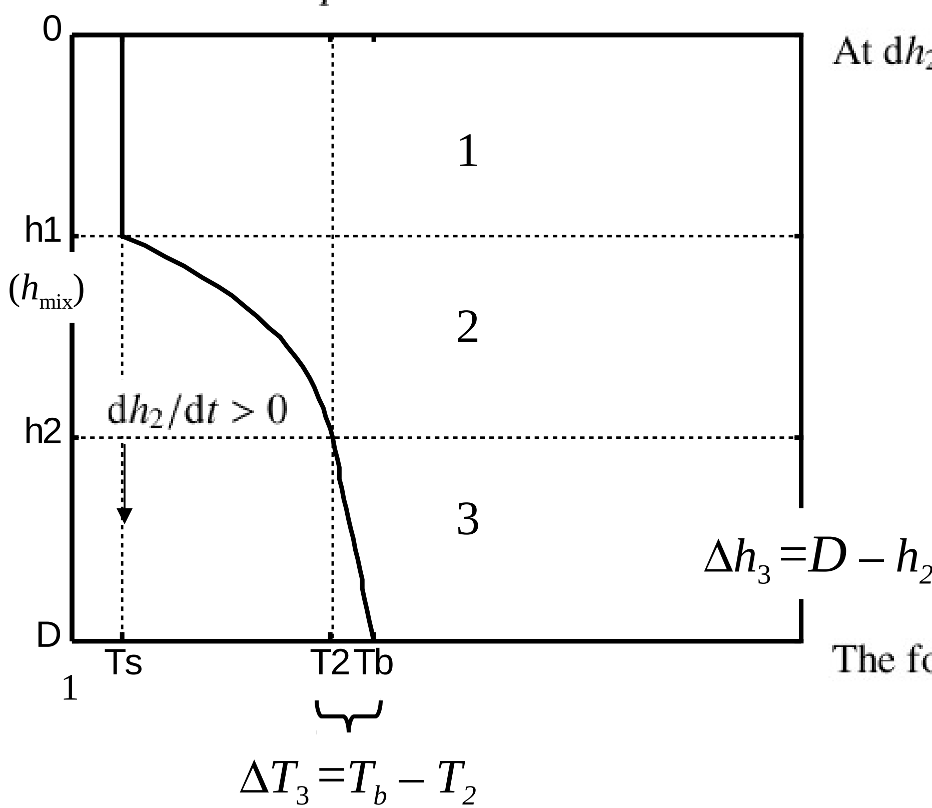
$$\boxed{h_2 \propto h_{eq} \propto \sqrt{\frac{h_1 h_2}{h_1 + h_2}}}$$

$$\bar{N} h_2 \sim C_g, \quad \text{where } C_g = \sqrt{g' h_{eq}}.$$



Closure at the lake bottom

Temperature at the thermocline's bottom, T_2 and bottom temperature, T_B



At $\mathrm{d}h_2/\mathrm{d}t > 0$:

$$\frac{\partial}{\partial t}\int_{h_2(t)}^{D} T(t,z)\mathrm{d}z + \frac{\mathrm{d}h_2(t)}{\mathrm{d}t} = \varkappa \frac{T_B(t) - T_2(t)}{D - h_2(t)}$$

$$T(t,z) = T_2(t) + \gamma(z - h_2(t))$$

$$\int_{h_2(t)}^{D} T(t,z)\mathrm{d}z = \bar{T}\Big|_{h_2}^{D} = \frac{1}{2}(T_2(t) + T_B(t))$$

$$\frac{\mathrm{d}T_B}{\mathrm{d}t} = -\frac{\mathrm{d}T_2}{\mathrm{d}t} + \gamma\left(\frac{\mathrm{d}h_2}{\mathrm{d}t} + \frac{2\varkappa_T}{\Delta h_3}\right)$$

The following is valid, if $\mathrm{d}h_2/\mathrm{d}t > 0$ only:

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \gamma\frac{\mathrm{d}h_2}{\mathrm{d}t},$$

Finally,

$$\frac{\mathrm{d}T_B}{\mathrm{d}t} = \gamma\frac{2\varkappa_T}{\Delta h_3} = 2\varkappa_T\frac{\Delta T_3}{\Delta h_3^2},$$

adjustable to take into account BBL turbulence

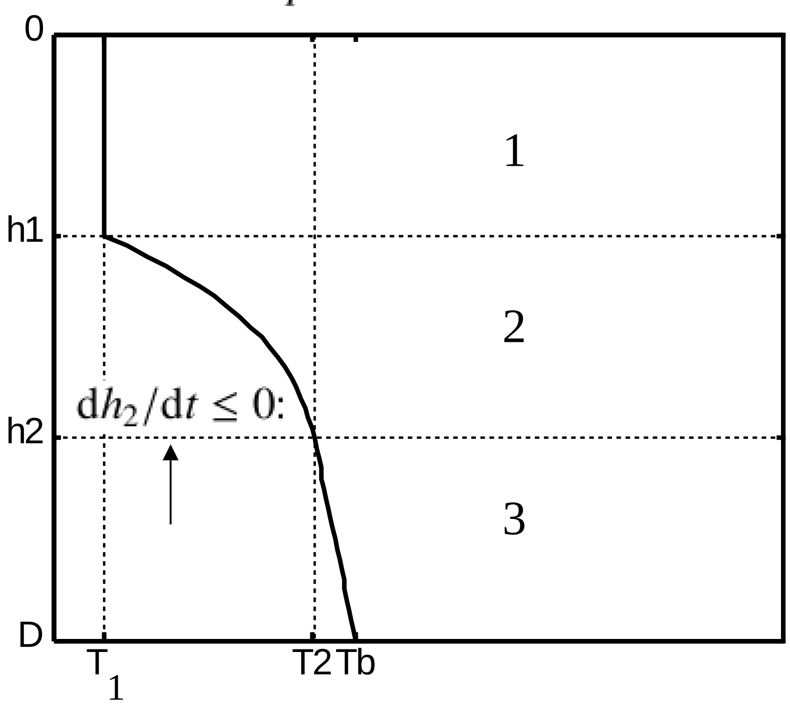
From typical orders of magnitude:
$\mathrm{d}T_B/\mathrm{d}t \sim 10^{-7}\mathrm{K/s} \sim O(1)\mathrm{K}/100\mathrm{days}$.

Summary

- When using Flake: think on the correct value of Crc! FLake can not predict stratification, it parameterizes the stratification
- An alternative for Crc is proposed allowing to avoid tuning
- Next step: a 3-layer FLake?

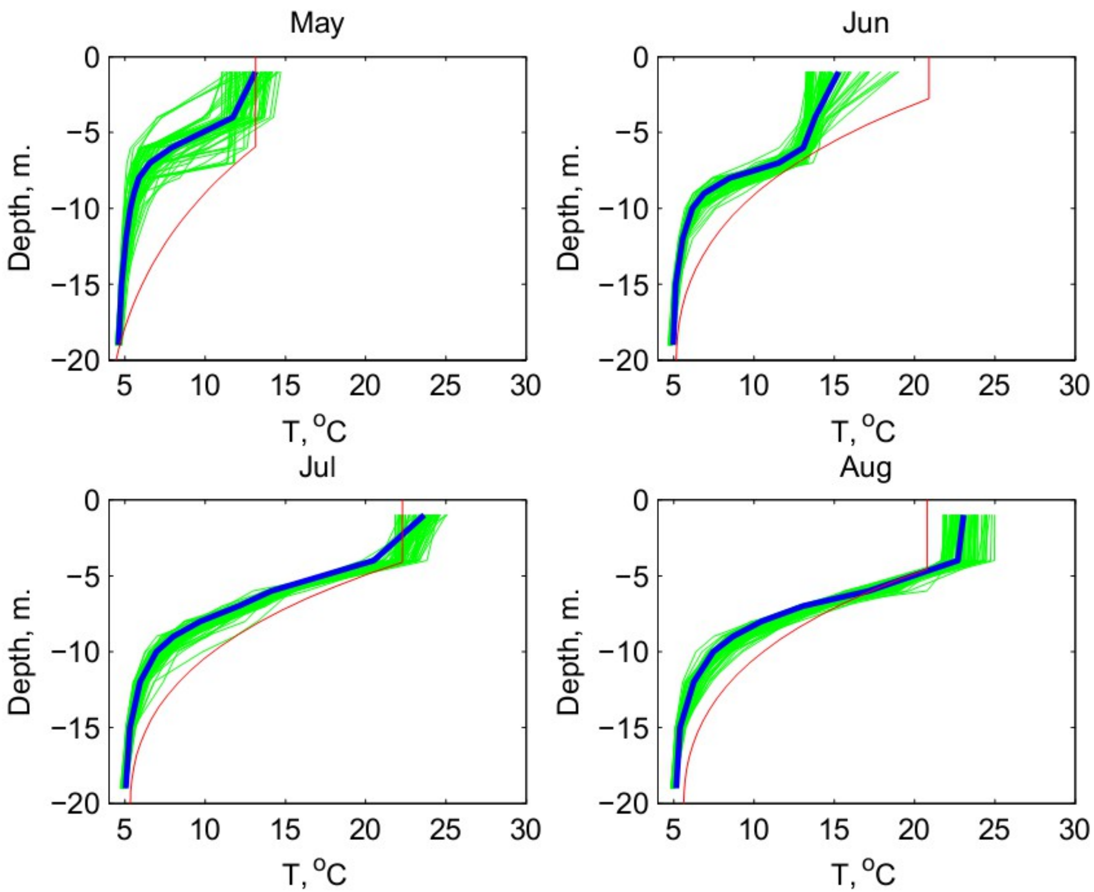
Closure at the lake bottom (continued) – remove?

Temperature at the thermocline's bottom, T_2 and bottom temperature, T_B



At $\mathrm{d}h_2/\mathrm{d}t \leq 0$:

$$\frac{\partial T_2}{\partial t} = \varkappa \frac{T_1 - T_2}{(h_2 - h_1)^2}$$


May
Jun
Jul
Aug
Depth, m.
0
−5
−10
−15
−20
5
10
15
20
25
30
T, °C