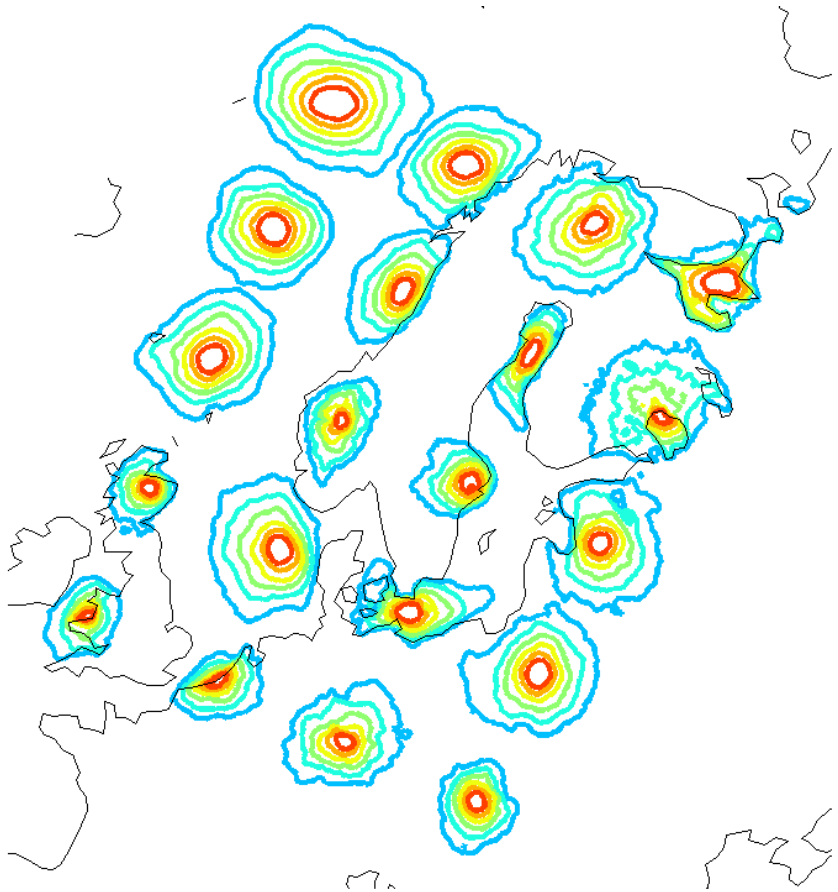
Wavelets as a framework for describing inhomogenity and anisotropy

Alex Deckmyn

Tomas Landelius

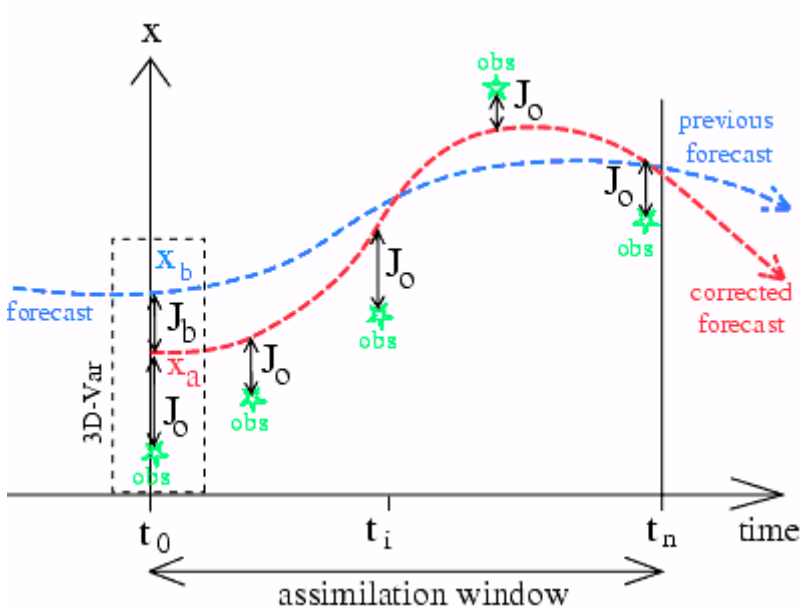
Anders Höglund



Variational data assimilation

Minimization of a cost function

$$J(x_0) = (x_0 - x_b)^T B^{-1} (x_0 - x_b) + \sum_{j=0}^{N} (y_j - H_j x_j)^T R^{-1} (y_j - H_j x_j)$$



SMHI

The importance of the *B* matrix

B contains the background error covariances:

$$B = E\left\{ (x_b - x_t)(x_b - x_t)^T \right\}$$

- **Weights background state against observations.**
- **Helps to impose balance to the assimilated fields.**
- **Smoothes out the observational information.**

Problems with the *B* matrix

It is HUGE: $\dim(B) = (n_x n_y n_z)^2 \approx 10^{14}$

- **Impossible to store explicitly.**

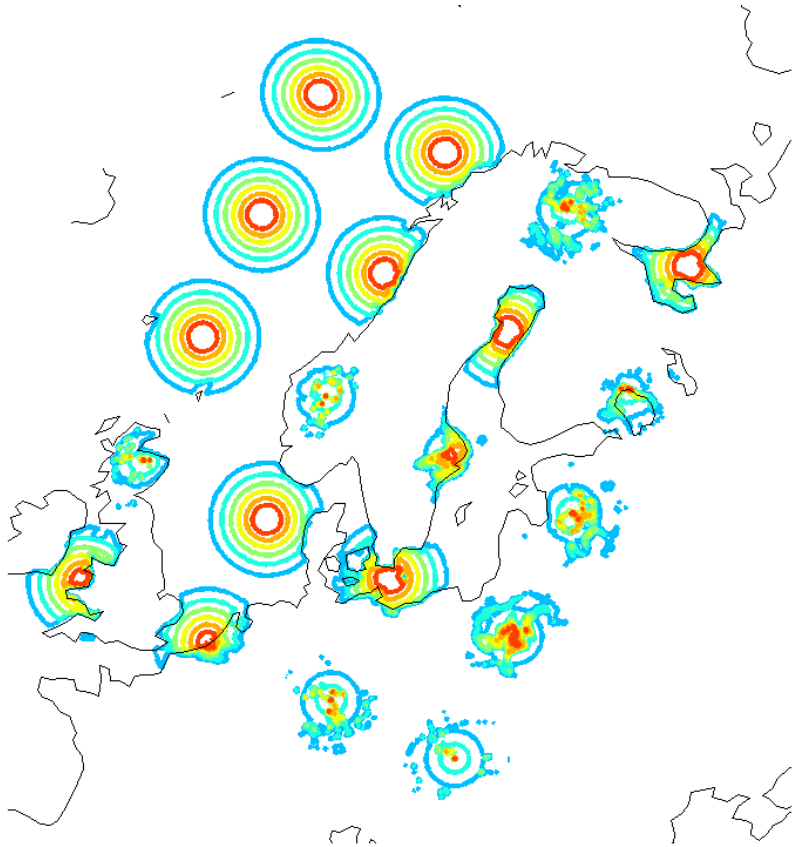
- **Impossible to fully determine.**

Optimal interpolation

Direct local solution to 3D-VAR

$$x_0(i) = x_b(i) + K_i\,(y - Hx_b)$$
$$K = BH^T(HBH^T + R)^{-1}$$

Local models of B that differ for different locations.



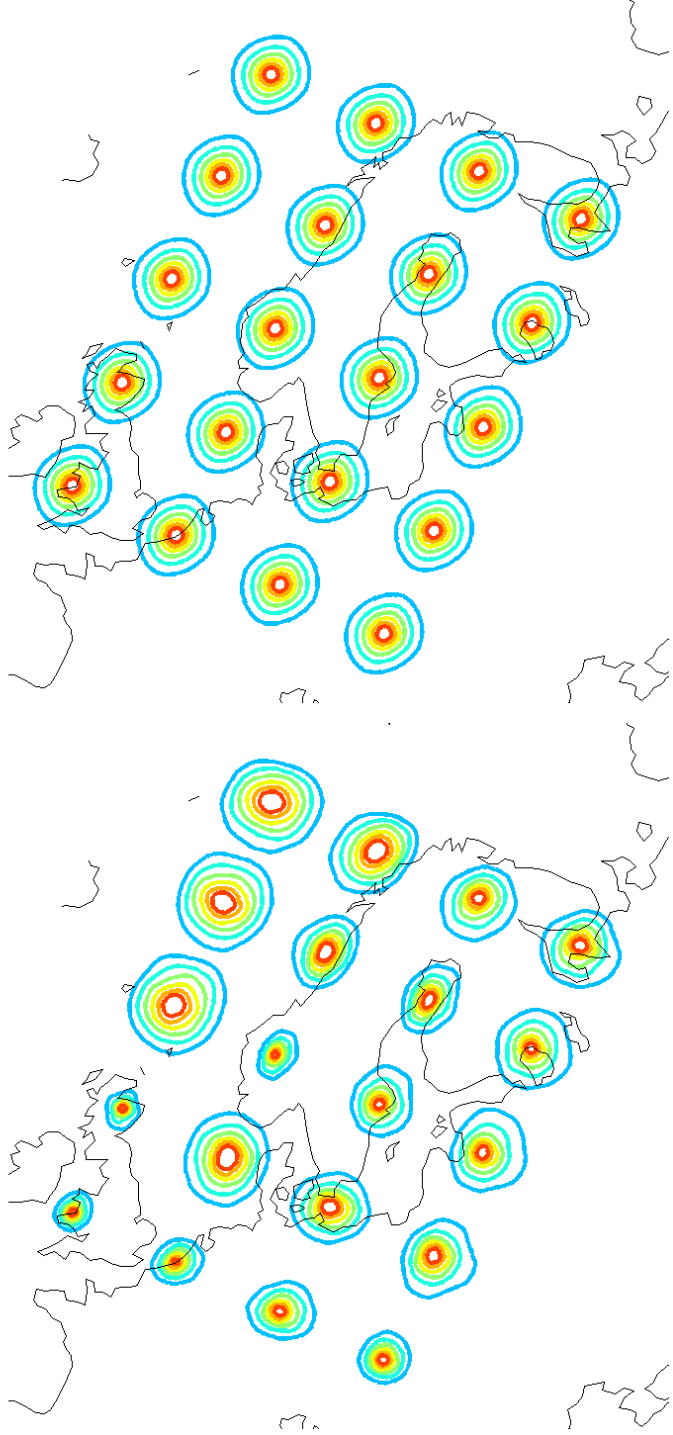
Need a global model for *B*

Variable substitution

$$J(x) = (x - x_b)^T B^{-1} (x - x_b) + J_o(x)$$
$$= / \; x - x_b = T^* u \;\; , \;\; T^* B^{-1} T = I \; /$$
$$= u^* u + J_o(T^* u + x_b)$$

Fourier/Wavelet transform

$$T^* = FD^{1/2} \Rightarrow B = FDF^*$$

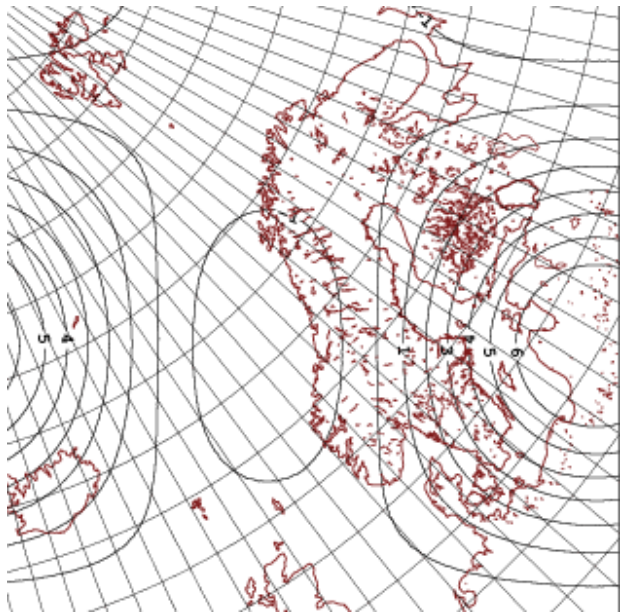


The Fourier transform

+ Fast implementation O(n log(n)).

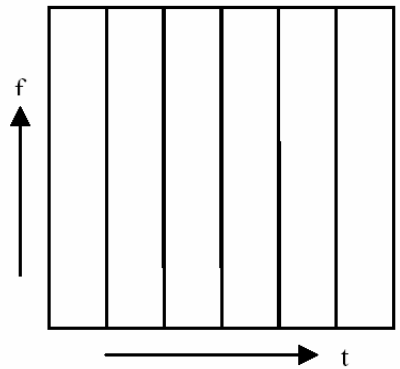
- Homogenous; same for all locations.

- Boundary problems; periodic.

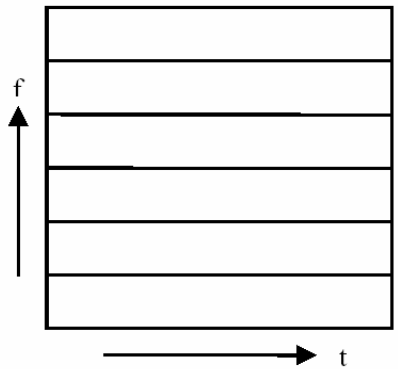


Tiling the time-frequency plane

Gridpoint



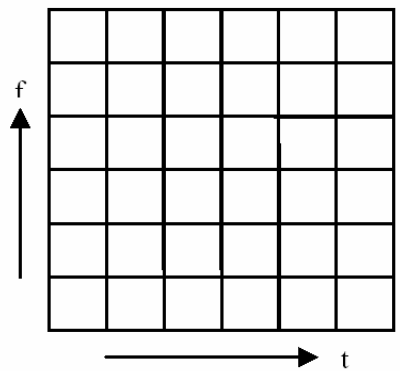
(a)



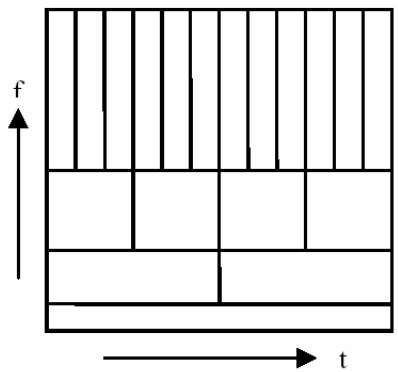
(b)

Fourier

STFT



(c)



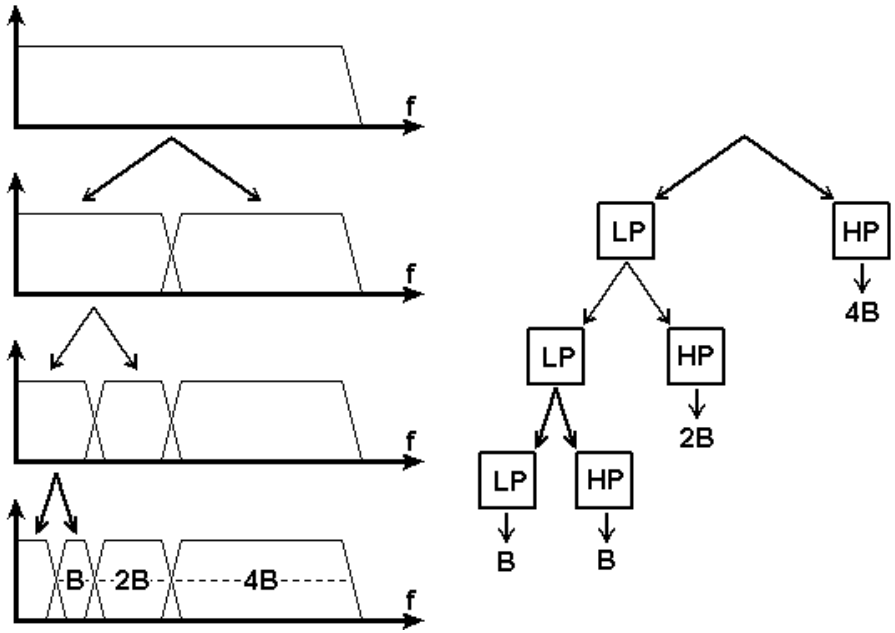
(d)

Wavelet

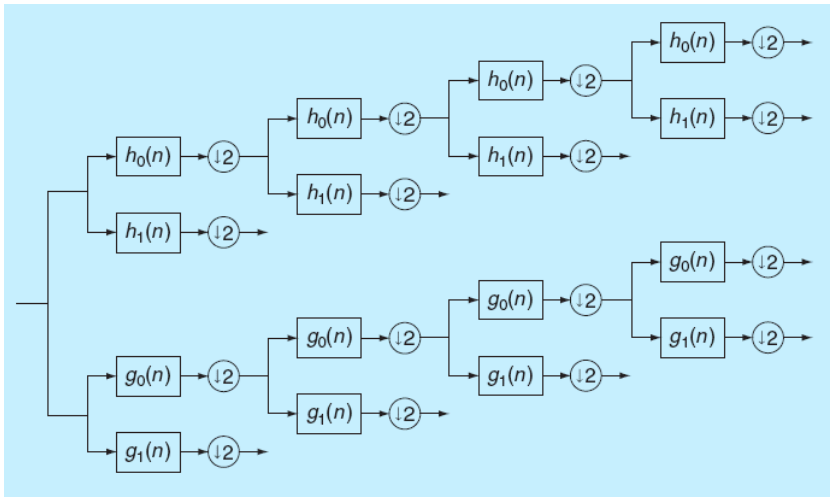
Discrete wavelet transform

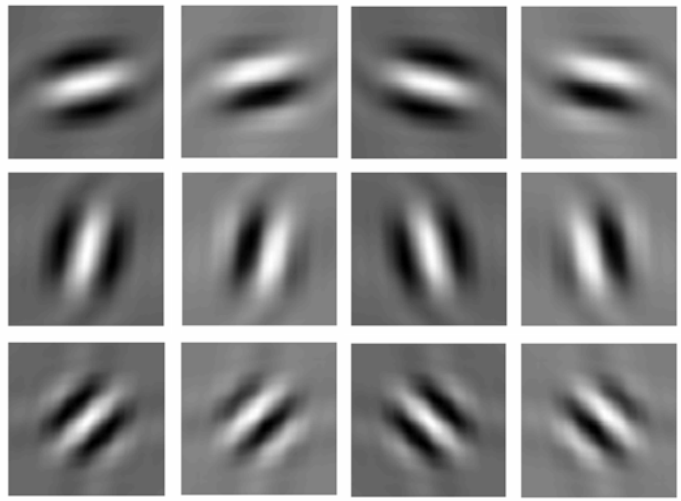
Some problems with the DWT

- **Oscillations near singularities.**
- **Shift variance.**
- **Sensitive to processing.**
- **Lack of directionality.**



Dual tree complex wavelet transform





$$\psi_1(x, y) = \phi(x)\,\psi(y) \qquad \text{(LH wavelet)},$$
$$\psi_2(x, y) = \psi(x)\,\phi(y) \qquad \text{(HL wavelet)},$$
$$\psi_3(x, y) = \psi(x)\,\psi(y) \qquad \text{(HH wavelet)}.$$

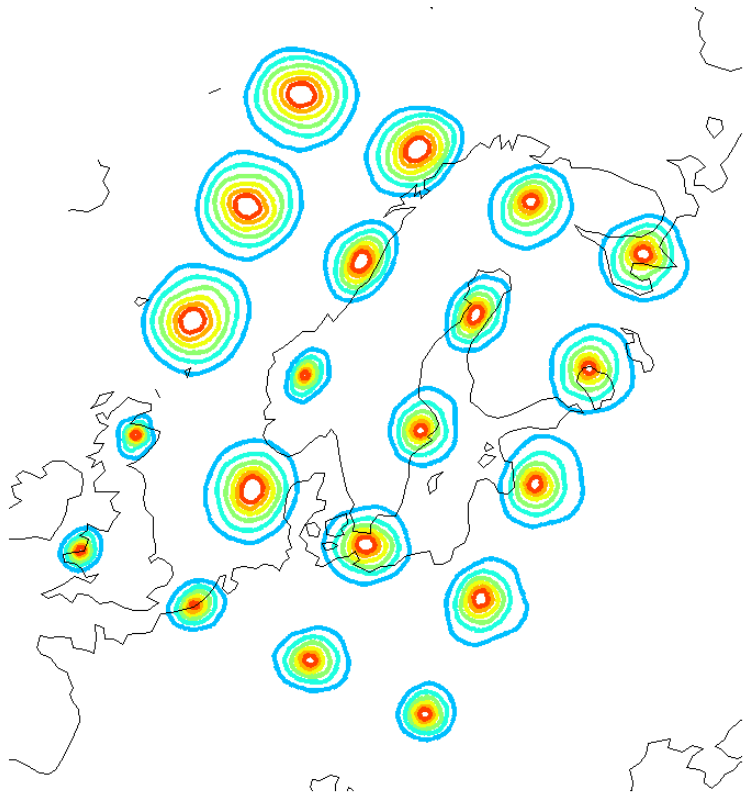
$$\mathrm{F}_{\mathrm{O2D}} = \frac{1}{\sqrt{8}} \begin{bmatrix} \mathrm{I} & -\mathrm{I} & & \\ \mathrm{I} & \mathrm{I} & & \\ & & \mathrm{I} & \mathrm{I} \\ & & \mathrm{I} & -\mathrm{I} \end{bmatrix} \begin{bmatrix} \mathrm{F}_{hh} \\ \mathrm{F}_{gg} \\ \mathrm{F}_{gh} \\ \mathrm{F}_{hg} \end{bmatrix}.$$

The wavelet transform

+ Very fast implementation O(n).

+ Some anisotropy and inhomogenity.

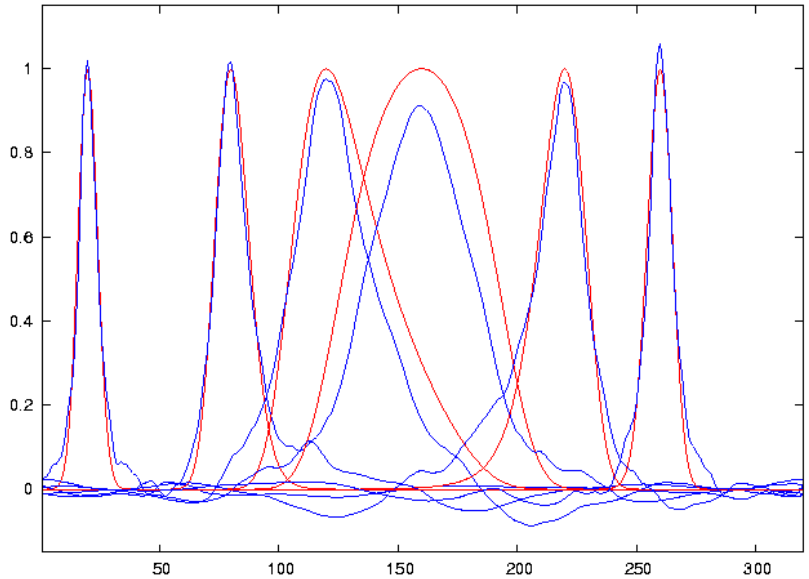
+ Boundary problem can be avoided.



Estimating *D*

Orthogonal T

$$d_i = E\left\{ t_i^* x x^T t_i \right\}$$

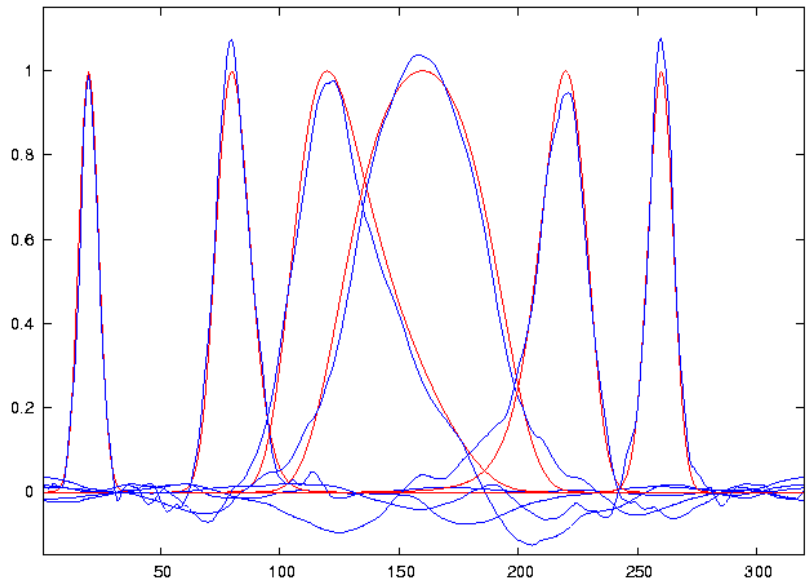


Non-orthogonal T

$$Ad = b$$

$$b_i = E\left\{ t_i^* x x^T t_i \right\}$$

$$A_{ij} = \left| t_i^* t_i \right|^2$$



SMHI

Work status and issues

ALADIN implementation

- Wavelet transform v0.1 (cy30t1_wavbec)

Software for estimation of *D*

- FESTATWAV assumes orthogonal *T*

- Treatment of vertical correlations

Design of boundary wavelets