A new source term in the parameterized TKE equation being of relevance for the stable boundary layer

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The circulation term

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Ref.: “Principals of a moist non local roughness layer extension of atmospheric turbulence closure”

M. Raschendorfer, 2006, DWD internal document, going to be published

Single column solution for turbulent flux densities:

1. Using closure assumptions:

→ 2-nd order budgets reduce to a 15X15 linear system of equations built of all second order moments of the variable set $\{\theta, q_v, u, v, w\}$

2. Using horizontal boundary layer approximation:

→ Flux gradient representation of the only relevant vertical flux densities:

turbulent master length scale

$$\boxed{\overline{\rho\phi''w''} \approx \overline{\rho}\,\overline{\phi'w'} \approx -\overline{\rho}K^{\phi}\partial_z\hat{\phi}}$$

$K^{\phi} := q\,\ell\,S^{\phi}$ turbulent diffusion coefficient

$\frac{1}{2}q^2$ TKE

stability function

The limit of the pure turbulent BL approximation (here of the ordinary TKE-equation) for stabile stratification:

$$D_t\left(\frac{1}{2}\overline{\rho}q^2\right) \approx -\overline{\rho u'' w''}\cdot\partial_z\overline{u} + \frac{g}{\overline{\theta}_v}\overline{\rho\theta_v'' w''} - \varepsilon$$

free atmospheric turbulence closure

$$\approx \underset{\substack{\downarrow \\ 0}}{\overline{\rho}K^M\left(\partial_z\overline{u}\right)^2} \underbrace{- \frac{g}{\overline{\theta}_v}\overline{\rho}K^H\partial_z\overline{\theta}_v}_{<0} - \varepsilon < 0$$

vanishing mean wind
stable thermal stratification

What processes can prevent TKE form decreasing to zero (what is missing)?

0. Prognostic extension
   - Sub grid scale transport

1. Moist extension
   - Sub grid scale condensation

2. Roughness layer extension
   - Canopy terms

3. Non turbulence scale (non local, multi scale) extension
   - Circulation terms

The moist extension - sub grid cloud processes:

- Inclusion of sub grid scale condensation:

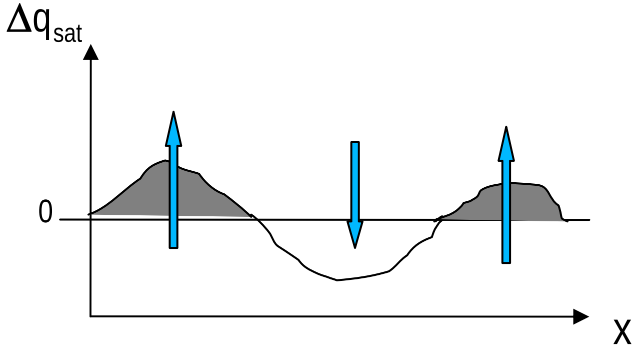
  -Using conservative variables with respect to condensation: $q_w = q_l + q_v \qquad \theta_l = \theta - \frac{L_c}{c_{p_d}} q_l$

  Correlations with condensation source terms are considered **implicitly** for non precipitating and not icing clouds.

- Solving for cloud water and cloud fraction by using the statistical condensation scheme (according to SD):

  -Normal distribution of **saturation deficiency** Δq_{sat}

  -Expressing variance of Δq_{sat} by variance of θ_l and q_w , both generated from the **turbulence scheme**

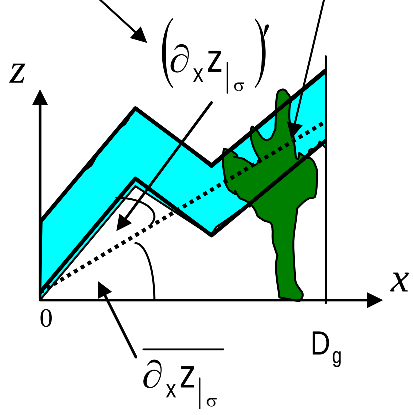


Mathematical reason for the canopy terms:

Sub grid scale slope of the model layers

or

Intersections of roughness elements



Averaging and differentiation in space (flux divergence) can no longer be commuted

Canopy terms in the averaged model equations

—

Wake turbulence production for TKE

The general budget of sub grid scale kinetic energy SKE:

$$D_t\left(\frac{1}{2}\overline{\rho}q^2\right) := \partial_t\left(\frac{1}{2}\overline{\rho}q^2\right) + \underbrace{\overline{\nabla}\cdot E^{v\cdot v} + \overbrace{E^{v\cdot v}\cdot\overline{\nabla}\,ln\,r^a}^{\text{grid volume reduction effect by intersections}}}_{\text{total flux convergence}} = Q^{v\cdot v}_{bod} \underbrace{\boxed{-\sum_{i=1,3}\left(\overline{\rho v_i''\tilde{v}''} + \overline{\tilde{e}^{v_i}}\right)\cdot\overline{\nabla}\hat{v}_i}}_{\text{total shear stress}} \underbrace{-\mu\sum_{i=1,3}\overline{\left|\nabla v_i\right|^2}}_{\text{molecular dissipation}} + \boxed{Q^{v\cdot v}_{prs}}$$

molecular friction

molecular strain on intersected bodies

≥ 0

total shear stress including effect of *sub grid scale surface slopes* ($\tilde{}$)

≥ 0

molecular dissipation

< 0

pressure source

pure atmospheric terms | roughness layer terms

$$Q_{prs}^{v\cdot v} = -\overline{v''\cdot\nabla p} = \underbrace{-\overline{\nabla}\cdot\overline{p'\tilde{v}''} \quad -\overline{p'\tilde{v}''}\,\overline{\nabla}\,ln\,r^a}_{\text{total pressure transport}} + \frac{1}{|G|}\hat{v}\int_{s\in B} p'\cdot n\,d^2s \rightarrow \geq 0$$

form drag wake and buoyant production due to intersections

$$0 \approx \leftarrow +\overline{p'\nabla\cdot v}$$

expansion work

$$+\overline{p'\left(\nabla_h z|_\sigma\right)'}\,\partial_z\hat{v}_h \rightarrow \geq 0$$

form drag wake production due to *sub grid scale slopes* of the grid box

$$\frac{g}{\hat{\theta}_v}\underbrace{\overline{\rho''w''\theta_v''}|_{turb}+\overline{\rho''w''\theta_v''}|_{circ}}_{\substack{\text{}}}\;,\quad \underbrace{-\bar{\rho}K\cdot\partial_z\bar{\theta}_v}_{}$$

$$\frac{g}{\hat{\theta}_v}\overline{\rho''w''\theta_v''} \approx -g\bar{\rho}\overline{w''} \approx \leftarrow -\overline{v''}\cdot\overline{\nabla}\,\bar{p}$$

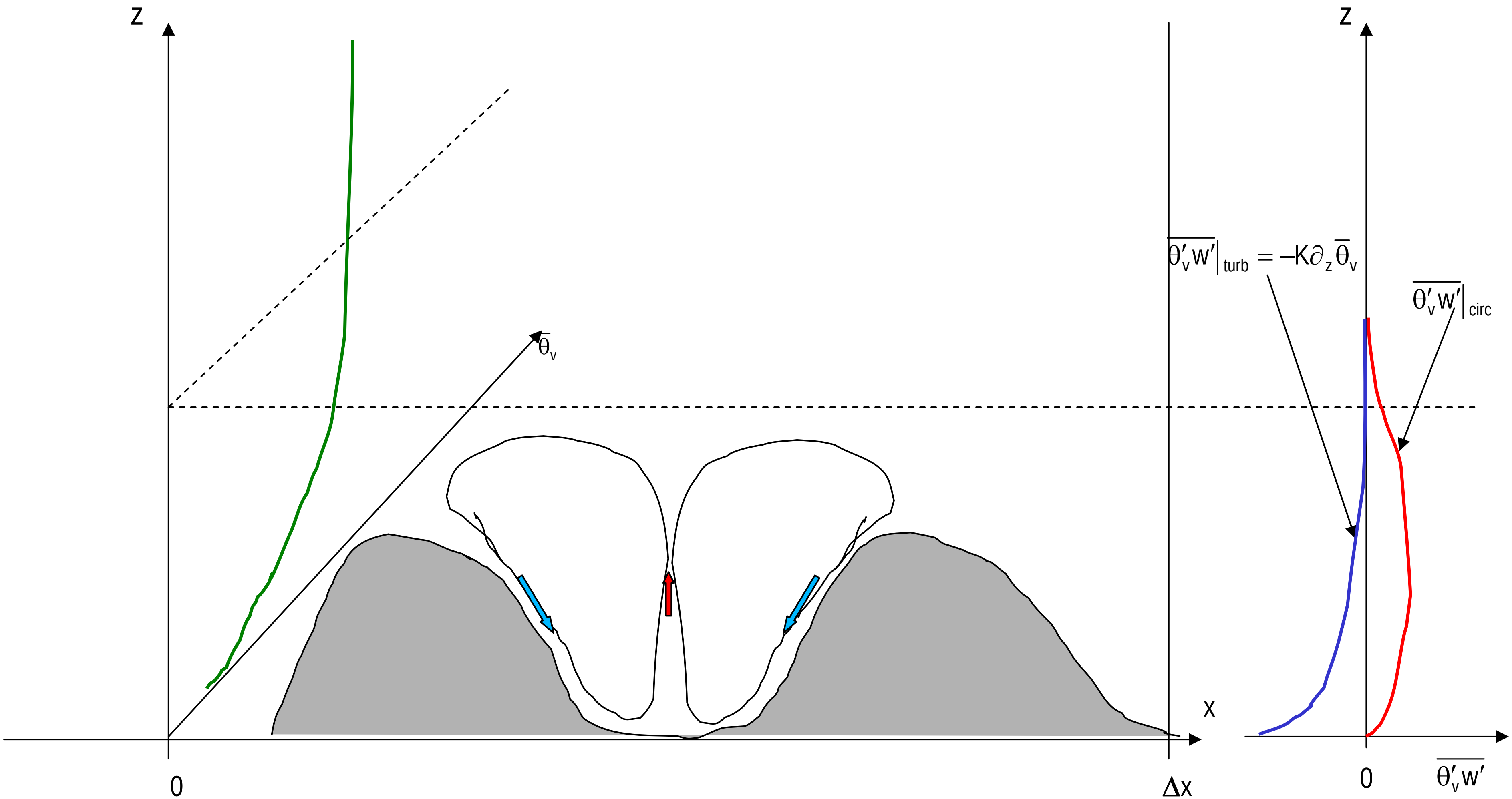
for stable stratification only the turbulent part $\propto -\partial_z\hat{\theta}_v$ is necessarily negative

buoyant production

in case of down hill density flows ≥ 0

$$+\overline{v_h''\cdot\left(\nabla_h z|_\sigma\right)'}\,\partial_z\bar{p}$$

buoyant production due to *sub grid scale slopes* of the grid box



- Even for vanishing mean wind and negative turbulent buoyancy there remains a positive definite source term

→ SKE will not vanish if there are thermal surface patterns

How to deal with the multi-scale problem?

• In general there are **non turbulent**, arranged circulation structures present as well:

-different **length scales** of sub grid structures

-turbulent scales

-circulation scales (convection, down hill density flows, body wakes)

→

More general closure assumptions **valid for both**, turbulence and circulations

Impossible without knowing moments of as much classes as we have length scales for

↓

Separation of turbulence and circulations with **adopted closure assumptions** for **each class** and related quite **easy particular solutions**

What are the postulates of turbulence:

- Equilibrium of the **source terms** in all second order budgets (except that for TKE):

→ - neglect of grid scale and **sub grid scale transport**

- **spectral density** of contributing modes follows a power law in terms of wave length in **each direction**:

inertial sub range spectrum

→ - whole spectrum in a given direction is determined by a single **peak wave length**

- the **peak wave length** is the same for samples in all directions: isotropic length scale

→ - pressure correlation and dissipation can be closed using a single turbulent master length scale for each location

Turbulence is that class of sub grid scale structures being in agreement with turbulence closure assumptions!

What is the remaining difficulty with circulations (beside sub grid icing and precipitation)?

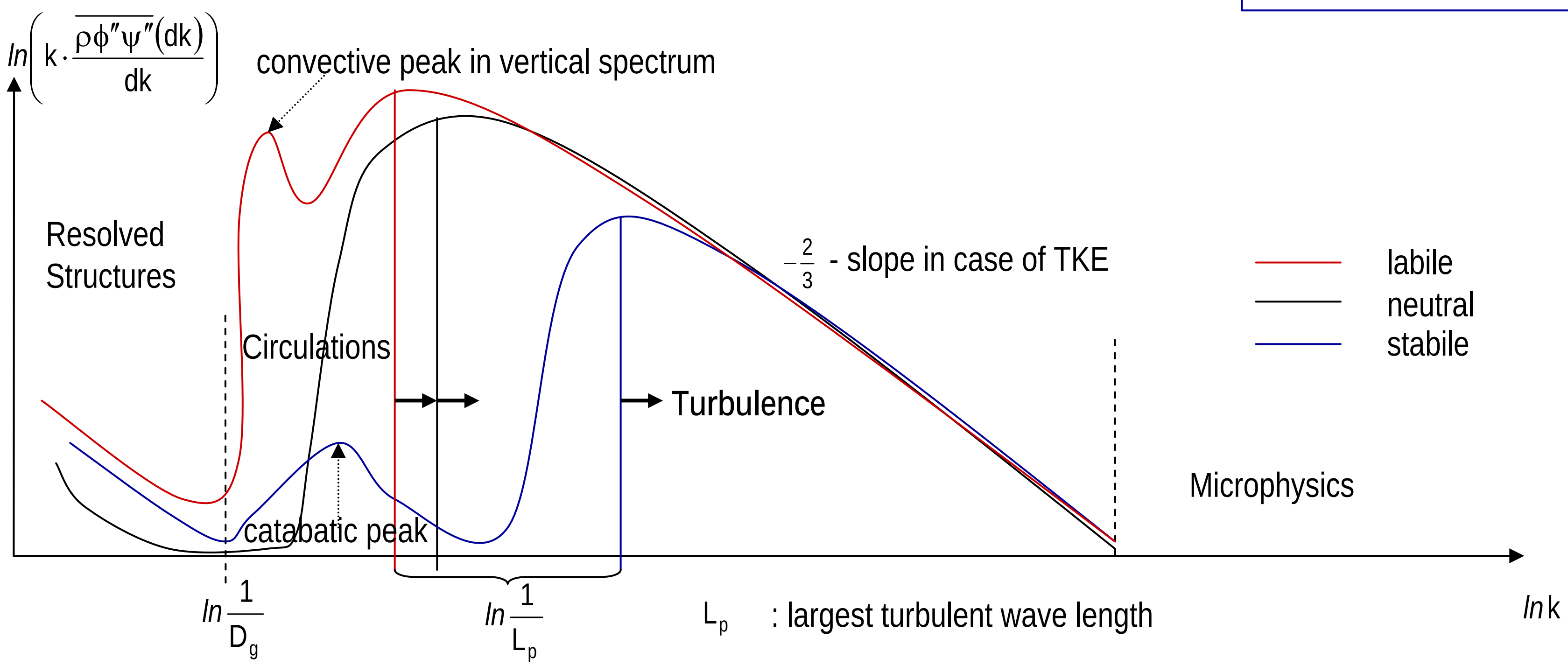
- they are related with at least one additional spectral peak

- or they cause different peak wavelengths in vertical direction compared to the horizontal directions:

anisotropic peak wave length

- **larger** peak wave length in **vertical** direction in case of **labile** stratification

at least a two scale problem



How to find a particular solution for turbulence?

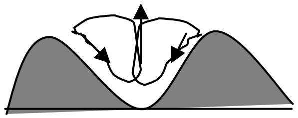
- turbulence closure is **only valid** for scales not larger **than**

  -the **smallest** peak wave length $L_{p,}$ in any direction

  -and the **largest** dimension D_g of the control volume

➡ Spectral separation by considering budgets with respect to the separation scale $L = min\{L_p, D_g\}$ and averaging these budgets along the whole control volume

-Additional scale interaction term in TKE equation due to non linear shear term



shear — turbulent buoyancy — dissipation

$$D_t\left(\frac{1}{2}\bar{\rho}\cdot\overline{q|_L}^{\,2}\right) \approx = -\bar{\rho}K^M\cdot\left[\left(\partial_z\hat{u}\right)^2 + \sum_{i=1,3}\overline{\left|\overline{\nabla}\hat{v}_i|_L{}'\right|^2}\right] + \frac{g}{\hat{\theta}_v}\overline{\rho\theta_v''w''|_L} - \bar{\rho}\frac{\overline{q|_L}^{\,3}}{\alpha^{MM}\ell}$$

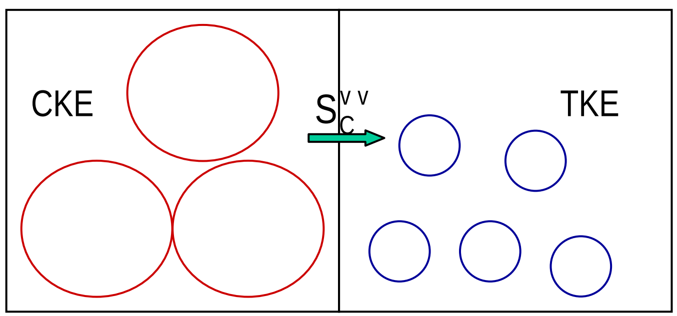
shear by circulation motions S_C^{vv}

Physical meaning of the circulation term:

Budgets for the **circulation** structures:

$$D_t\left(\overline{\rho\phi''\psi''}-\overline{\overline{\rho\phi''\psi''}\big|_L}\right)=\ldots+\underbrace{\left[\overline{\overline{\rho\phi''\mathbf{v}''}\big|_L{}'\cdot\left(\overline{\nabla}\hat{\psi}\right)\big|_L{}'}+\overline{\overline{\rho\psi''\mathbf{v}''}\big|_L{}'\cdot\left(\overline{\nabla}\hat{\phi}\right)\big|_L{}'}\right]}_{-S_C^{\phi\psi}}+\ldots$$

Circulation term is the **scale interaction term** shifting SKE or any other variance form the circulation part of the spectrum (CKE) to the turbulent part (TKE) **by virtue of shear** generated by the circulation flow patterns.



Parameterization of the buoyant circulation term for thermal driven (direct) circulations:

- a first attempt -

1. In all budgets:

- circulation structures are **negligible** during **neutral stratification**

→ Since local **shear production** of circulation motions acts also for neutral stratification, this should be **negligible**:

- a **vertical constant** circulation **time scale** for expressing **scale interaction** loss and **pressure** destruction
- a **mass flux approach** for circulation patterns

2. In the CKE budget:

- **scale interaction** loss = **buoyant** production

3. In the budget for circulation scale **heat and moisture flux** :

- **scale interaction** loss + **pressure** destruction = **buoyant production**

4. In the budget for circulation scale **temperature variance** :

- **scale interaction** loss = **vertical flux divergence** from the surface
- **flux gradient form** of temperature variance flux with a **vertical constant** circulation scale **diffusion coefficient**

Circulation term ~ circulation scale temperature variance ~ circulation scale buoyant heat flux

difference between updraft and downdraft

horizontal updraft fraction

$$\overline{\bar{\rho}|_L \hat{\theta}_V|_L{}''^2} \propto S_{CB}^{vv} \approx \frac{g}{\hat{\theta}_v} \cdot \overline{\bar{\rho}|_L \hat{w}|_L{}'' \hat{\theta}_V|_L{}''}(z) \approx \frac{g}{\hat{\theta}_v} \cdot \bar{\rho}\, w_C\, \Delta\hat{\theta}_v \cdot exp\left(-\frac{z}{H_C}\right) \geq 0$$

$$\approx -\partial_z \left[\bar{\rho} L_{pat}{}^3 \left| \frac{g}{\hat{\theta}_v} \partial_z \hat{\theta}_v \right|^{\frac{3}{2}} \right]$$

$$w_C = \gamma_C \cdot \frac{a \cdot (1-a)}{(1-2a)^{1/2}} \cdot \left(g \frac{\Delta\hat{\theta}_v}{\hat{\theta}_v} H_C \right)^{1/2}$$

circulation boundary layer height

$\leq$

pattern length

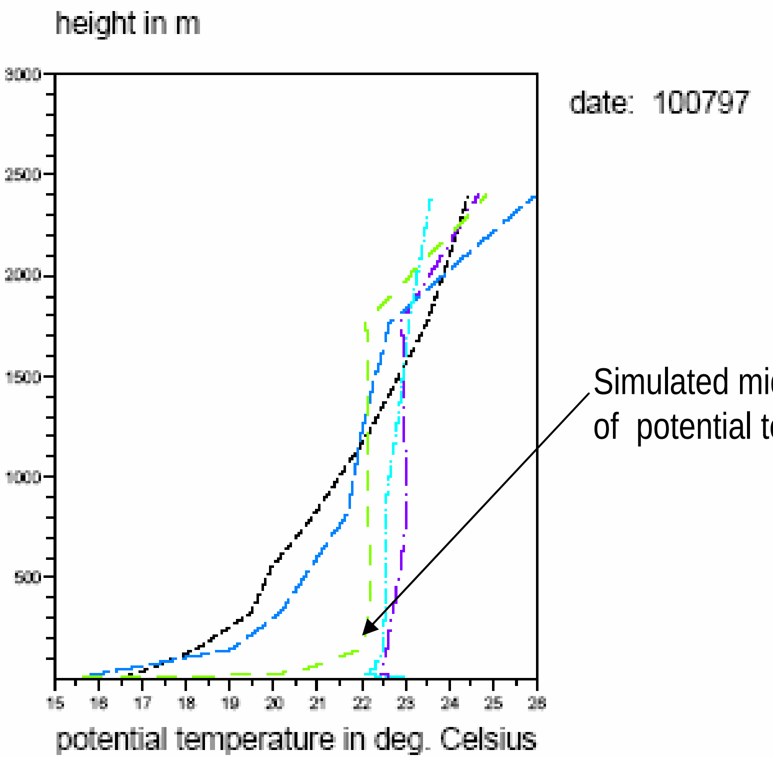
- **other circulation scale 2-nd order moments** have a similar representation

- turbulent and circulation scale flux densities **can be added**:
  - in the **first order budgets**
  - as input for the **statistical condensation scheme**

- in general a form drag circulation term S_{CD}^{vv} has to be considered as well:

$$S_C^{vv} = S_{CB}^{vv} + S_{CD}^{vv}$$

1-d simulations without the circulation term



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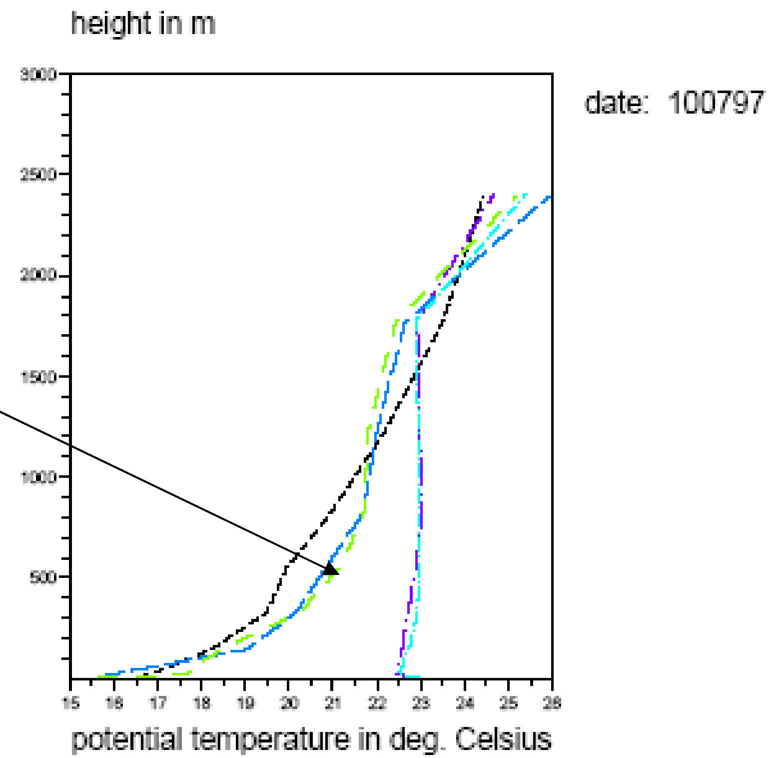
Simulated midnight profile of potential temperature

1-d simulations including the circulation term



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Conclusions:

1. Even for vanishing mean wind shear and stable thermal stratification there are source terms for TKE in general:

   • transport from other locations (by the mean wind, by turbulent transport e.g. from the surface layer)

   • sub grid scale condensation

   • wake turbulence production

   • shear production from circulation scale sub grid motions (circulation term)

     - here: direct thermal circulations, driven by differential heating or cooling

2. Volume averaging along a grid box with solid intersections or sub grid scale slopes along the earth surface leads to the wake turbulence terms:

   • non commutability of differentiation in space and averaging

3. Scale separation approach leads to a parameterization of the circulation term:

   • transport of circulation scale temperature variance from the surface
     ~ circulation scale upward heat flux
     ~ thermal production of CKE
     ~ shear production of TKE by circulation motions

Some remaining problems:

1. Inclusion of **deep convection** seems to be **impossible**

   • sub grid scale precipitation, - radiation, further different length scales

   ⇒ We do it with a **separate deep convection scheme**, unless this process is already grid scale

2. Closure of **circulation scale budgets** is not yet very **sophisticated**

   • Introduction of at least an equation for the **skewness in** order to solve for the **horizontal updraft fraction or** without an additional mass flux approach and:

   • A full set of 2-nd order equations

   • Solving of at least one **prognostic equation** (circulation scale **temperature variance**)

   • Proper parameterization of third order moments

3. Adaptation of the **statistical condensation scheme** to circulations

   •Other **distribution function of saturation deficiency** for circulation class

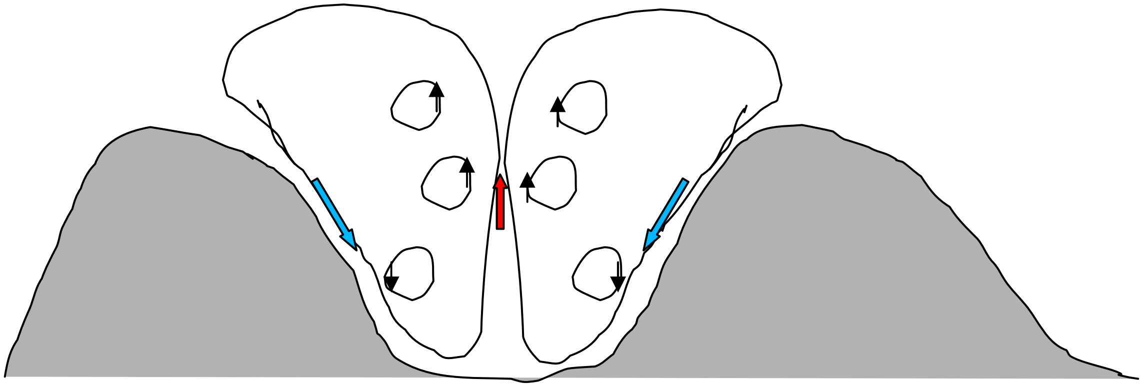
4. Adaptation of **deep convection** and **microphysical processes**

   •**Separation** of turbulent and circulation **condensation** from

   -Condensation due to deep convection:

   -Microphysical processes, **like cloud ice formation** and **precipitation**:

Thanks for your attention!



$$\hat{\zeta} := \frac{\overline{\rho\zeta}}{\overline{\rho}} \qquad \overline{\rho\zeta''} = 0 \qquad \tilde{v} := v - \left[v_h \cdot \left(\nabla_h z_{|_\sigma} \right)' \right] \vec{z} \qquad \overline{\tilde{e}^{\phi}} = -a^{\phi} \left[\overline{\nabla\phi} - \overline{\nabla_h \phi \cdot \left(\nabla_h z_{|_\sigma} \right)'} \right]$$

$$e_{kin} = \frac{1}{2} \overline{\rho |v''|^2} = \frac{1}{2} \overline{\rho}\, q^2 \qquad q := \sqrt{2 \frac{e_{kin}}{\overline{\rho}}} = \sqrt{\frac{1}{\overline{\rho}} \overline{\rho |v''|^2}} \qquad E^{v\cdot v} := \frac{1}{2} \sum_{i=1,3} E^{v_i v_i} = \frac{1}{2} \overline{\rho} q^2\, \hat{\tilde{v}} + \frac{1}{2} \overline{\rho |v''|^2\, \tilde{v}''} + \sum_{i=1,3} \overline{v_i'' \tilde{e}^{v_i}}$$

$$Q^{v\cdot v}_{prs} := \frac{1}{2} \sum_{i=1,3} Q^{v_i v_i}_{prs} = -\overline{v'' \cdot \nabla p} \qquad Q^{v\cdot v}_{bod} := \frac{1}{2} \sum_{i=1,3} Q^{v_i v_i}_{bod} = -\frac{1}{|G|} \int_{s\in B} \left(\sum_{i=1,3} v_i'' e^{v_i} \cdot n \right) d^2 s = \frac{1}{|G|} \sum_{i=1,3} \hat{v}_i \int_{s\in B} e^{v_i} \cdot n\, d^2 s$$

$$D_t \left(\frac{1}{2} \overline{\rho} q^2 \right) := \partial_t \left(\frac{1}{2} \overline{\rho} q^2 \right) + \overline{\nabla} \cdot E^{v\cdot v} + E^{v\cdot v} \cdot \overline{\nabla}\, ln\, r^a - Q^{v\cdot v}_{bod} = -\sum_{i=1,3} \left(\overline{\rho v_i'' \tilde{v}''} + \overline{\tilde{e}^{v_i}} \right) \cdot \overline{\nabla} \hat{v}_i - \mu \sum_{i=1,3} \overline{|\nabla v_i|^2} + Q^{v\cdot v}_{prs}$$
$$+ \sum_{i=1,3} \overline{v_i'' Q^{v_i}}$$